

FUNDAMENTAL STUDY ON INERTIAL SURVEYING

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Abstract:

Inertial surveying is recently developed as an application of inertial navigation system, which has been applied to military and civil fields. The fundamental principle of inertial surveying is based on simple mechanics. The purpose of the present study is to develop a surveying system, which consists of a gyroscope and an accelerometer along with a software program. In this paper, the authors performed leveling and traverse survey by using the characteristic experimental results of a vibration gyro, an accelerometer, and a device manufactured.

1. Introduction

Inertial survey has been obtained by applying an inertial navigation unit, and has advanced as topographical survey technique in recent years. An inertial navigation unit is installed onto a vehicle, and is used as a sensor for navigation. This system measures a speed and angular acceleration during movement. Accordingly, unlike the conventional optical method, it is capable of comparatively freely selecting a necessary point without being influenced by weather and visibility.

In this paper, the authors performed leveling and traverse survey by using the characteristic experimental results of a vibration gyro, an accelerometer, and a device manufactured.

For the application to photographic surveying, since information on the position and attitude of a camera is obtained, the photographic surveying without using any reference points becomes possible, thus probably increasing the photographic surveying efficiency rapidly.

2. Vibration Gyro

2.1 Experiment of vibration gyro

In this measurement, a piezoelectric vibration gyro model ENV-05S manufactured by Murata Mfg. Co., Ltd. was used. Unlike a conventional method in

which the angular momentum of a body of rotation of a rate gyro is used, it is said to have long service life, short starting time, and good follow-up performance, and be resistant to a dynamic environment. For the characteristic experiment, a turn table ($0.01-720^\circ/\text{sec}$) was used. For the turn table, a rate of turn table model C-181 manufactured by Genisco Inc. was used. Fig. 1 shows a graph of the output characteristics of a vibration gyro to the rotational angular speed. For the output voltage of the vibration gyro, the output scale factor to the angular speed at any position on the turn table, becomes $22.77\text{mV}/\text{deg}/\text{sec}$ from Fig. 1. The normal linearity of the vibration gyro was confirmed.

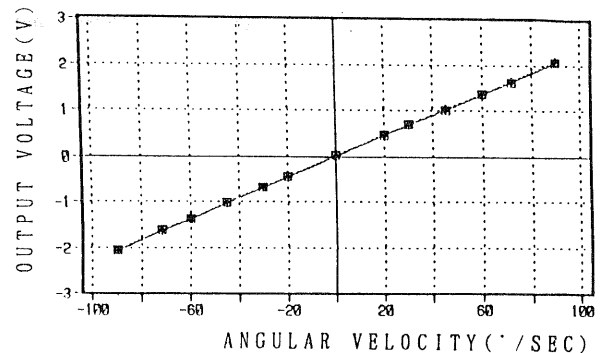


Fig. 1 Relation between the output voltage and vibration gyro

Next, Fig. 2 shows a graph of the DC drift for the vibration gyro. In this experiment, the output of the vibration gyro was measured every ten minutes for 120 minutes in a stationary state. For the change in the output due to elapsed time, 0 to about 20mV was observed, and it was confirmed that it is within the range of the DC drift specified in the performance data in the vibration gyro used.

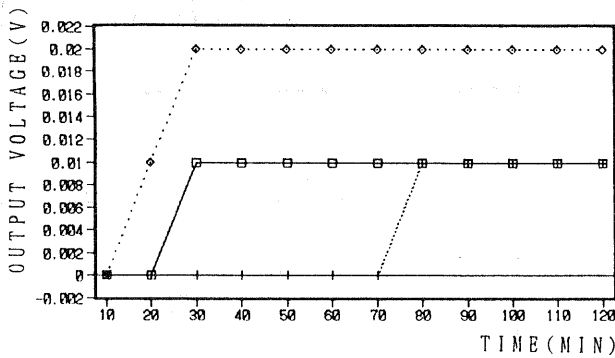


Fig. 2 DC drift

Fig. 3 shows a graph for measurement result for the temperature drift. This experiment was performed at temperatures within a range of 0 degree to about 35 degrees assuming the temperatures which can be usually considered. First, ice and water were put into expanded styrene to make the water temperature constant at 0 degree, and then a vibration gyro with constant output was wrapped in vinyl and was put into the ice and water to measure, after a while, the surface temperature of the vibration gyro. For hot water, the surface temperature of the vibration gyro in the hot water was measured similarly.

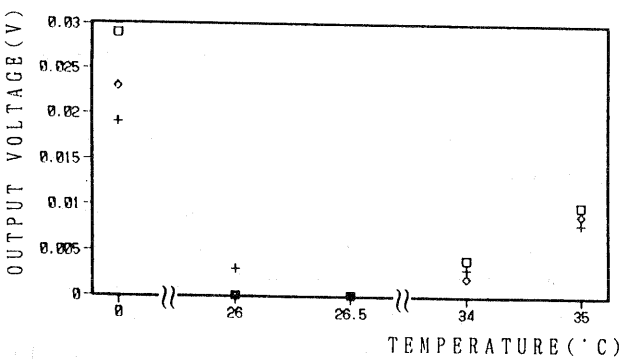


Fig. 3 Temperature drift

When a vibration gyro was put in the ice and water, the output rose 0.019V to 0.029V (measured three times) as compared with the output (2.572V) at normal temperature (26.0°C), and the rate of change was 0.73% to 1.13% (0.019V/2.572V to 0.019V/2.572V). Also when it was put into hot water, the output rose 0.004V to 0.010V as compared with the output (2.518V) at

normal temperature, the rate of change being 0.15% to 0.39% (0.004V/2.518V to 0.010V/2.518V). Either temperature had error of about 1%.

From the foregoing experimental result, the DC drift for the vibration gyro changes 10mV at maximum per ten minutes from Fig. 2, but it comes to about 13' when converted into angle. In this research, therefore, the experiment was performed so that the measurement is started in five minutes after the switch is finished within about 1 to 2 minutes, and the observed value is not greatly affected by the DC drift as an error.

Also, the temperature drift changes 0.005V at maximum within a range of about $\pm 8^\circ\text{C}$ from the temperature during the measurement. This voltage variation produces an error of about 40". In the experiment of this research, it was performed where there is no temperature change more than $\pm 8^\circ\text{C}$ during measurement so that it is not affected by the temperature drift as much as possible. In this respect, it is necessary to take into consideration the effect by these drifts during measurement for a long period of time.

Incidentally, since this system uses three gyros altogether, graphs for the respective gyros are shown.

2.2 Performance of measuring angular speed

In the experiment, a vibration gyro was placed on a turn table, and the output voltage when the rotational speed was changed to 90° was measured. The graph in Fig. 4 shows the relation between the elapsed time required to rotate 90° and angle. From this figure, the converted angles when the 90° rotation was performed at each rotational speed are $90^\circ 29' 27''$ at $9^\circ/\text{sec}$, $90^\circ 4' 57''$ at $16^\circ/\text{sec}$, and $89^\circ 29' 12''$ at $40^\circ/\text{sec}$ as shown by mark \bullet in the figure, and the error at 90° was about $\pm 1\%$.

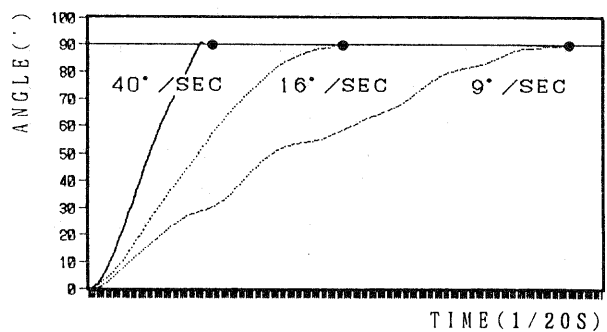


Fig. 4 Relation between elapsed times and turned angle

3. Accelerometer

3.1 Experiment for characteristics of accelerometer

For the accelerometer, a servo-type accelerometer JA-VC1 manufactured by Japan Aviation Electronics Industry, LTD was used.

The features of the accelerometer are to be capable of measuring microacceleration with good precision, free from mechanical friction, wear looseness, and stable, small-size and light-weight.

In order to grasp the characteristics of an accelerometer, an accelerometer was placed on a horizontal pedestal, and it was manually moved 50cm in the detection shaft direction of the accelerometer. The graph in Fig.5 shows the acceleration, speed and change in distance from the beginning of the movement till the stop of the accelerometer.

The graph for acceleration in Fig.5(A) shows the change in acceleration from the beginning of the movement till the stop of the accelerometer. The voltage becomes (+) during acceleration, while it becomes (-) during deceleration. It is because the accelerometer has been manually moved that the shape of the graph changes irregularly.

The graph for speed in Fig.5(B) represents change in speed from the beginning of the movement till the stop of the accelerometer by a value obtained by integrating the voltage of the acceleration. It can be seen that the speed gradually increases on the contrary in the second half. This corresponds to a state when the accelerometer was moved.

Fig.5(C) represents a distance obtained by integrating a speed. This graph shows a distance along which it has moved from the beginning of the movement, and the portion having a constant voltage value in the second half of the graph has a voltage (195.2V) corresponding to the moving distance of 50 cm.

This characteristic experiment was performed in order to make sure that a voltage corresponding to the distance when it was stopped by changing the acceleration on the way after an accelerometer was moved becomes constant. The graph shows that the value obtained by double-integrating the acceleration data into a distance is constant where the movement was stopped.

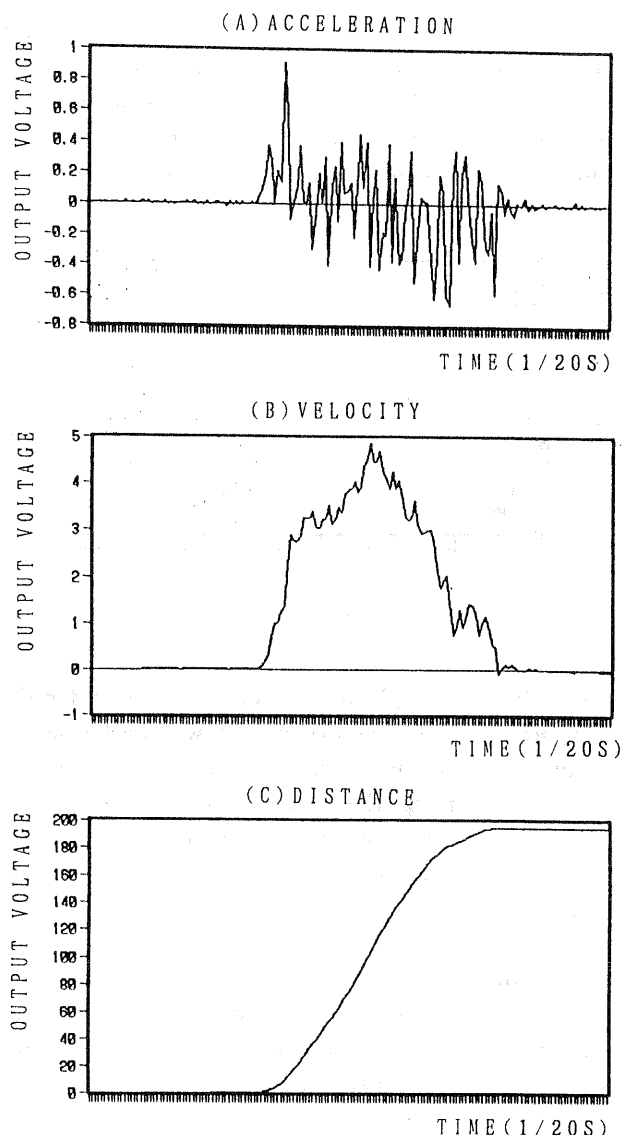


Fig. 5 Acceleration, Velocity, Distance

3.2 Distance measuring experiment in the plane direction

In order to practically calculate the moving distance in the plane direction, survey points were provided at intervals of about 10m, 20m and 30m with ethron tape on a horizontal corridor on the 7th floor of large building No.4 of Chiba Institute of Technology, and the carriage was moved in the direction of detection shaft (X-axis direction in this experiment) of the accelerometer to allow only the accelerometer in the X-axis direction to output.

In the experiment, the carriage was assumed to be horizontal all the time, an angle of rotation (κ) in the horizontal direction was calculated by using a vibration gyro installed to the Z-axis, and coordinate

transformation was performed by using an expression of $X = (\text{acceleration value of X-axis}) \times \cos \kappa$ to calculate the plane coordinates. The accelerometer and vibration gyro were provided on the carriage, a plumb bob was attached at the rear of the carriage for centering.

The measuring time was about 30 seconds for 10m, about 50seconds for 20m, and about 1 minute for 30m. The error for the moving distance of 10m was 1.32% (precision 1/77), the error for the moving distance of 20m was 1.75% (precision 1/57), the error for the moving distance of 30m was 2.56% (precision 1/40).

3.3 Distance measuring experiment in the height direction

Next, in order to calculate a height, a leveling rod was vertically installed, and an accelerometer was moved along the leveling rod in the vertical direction. An accelerometer installed on a block was mounted on a pedestal which had been placed to be horizontal along the leveling rod with a level, and was manually moved up to a predetermined height along the graduation on the leveling rod. The measuring time was about 10 seconds for 50cm, about 10 seconds for 100cm, and about 15 seconds for 150cm. The error for 50cm was 0.60% (1/170), that for 100cm was 1.28% (1/80), and that for 150cm was 1.53% (1/70).

The accelerometer for use with this research always outputs the gravitational acceleration, and when the detection axis direction of the accelerometer is in the vertical direction, a voltage of about 5V is outputted.

According to the experiment, it could be confirmed that when an accelerometer, which always detects gravitational acceleration, is used, and has been moved in the vertical direction, the height along which it has been moved can be calculated. It is smaller than the above-described error when it has been moved in the horizontal direction. As the causes therefor, the following may be considered: ① there were few data owing to the short measuring time, and the accumulated error for the integrated values was small; ② there was less vibration (noise), etc. in the vertical direction during the movement; and ③ no coordinate transformation was performed because it was assumed that it was moved in the gravitational direction.

4. Application to Survey

4.1 Leveling experiment and consideration

In order to perform an experiment for calculating a height and a moving distance by using an inertial device, a wheel-barrow made of aluminium mounted with an inertial device was moved.

For the experimental method, a difference in level of 0.469m in a longitudinal level difference was provided

on a horizontal distance of 4.765m shown in Fig. 6, and the wheel-barrow mounted with an inertial device was moved to compare with the leveling using a level. The accelerometer in the X-axis direction, which is the traveling direction, was turned toward the next survey point, and was moved linearly between the survey points maintaining it horizontal with a level mounted on the wheel-barrow. As regards the survey points, a plumb bob was attached on one side of the tire of the wheel-barrow for centering.

The accelerometers and the vibration gyros installed to the three axes (See Fig. 7).

The data used for the analysis are X-axis accelerometer (V_x), X-axis vibration gyro (ϕ), Y-axis accelerometer (V_y), Y-axis vibration gyro (ω), Z-axis accelerometer (V_z), Z-axis vibration gyro (κ).

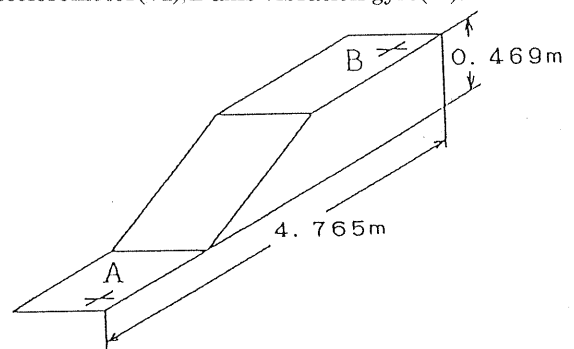


Fig. 6 Experiment place

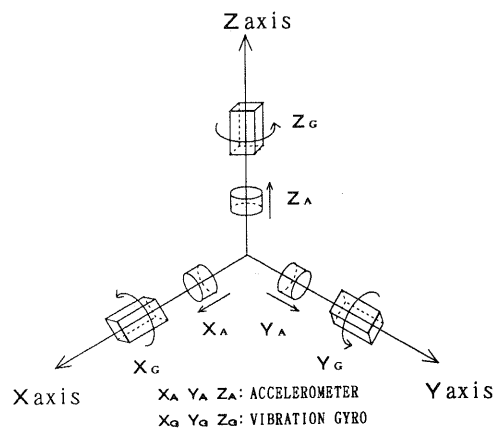


Fig. 7 Three axes and sensors

When rotary movement was performed by κ clockwise with respect to the positive direction of Z-axis in an absolute coordinate system XYZ before movement, next similarly by ϕ around the Y-axis, and further by ω around the X-axis in order, it is assumed that it can be converted to an inclined device coordinate system after the movement.

For simplification, as regards X-coordinate value, Y-coordinate value, and Z-coordinate value in a device coordinate system which has inclined when the rotations by κ , ϕ , and ω were rotated independently

and respectively, the values which those coordinate values should have in an absolute coordinate system which is not inclined will be determined.

Since the rotation amount of κ has often a greater value than ϕ and ω , it is first corrected as a third axis. ϕ and ω are comparatively small values, and are taken on the second and first axes. Then, rotational transformation is performed in the order of the third, second and first axis, and an inclination composed by the inclination of the third axis is provided.

The composed inclination is represented by a product of rotation matrixes of three axis, and a rotation matrix consisting of the following 3×3 elements can be obtained.

$$R = R_{\omega} R_{\phi} R_{\kappa} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

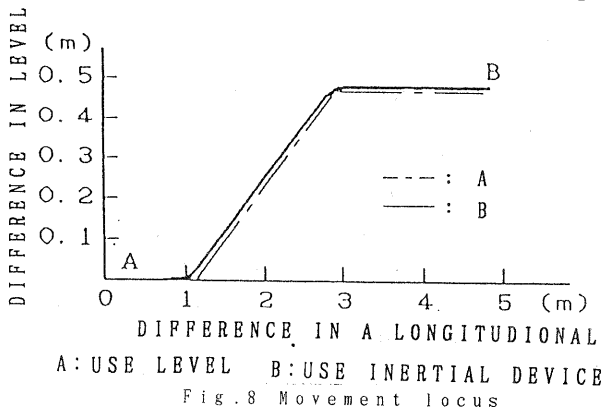
$$\begin{aligned} a_{11} &= \cos \phi \cos \kappa, & a_{12} &= -\cos \phi \sin \kappa \\ a_{13} &= \sin \phi, & a_{21} &= \cos \omega \sin \kappa + \sin \omega \sin \phi \cos \kappa \\ a_{22} &= \cos \omega \cos \kappa - \sin \omega \sin \phi \sin \kappa \\ a_{23} &= -\sin \omega \cos \kappa, & a_{31} &= \sin \omega \sin \kappa - \cos \omega \sin \phi \cos \kappa \\ a_{32} &= \sin \omega \cos \kappa + \cos \omega \sin \phi \sin \kappa, & a_{33} &= \cos \omega \cos \phi \end{aligned}$$

After all, as regards the coordinate values for each axis of a device coordinate system which has inclined by the rotation around these three axes by rotation matrix, the coordinate values which those coordinate values should take in an absolute coordinate system which is not inclined can be obtained in the following manner:

$$\begin{pmatrix} X_B \\ Y_B \\ Z_B + g \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Fig.8 shows the comparison between a locus along which a wheel-barrow has moved. The average error with respect to the level was 7.7mm.

From this experiment, the distances in the horizontal and vertical directions could be calculated at the same time by an inertial device even when moved on a slope.



4.2 Experiment and consideration for traversing

From the experiment in the research so far, it has been found out that the measurement within a narrow range can be performed even by the present system, and therefore, a wide range to a certain degree was surveyed for traversing.

In the experiment, four traverse points were provided on comparatively flat asphalt pavement within Chiba Institute of Technology so that a quadrilateral of about $60\text{m} \times \text{about } 40\text{m}$ is formed, and the rear wheel of a wheel-barrow made of aluminum was provided with an auxiliary wheel so that the inertial device was placed almost horizontal. In this respect, assuming that these four traverse points a plane in in this this experiment. The measurement between each traverse point was performed within a range of about 40 seconds to 1 minute.

In addition, since the distance to be surveyed is long, a battery power supply was mounted on a carriage, which was moved to run side by side with the wheel-barrow during measurement. The accelerometers installed to the three axes were provided on the front of the wheel-barrow. For the installation method, two ways were adopted: when the X-axis and the Y-axis are inclined substantially 45° respectively. A plumb bob was attached below the sensor. For the survey points, a plumb bob was attached to one side of the tire of the wheel-barrow for determining the vertical direction, but it is difficult to accurately center in the construction of the carriage, and the error in centering is considered to be about 1 to 2 cm per point.

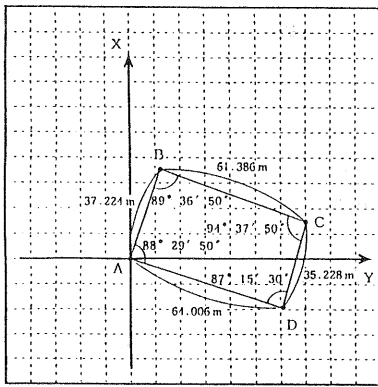
Fig.9 shows traverse points. In the survey using a transit and a steel tape, the ratio of closure is $1/25,000$. The ratios of closure in the respective experiments were obtained by using the following expressions:

$$\text{Ratio of closure} = \frac{\text{Difference in closure (m)}}{\text{Full length of course of traverse calculated (m)}}$$

$$\text{Difference in closure (m)} = AA'$$

Assuming differences in X and Y between A' and A (origin) when A' was returned to the absolute coordinate system before the movement by the rotation of coordinate to be X1 and Y1 respectively,

$$AA' = \sqrt{X1^2 + Y1^2}$$



● : TRAVERSE POINT

Fig. 9 Traverse points

(1) Experiment No.1 (movement in X-axis direction)

movement by a wheel-barrow

-X,Y-axes accelerometers,and Z-axis gyro for use,no vibration isolation-

For the calculation expression,the following expression was used.

• κ rotation(around Z-axis)

$$\begin{bmatrix} X_B \\ Y_B \end{bmatrix} = \begin{bmatrix} \cos \kappa & -\sin \kappa \\ \sin \kappa & \cos \kappa \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

The ratio of closure was 1/5.

From these data,it has turned out that the value of the Y-axis accelerometer is outputted although it was moved in the X-axis direction.As the reason,it is thinkable that the Y-axis accelerometer,in which the vibration during the measurement,or the change in acceleration is hardly considered to exist,includes an error due to resolution in the A/D conversion or that there is vibration from the ground.

(2) Experiment No.2 (movement in X-axis direction)

movement by a wheel-barrow

-X-axis accelerometers,and Z-axis gyro for use,no vibration isolation-

This result has been obtained when the analysis was performed with a X-axis accelerometer and Z vibration gyro using the same data as that used in the Experiment No.1,and without using the value of the Y-axis accelerometer which ought to have hardly changed.For the calculation expression,the following expression was used.

• κ rotation(around Z-axis)

$$\begin{cases} X_B = X \cdot \cos \kappa \\ Y_B = Y \cdot \sin \kappa \end{cases}$$

From this experiment,it can be seen that the ratio of closure is increased by the influence of the integral error of the Y-axis accelerometer on the ratio of

closure.

(3) Experiment No.3 (movement with X and Y-axes inclined 45° respectively) movement by a wheel-barrow

-X-axis accelerometer Y-axis accelerometer and Z-axis gyro for use,no vibration isolation-

A X-axis accelerometer and a Y-axis accelerometer were inclined at an angle of about 45° toward the traveling direction respectively for the experiment.For the calculation expression,the following expression was used.

• κ rotation(around Z-axis)

Expressing by using rotation matrix of 2×2

$$\begin{bmatrix} X_B \\ Y_B \end{bmatrix} = \begin{bmatrix} \cos \kappa & -\sin \kappa \\ \sin \kappa & \cos \kappa \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

In this method,the accelerometers for two X and Y-axes output the voltage value to share it in the traveling direction.In this case,the ratio of closure was 1/80 .

When the measurement is performed with two X and Y-axes in this way,the percentage of the error in the measured acceleration voltage value reduces and the ratio of closure increases.

Next,since the vibration received from the ground during measurement was considerably great,the vibration was insulated for the experiment.In the experiment,an inertial device was provided on an automobile (station wagon),and the other battery,personal computer,etc. ware provided at the rear of the automobile.Also,a plumb bob was attached to the rear bumper of the automobile about 20cm behind the inertial sensor and a circle of about 10cm in radius was drawn at the survey point so that the plumb bob comes to the center of the circle.

(4) Experiment No.4 (movement in X-axis direction) movement by an automobile.

-X-axis accelerometer,and Z-axis gyro for use,with vibration isolation-

When the vibration was isolated,and it was moved in the X-axis direction,the experiment was performed using the X-axis accelerometer and Z-axis vibration gyro.For the calculation expression,the following expression was used.

• κ rotation(around Z-axis)

$$\begin{cases} X_B = X \cdot \cos \kappa \\ Y_B = Y \cdot \sin \kappa \end{cases}$$

The ratio of closure was 1/30.

From this experiment,the ratio of closure was increased as compared with the experiment performed under the same conditions,and the vibration-proof effect was seen.

(5)Experiment No.5 (movement with X and Y-axes inclined 45°) movement by an automobile -X-axis accelerometerY-axis accelerometer and Z-axis gyro for use,with vibration isolation-

The vibration was insulated,and a X-axis accelerometer and a Y-axis accelerometr were provided to incline at an angle of about 45° toward the traveling direction respectively for the moving experiment.

The following calculation expression was used for calculation.

• κ rotation(around Z-axis)

Expressing by using rotation matrix of 2×2

$$\begin{bmatrix} X_B \\ Y_B \end{bmatrix} = \begin{bmatrix} \cos \kappa & -\sin \kappa \\ \sin \kappa & \cos \kappa \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

The ratio of closure was 1/100.

The increase in the ratio of closure is the same tendency as when the vibration is not insulated.Table1 shows the respective experimental results.

Table.1 TRAVERSING BY INERTIAL DEVICE

MOVEMENT DIRECTION	USED DATA	VIBRATION ISOLATION	RATIO OF CLOSURE
X	X ACC, Y ACC, Z GYA	x	1/5
X	X ACC, Z GYA	x	1/20
X	X ACC, Z GYA	○	1/30
X, Y (45°)	X ACC, Y ACC, Z GYA	x	1/80
X, Y (45°)	X ACC, Y ACC, Z GYA	○	1/100

ACC:ACCELEROMETER
GYA:VIBRATION GYRO

From the experimental result, as a method of moving in a certain direction on a plan,accelerometers were installed in an oblique direction so that the acceleration during movement can be measured by accelerometers on two axes(X and Y) for moving.Thus,the influence of the A/D conversion on the resolution becomes relatively less.

As regards the influence due to vibration during measurement,there was shown a tendency to increase the ratio of closure by vibration insulation.Therefore,it is necessary to consider a method to soften the minute vibration,etc. by using damping material for the sensor portion of an inertial device so that the vibration does not affect the measured value.

5. Conclusion

This research has, as the basic research on the inertial survey, studied the principle of the inertial survey, the

characteristics of the sensor for use, and the calculation of an angle and a distance by actually operating a sensor by using an A/D conversion board of 12 bits. As the result of the experiment, the basic data for developing the research could be collected such as enabling continuous measurement to be performed by use of the inertial device although the error is too great to be used for survey because the resolution of the A/D conversion was 12 bits.

From the experiment, the following items can be said to be effective for reducing the error on using an A/D conversion board of 12bits:

- ① The movement should be made as smoothly as possible,and any abrupt start and stop should be avoided.
- ② For the measuring place, any exceedingly rolling places and avoided.
- ③ The measurement should not be performed only with one axis in the traveling direction, but accelerometers should be installed to detect the acceleration with two axes. (In the two-dimensional measurement, the X- and Y-axes accelerators are inclined.)
- ④ For the sensor portion in the inertial device, damping material is used.
- ⑤ In the measurement at the same time, the number of times for sampling should be increased as many as possible.

For the application to photographic surveying, since information on the position and attitude of a camera is obtained, the position and attitude of a camera is obtained, the photographic surveying without using any reference points becomes possible, thus probably increasing the photographic surveying efficiency rapidly.