Linear feature extraction with LSB-Snakes from multiple images

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ABSTRACT

In general, the snakes or active contour models feature extraction algorithm integrates both photometric and geometric constraints, with an initial estimate of the location of the feature of interest, by an integral measure referred to as the total energy of snakes. The local minimum of this energy defines the feature of interest. To improve the stability and convergence of the solution of snakes, we propose a new implementation based on parametric B-spline approximation. Furthermore, the energies and solutions are formulated in a least squares approach and extended to integrate multiple images in a fully 3-D mode. This novel concept of LSB-Snakes (Least Squares B-spline Snakes) improves considerably active contour models by using three new elements: (i) the exploitation of any a priori known geometric (e. g. splines for a smooth curve) and photometric information to constrain the solution, (ii) the simultaneous use of any number of images through the integration of camera models and (iii) the solid background of least squares estimation. The mathematical model of LSB-Snakes is formulated in terms of a combined least squares adjustment. The observation equations consist of the equations formulating the matching of a generic object model with image data, and those that express the geometric constraints and the location of operator-given seed points. By connecting image and object space through the camera models, any number of images can be simultaneously accommodated. Compared to the classical two-image approach this multi-image mode allows to control blunders, like occlusions, which may appear in some of the images, very well. The issues related to the mathematical modelling of the proposed method are discussed and experimental results are shown in this paper.

1. Introduction

This paper deals with semi-automatic linear feature extraction from digital images for GIS data capture, where the identification task is performed manually on a single image, while a special automatic digital module performs the high precision line extraction. A human operator is used to identify the object from an on-screen display of a digital image, selects the particular class this object belongs to and provides some very few seed points coarsely distributed. This is done through activation of a mouse in a convenient interactive graphics-image user interface. Subsequently, with these seed points as approximation of the position and shape, the linear feature will be extracted automatically. There are several techniques available to solve this problem. These techniques can be either used in a monoplotting mode (combining one image with the underlying DTM) or in a multi-image mode. This semi-automatic feature extraction scheme is shown in Figure 1. The monoplotting mode based on wavelet transform and dynamic programming has been well demonstrated and documented in our previous publications (Gruen, Li, 1995). We will focus here on the multi-image mode based on LSB-Snakes, which provides for a robust and mathematically sound fully 3-D approach.

In general, the snakes or active contour models feature extraction algorithm integrates both photometric and geometric constraints, with an initial estimate of the location of the feature, by an integral measure referred to as the total energy of snakes (Kass, et al., 1988). The local minimum of this energy defines the feature of interest. It has two advantages: Geometric constraints are directly used to guide the search for the feature, and global information is used through integration of the energy along the whole length of the curve (Fua, Leclerc, 1990). The mathematical basis of the existing optimization approaches, however, is not well formulated and we diagnose a lack of investigations on issues with regard to optimality, existence and uniqueness of the solution (Amini, et al., 1990), and the balance between different part of the energy (Samadani, 1991). Also, the internal quality assessment of the results is not possible.

In this paper, active contour models are formulated in a least squares approach and extended to integrate multiple images for feature extraction in a fully 3-D mode. With such a development, the various tools of least squares estimation with their familiar and well established mathematical formulations can be favourably utilized for the statistical analysis of the obtained results and the realistic evaluation of its performance, e. g. through the use of the covariance matrix of the estimated parameters. This is in clear contrast to conventional snakes, which due to their particular theoretical background and formulation, do not provide any measures for the qualitative control of their results. At the same time, it can be considered as a new application and extension of the least squares template matching (LSM) technique (Gruen, 1985). Also, through the integration of camera models and a multiple-image approach redundant image information becomes available, which stabilizes the solution and allows to deal with partial occlusions and similar distortions.

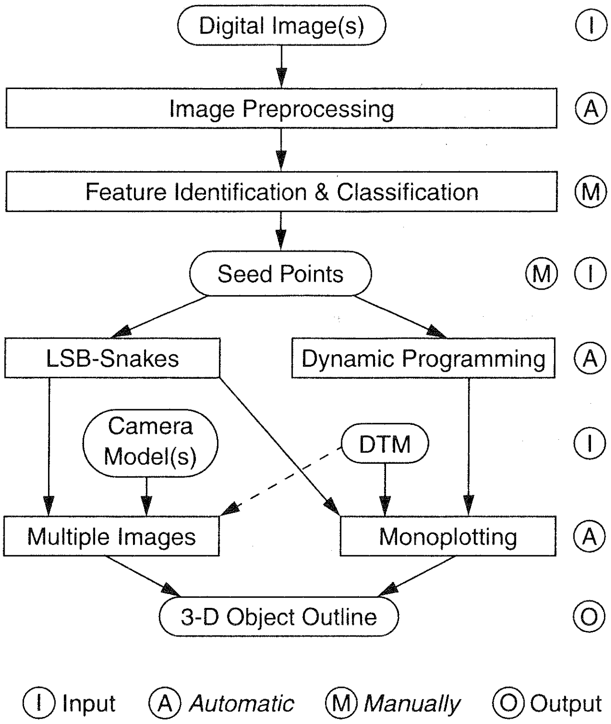


Fig. 1. A semi-automatic feature extraction scheme.

2. LSB-Snakes

LSB-Snakes derive their name from the fact that they are a combination of least squares template matching (Gruen, 1985) and B-spline snakes (Trinder, Li, 1995). In least squares notation we use three types of observations. These can be divided in two classes, photometric observations that formulate the grey level matching of images and the object model and geometric observations that express the geometric constraints and the a priori knowledge of the location and shape of the feature to be extracted.

2.1 Photometric observation equations

Assume a template and image region are given as discrete two dimensional functions $PM(x, y)$ and $g(x, y)$, which might have been derived from the a priori knowledge of the feature of interest and a discretization of continuous functions (analogue photographs). They can be considered as the conjugate regions of a stereopair in the 'left' and the 'right' photograph respectively. An ideal situation gives

$$PM(x, y) = g(x, y) . \qquad (2\text{-}1)$$

Taking into consideration the noise and assuming that the template is noise free or its noise is independent of the image noise, equation (2-1) becomes,

$$PM(x, y) - e(x, y) = g(x, y) , \qquad (2\text{-}2)$$

where $e(x, y)$ is a true error function.

In terms of least squares estimation, equation (2-2) can be considered as a nonlinear observation equation which models the observation vector of $PM(x, y)$ with a discrete function $g(x, y)$. Applying Taylor's series to equation (2-2), dropping second and higher order terms, with notations

$$G_x = \frac{\partial}{\partial x} g(x, y) , \quad G_y = \frac{\partial}{\partial y} g(x, y) , \qquad (2\text{-}3)$$

the linearized form of the observation equation becomes

$$-e(x, y) = G_x(x^0, y^0)\Delta x + G_y(x^0, y^0)\Delta y + (g(x^0, y^0) - PM(x, y)) . \qquad (2\text{-}4)$$

The relationship between the template and the image patch needs to be determined in order to extract the feature, i. e. the corrections Δx, Δy in equation (2-4) have to be estimated. In the conventional least squares template matching applied to feature extraction, an image patch is related to a template through a geometrical transformation, formulated normally by a six parameter affine transformation to model the geometric deformation (Gruen, 1985). The template is typically square or rectangular and sizes range from 5×5 to 25×25 pixels. Originally the LSM technique is only a local operator used for high precision point measurement. It was extended to an edge tracking technique to automatically extract edge segments (Gruen, Stallmann, 1991), and a further extension was made through the introduction of object-type-dependent neighbour-

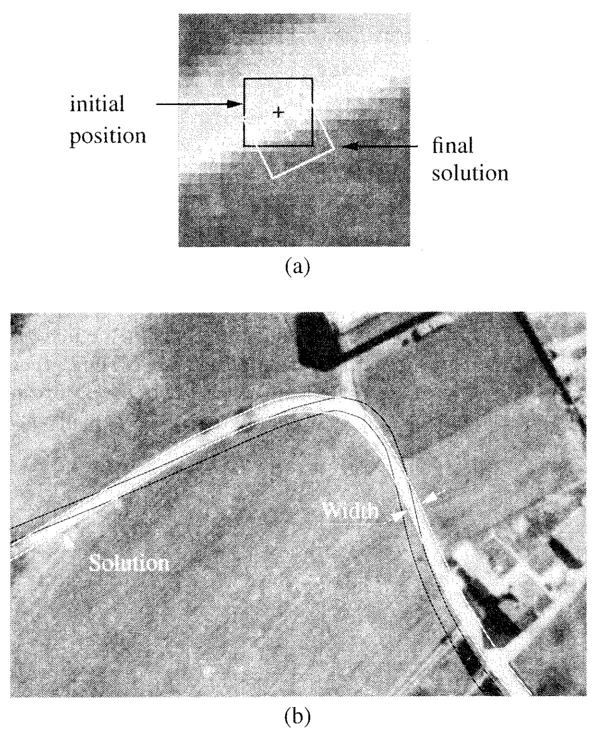


Fig. 2. Visualization of (a) least squares template matching of an edge and (b) LSB-Snakes of a road segment, with the initial position (black) and final solution (white).

hood constraints between individual patches to enforce the local continuity in a global solution (Gruen, Agouris, 1994). In another approach least squares template matching was combined with Kalman filtering of the parameters of the road for road tracking (Vosselman, Knecht, 1995).

Instead of a square or rectangular template, we extend the least squares template matching technique into LSB-Snakes by using a deformable contour as the template. This is shown in Figure 2. The ribbon with black outline is our initial template and the white one is the final solution. Its centre line is the extracted feature, in the example of the figure the centre line of a road segment.

Suppose a linear feature, the centre line of the template is approximated by a spline curve and represented in parametric form as

$$x(s) = \sum_{i=1}^{n} N_i^m(s) X_i \ , \qquad y(s) = \sum_{i=1}^{n} N_i^m(s) Y_i \ , \qquad (2\text{-}5)$$

where X_i and Y_i are the coefficients of the B-spline curve in x and y direction respectively. In terms of the B-spline concept, they form the coordinates of the control polygon of the curve. $N_i^m(s)$ is the normalized mth B-spline between knots u_i and u_{i+m+1} (Bartels, et al., 1987). While the knot sequence is given, feature extraction can be treated as a problem of estimation of the coefficients X_i and Y_i of the spline curve.

The first order differentials of the B-spline curve can be obtained as

$$\Delta x = \sum_{i=1}^{n} N_i^m(s) \Delta X_i \ , \qquad \Delta y = \sum_{i=1}^{n} N_i^m(s) \Delta Y_i \ . \qquad (2\text{-}6)$$

Substituting the terms in equation (2-4), we obtain the linearized photometric observation equations with respect to the coefficients of the B-splines. The linearization of the observation equations for all involved pixels can be expressed in matrix form as

$$-\boldsymbol{e}_m = \boldsymbol{G}_x \boldsymbol{N} \Delta \boldsymbol{X} + \boldsymbol{G}_y \boldsymbol{N} \Delta \boldsymbol{Y} - \boldsymbol{l}_m \ ; \qquad \boldsymbol{P}_m \qquad (2\text{-}7)$$

with

$$\boldsymbol{N} = \left[N_0^m(s) \ N_1^m(s) \ \dots \ N_n^m(s) \right] . \qquad (2\text{-}8)$$

Since a linear feature is essentially unidirectional, the template would slide along it during matching. To ease this problem and simplify the implementation, the above equations are changed to

$$-\boldsymbol{e}_{mx} = \boldsymbol{G}_x \boldsymbol{N} \Delta \boldsymbol{X} - \boldsymbol{l}_{mx} \ ; \qquad \boldsymbol{P}_{mx} \ , \qquad -\boldsymbol{e}_{my} = \boldsymbol{G}_x \boldsymbol{N} \Delta \boldsymbol{Y} - \boldsymbol{l}_{my} \ ; \qquad \boldsymbol{P}_{my} \ . \qquad (2\text{-}9)$$

A pair of independent observation equations are thus formed for the x and y directions. The observation vectors $\boldsymbol{l}_{mx}$ and $\boldsymbol{l}_{my}$ contain the differences of conjugate pixels, $\boldsymbol{P}_{mx}$ and $\boldsymbol{P}_{my}$ are the corresponding weight matrices, which are introduced as diagonal matrices.

2.2 Geometric observation equations

In a semi-automatic feature extraction scheme, a set of seed points near the feature of interest is given by a human operator or other preprocessing procedures. In terms of least squares adjustment, these seed points can be interpreted as the control points which determine the location of the feature to be extracted. Because they are only coarsely given, a correction has to be estimated. Therefore they should be considered as observations. Thus the second type of observation equations can be established as

$$-\boldsymbol{e}_{cx} = \boldsymbol{x} - \boldsymbol{x}_0 \ ; \qquad \boldsymbol{P}_{cx} \ , \qquad -\boldsymbol{e}_{cy} = \boldsymbol{y} - \boldsymbol{y}_0 \ ; \qquad \boldsymbol{P}_{cy} \ , \qquad (2\text{-}10)$$

where $\boldsymbol{x}_0$ and $\boldsymbol{y}_0$ are the observation vectors of coordinates of the seed points in x and y direction respectively, $\boldsymbol{P}_{cx}$ and $\boldsymbol{P}_{cy}$ are the corresponding weight matrices, introduced as diagonal matrices. The linearization of the coordinates with respect to the coefficients of the B-splines can be expressed in matrix form as

$$-\boldsymbol{e}_{cx} = \boldsymbol{N} \Delta \boldsymbol{X} - \boldsymbol{t}_{cx} \ ; \qquad \boldsymbol{P}_{cx} \ , \qquad -\boldsymbol{e}_{cy} = \boldsymbol{N} \Delta \boldsymbol{Y} - \boldsymbol{t}_{cy} \ ; \qquad \boldsymbol{P}_{cy} \ , \qquad (2\text{-}11)$$

in which $\boldsymbol{t}_{cx}$ and $\boldsymbol{t}_{cy}$ are given by

$$\boldsymbol{t}_{cx} = \boldsymbol{x}^0 - \boldsymbol{x}_0 = \boldsymbol{N} \boldsymbol{X}^0 - \boldsymbol{x}_0 \ , \qquad \boldsymbol{t}_{cy} = \boldsymbol{y}^0 - \boldsymbol{y}_0 = \boldsymbol{N} \boldsymbol{Y}^0 - \boldsymbol{y}_0 \ . \qquad (2\text{-}12)$$

With the seed points an initial curve is formed as a first shape approximation of the feature. In order to stabilize the local deformation of the template we introduce the following smoothness constraints. Assume the initial curve is expressed by $x^0(s)$ and $y^0(s)$. We establish the third type of observation equations based on the first and second derivatives of the curve as

$$-e_{sx} = x_s(s) - x_s^0(s) \ ; \qquad P_{sx} \ , \qquad -e_{sy} = y_s(s) - y_s^0(s) \ ; \qquad P_{sy} \ , \qquad (2\text{-}13)$$

$$-e_{ssx} = x_{ss}(s) - x_{ss}^0(s) \ ; \qquad P_{ssx} \ , \qquad -e_{ssy} = y_{ss}(s) - y_{ss}^0(s) \ ; \qquad P_{ssy} \ . \qquad (2\text{-}14)$$

Linearizing them with respect to the coefficients of the B-spline they can be expressed in matrix form as

$$
\begin{aligned}
-e_{sx} &= N_s \Delta X - t_{sx} \; ; \qquad P_{sx} \,, \\
-e_{sy} &= N_s \Delta Y - t_{sy} \; ; \qquad P_{sy} \,,
\end{aligned}
\tag{2-15}
$$

$$
\begin{aligned}
-e_{ssx} &= N_{ss} \Delta X - t_{ssx} \; ; \qquad P_{ssx} \,, \\
-e_{ssy} &= N_{ss} \Delta Y - t_{ssy} \; ; \qquad P_{ssy} \,.
\end{aligned}
\tag{2-16}
$$

Where N_s and N_{ss} are the first and second derivatives of N defined in equation (2-8), and the terms t are given by

$$
\begin{aligned}
t_{sx} &= N_s X^0 - x_s^0 \,, \\
t_{sy} &= N_s Y^0 - y_s^0 \,,
\end{aligned}
\tag{2-17}
$$

$$
\begin{aligned}
t_{ssx} &= N_{ss} X^0 - x_{ss}^0 \,, \\
t_{ssy} &= N_{ss} Y^0 - y_{ss}^0 \,.
\end{aligned}
\tag{2-18}
$$

Any other a priori geometric information of the feature can be formulated in this manner. A joint system is formed by all of these observation equations (2-9), (2-11), (2-15) and (2-16).

2.3 Solution of LSB-Snakes

In our least squares approach linear feature extraction is treated as the problem of estimation of the unknown coefficients X and Y of the B-spline curve. This is achieved by minimizing a goal function which measures the differences between the template and the image patch and which includes the geometrical constraints. The goal function to be minimized in this approach is the $L_2 - norm$ of the residuals of least squares estimation. It is equivalent to the total energy of snakes and can be written as

$$
\begin{aligned}
v^T P v &= (v_s^T P_s v_s + v_{ss}^T P_{ss} v_{ss}) + v_m^T P_m v_m + v_c^T P_c v_c = \\
&= E_I + E_X + E_C \Rightarrow \text{Minimum} \,.
\end{aligned}
\tag{2-19}
$$

In terms of snakes, E_I denotes the internal (geometric) energy of the snakes derived from smoothness constraints, E_X denotes the external (photometric) energy derived from the object model and the image data, and E_C represents the control energy which constrains the distance between the solution and its initial location.

To minimize this goal function (total energy of snakes), we have the following necessary conditions

$$
\frac{\partial}{\partial \Delta X} v^T P v = \frac{\partial}{\partial \Delta Y} v^T P v = 0 \,. \tag{2-20}
$$

A further development of these formulae will result in a pair of normal equations used for estimation of ΔX and ΔY respectively. Because of the local support property of B-splines, it can be shown that the normal equations are banded (bandwidth $b = m + 1$) and the solution can be efficiently computed. The various tools of least squares estimation with their familiar and well established mathematical formulations can be favourably utilized for the statistical analysis of the obtained results and the realistic evaluation of the algorithmic performance. So one can evaluate the covariance matrix of the estimated parameters and derived quantities therefrom. Also, one obtains an estimate of the system noise. In addition to the traditional least squares estimation, a robust estimation can be efficiently computed too, if required.

3. LSB-Snakes with multiple images

If a feature is extracted from more than one image, its coordinates in 3-D object space can be derived. The 3-D coordinates of the feature point in object space can be directly obtained by the MPGC matching technique (Gruen, 1985, Gruen, Baltsavias, 1985) or an object-space correlation algorithm (Wrobel, 1987, Helava, 1988).

Suppose a linear feature in 3-D object space can be approximated by a spline curve and represented in B-spline parametric form as

$$
\begin{aligned}
X_T(s) &= NX \,, \\
Y_T(s) &= NY \,, \\
Z_T(s) &= NZ \,,
\end{aligned}
\tag{3-1}
$$

where N is defined in (2-8), X, Y and Z are the coefficient vectors of the B-spline curve in 3-D object space and X_T, Y_T and Z_T are the object space coordinates of the feature. If multiple images are available, there are two main ways to perform the multiphoto matching. The first method is to connect the photometric observation equations of every image by means of external geometrical constraints. One class of the most important constraints is generated by the imaging rays intersection conditions (Gruen, 1985). The second method is object-space correlation, which is performed in object space by matching densities assigned to "groundels" (ground elements). Both methods can be applied for LSB-Snakes. Since LSB-Snakes deal with a curve instead of an individual point, direct use of the MPGC algorithm will introduce much more unknowns than necessary. The method of object-space correlation is definitely of theoretical interest, however, it cannot be easily applied to LSB-Snakes without extension of the algorithm, since we are facing here a truly 3-D problem. Our 3-D LSB-Snakes can be interpreted as the object-space analogy of MPGC for multiple points defined on a deformable spline curve.

Assume patches are used from $k > 1$ images. If the image forming process followed the law of perspective projection, a pair of collinearity conditions in parametric form can be formulated for each of the image patches as

$$
\begin{aligned}
x_i &= -c \frac{a_{11}(X_T - X_0) + a_{21}(Y_T - Y_0) + a_{31}(Z_T - Z_0)}{a_{13}(X_T - X_0) + a_{23}(Y_T - Y_0) + a_{33}(Z_T - Z_0)} \,, \\
y_i &= -c \frac{a_{12}(X_T - X_0) + a_{22}(Y_T - Y_0) + a_{32}(Z_T - Z_0)}{a_{13}(X_T - X_0) + a_{23}(Y_T - Y_0) + a_{33}(Z_T - Z_0)} \,.
\end{aligned}
\tag{3-2}
$$

If the interior and exterior orientation parameters of each image are given or can be derived, the unknowns in equation (3-2) to be estimated are the coefficient vectors of a B-spline curve. The first order differentials can be obtained as

$$\Delta x_i = \frac{\partial x_i}{\partial X_T} N\Delta X + \frac{\partial x_i}{\partial Y_T} N\Delta Y + \frac{\partial x_i}{\partial Z_T} N\Delta Z \ ,$$

$$\Delta y_i = \frac{\partial y_i}{\partial X_T} N\Delta X + \frac{\partial y_i}{\partial Y_T} N\Delta Y + \frac{\partial y_i}{\partial Z_T} N\Delta Z \ . \qquad (3\text{-}3)$$

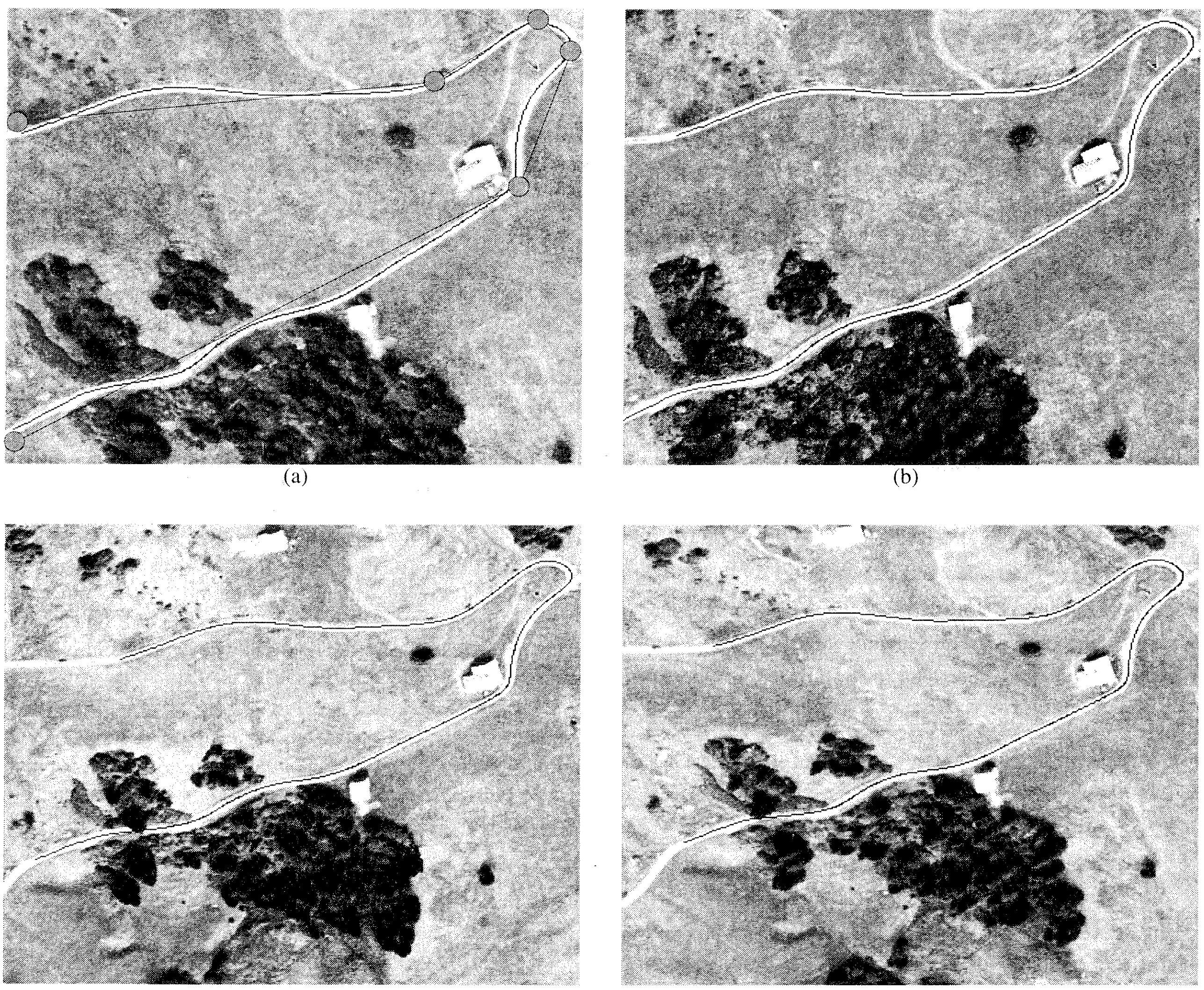
Substituting equation (3-3) in (2-4) the linearization of the observation equations with respect to the coefficient vectors of a 3-D B-spline curve can be obtained. For the same reasons as for 2-D LSB-Snakes the equations are changed such that they can be expressed in matrix form as

$$\begin{aligned} -\boldsymbol{e}_{mx} &= \boldsymbol{F}_X N\Delta \boldsymbol{X} - \boldsymbol{l}_{mx} \ ; \qquad \boldsymbol{P}_{mx} \ , \\ -\boldsymbol{e}_{my} &= \boldsymbol{F}_Y N\Delta \boldsymbol{Y} - \boldsymbol{l}_{my} \ ; \qquad \boldsymbol{P}_{my} \ , \\ -\boldsymbol{e}_{mz} &= \boldsymbol{F}_Z N\Delta \boldsymbol{Z} - \boldsymbol{l}_{mz} \ ; \qquad \boldsymbol{P}_{mz} \ . \end{aligned} \qquad (4\text{-}1)$$

$\boldsymbol{F}_X$, $\boldsymbol{F}_Y$ and $\boldsymbol{F}_Z$ are partial derivatives which can be written as

$$\begin{aligned} F_X &= \frac{\partial}{\partial x} g(x, y) \frac{\partial x_i}{\partial X_T} + \frac{\partial}{\partial y} g(x, y) \frac{\partial y_i}{\partial X_T} \ , \\ F_Y &= \frac{\partial}{\partial x} g(x, y) \frac{\partial x_i}{\partial Y_T} + \frac{\partial}{\partial y} g(x, y) \frac{\partial y_i}{\partial Y_T} \ , \\ F_Z &= \frac{\partial}{\partial x} g(x, y) \frac{\partial x_i}{\partial Z_T} + \frac{\partial}{\partial y} g(x, y) \frac{\partial y_i}{\partial Z_T} \ . \end{aligned} \qquad (4\text{-}2)$$

The geometric observation equations (2-11), (2-15), (2-16) can be extended into three dimensions by introducing a new component for the Z-direction. Then the 3-D LSB-Snakes can again be solved by a combined least squares adjustment. That is, a 3-D linear feature is extracted directly from multiple images. The statistical analysis of the obtained results and the realistic evaluation of the algorithmic performance can be done through the use of the covariance matrix of the estimated parameters.



(a) (b) (c) (d)

Fig. 3. Simultaneous 3-D extraction of a road segment from four images. Upper left: Location of seed points (0-iterations) and extracted road centre line. Rest: Extracted road centre line (after ca. 5 iterations).

4. Experiments with road extraction

The LSB-Snakes method described in this paper has been successfully implemented on computer workstations. We have tested the algorithm on a number of real images. In this section, some experimental results of road extraction will be given.

An imaged object is defined and identified by its characteristics, which can be classified into five groups: Photometric, geometric, topological, functional and contextual characteristics. In our semi-automatic feature extraction scheme the high level knowledge, which requires quite some intelligence for the image interpretation process, is used by the human operator to identify and classify the object. The generic object model involved in the model driven feature extraction algorithms consists of some photometric and geometric characteristics. Examples of a generic road model are (Gruen, Li, 1995):

- a road surface often has a good contrast to its adjacent areas,
- a road surface usually is homogeneous (at least in a certain portion of the image),
- a road is a continuous and narrow region or linear feature,
- a road is smooth and does in general not have small wiggles,
- the local curvature of a road has an upper bound,
- the width of a road or road segment does not change significantly.

Some of these properties are mathematically formulated and used to generate the template and define the weight functions. For instance, the grey values of the template can be derived from the images through computations of the local contrast according to the first property, while the second property suggests that the weights of the photometric observations should be related to the local changes of the grey levels along the road.

In the current implementation, only one image is displayed in the user interface. After some very few seed points have been given by the operator in the displayed image, the camera model or projection equation is applied to project them into object space. This is done in an iteration procedure for the computation of the *X*, *Y*-coordinates and interpolation of the *Z*-component with a very coarse DTM. Then the 3-D feature is extracted automatically and precisely by the LSB-Snakes algorithm.

Figure 3 shows one example of simultaneous 3-D extraction of a road segment from four images, which are portions of aerial images of Heinzenberg, Switzerland. The four images are from two strips with about 60% end and side lap. The scale of the original photograph is about 1:15,000. The images were taken in a mountainous area and the height differences in the test region are about 1,000 meters. The negative films were scanned at a Leica/Helava scanner with 10 microns and were later subsampled to 40 microns pixel size. Thus the footprint of the images is about

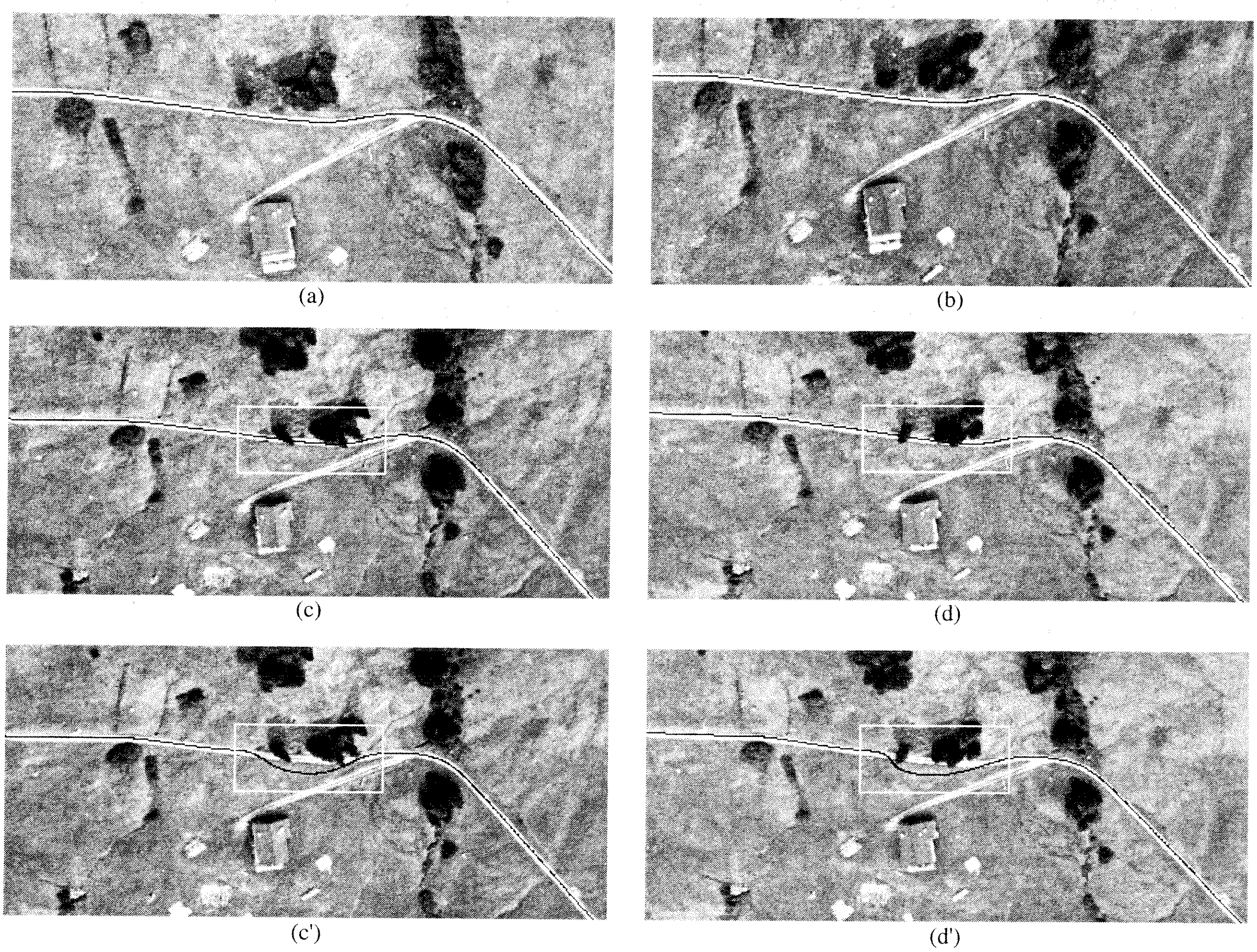


Fig. 4. Simultaneous 3-D extraction of a road segment from four images (a), (b), (c) and (d). The white rectangle denotes the problematic area of occlusion and the extracted road segments are displayed as a black curve. (c') and (d') show the results without blunder detection.

0.6 meters. Most roads in the test region belong to the 3rd or 4th class and are about 5 pixels wide on the images. The exterior orientation parameters are taken from the results of a block adjustment. The VirtuoZo software was used to perform the interior orientation of the digitized images and to generate a very coarse DTM (interval 30 m, ca. 50 pixels). The seed points (initial polygon), provided manually by the operator, are displayed in black overlaid on the first image. The extracted road centre line is shown as a black curve. Visual tests prove the successful performance of the algorithm.

Figure 4 focuses on another portion from the same aerial images. At this time, the problematic areas marked with white rectangles show occlusions in two out of four images caused by trees. In terms of least squares adjustment, the photometric observations in the problematic areas are blunders. They have to be detected and rejected. This is achieved in our implementation by using adaptive weight functions, in which the weights of observations are related with the ratio of their residuals and the variance factor. To get a large pull-in range the algorithm starts with a flat weight function. To reduce the influence of blunders it becomes steep after three iterations. In such a way, the weights of observations with big residuals will become smaller and smaller. The results shown in Figure 4 prove that the blunders are successfully rejected and the algorithm bridges gaps in a robust manner. For comparison the results without blunder detection are shown in Figure 4 as (c') and (d'). Since the extracted road on images (a) and (b) is the same in both cases they are not displayed again. It is also verified by this example that more than two images are required for 3-D linear feature extraction. Using only two images cannot give reliable 3-D results.

5. Conclusions

In this paper we have presented a new approach for (linear) feature extraction (LSB-Snakes). The method of active contour models (Snakes) is formulated in a least squares approach and, at the same time, the technique of least squares template matching is extended by using a deformable contour instead of a rectangle as the template. Through the integration of camera models any number of images can be simultaneously used and the feature can be extracted in a fully 3-D mode. Thus blunders in image data, like occlusions, can be controlled very well. Instead of a set of points on the feature, a B-spline representation of the linear feature is estimated. The results obtained so far are very encouraging. Further studies will make use of more extensive data sets and will focus on the quality assessment and automated performance evaluation.

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