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ABSTRACT

An orientation theory of two-dimensional affine imagery was first presented by Okamoto (1992), which can be applied for ultra-precise measurement of very small objects. This theory may be employed for the analysis of satellite CCD camera imagery by clarifying the geometrical characteristics of the model connection problem with adjacent stereo models and transforming the central-perspective images into affine ones. Therefore, in this paper, the orientation and model connection problems of two-dimensional affine images are first discussed in detail. Then, the transformation of central-perspective images into affine ones is explained with correction of the image transformation errors due to height differences in the terrain. The proposed method was applied for space triangulation of simulated satellite CCD camera images taken consecutively and proved to have a fairly high accuracy.

INTRODUCTION

In satellite photogrammetry, imaging devices (CCD camera and CCD line-scanner) have usually very narrow field angle. Therefore, we encounter great difficulty to apply the conventional orientation theory based on projective transformation for the analysis of satellite imagery. In order to overcome this problem, we should develop another orientation theory. Affine transformation may be a very promising one, because affine transformation pertains to parallel projection and thus the flying height of the satellite plays no role in the geometry of an affine image.

The orientation theory of two-dimensional affine images was first derived by Okamoto in 1992. In order to employ this theory for the analysis of satellite CCD camera imagery, the central-perspective imagery must be transformed into an affine one by using the approximations of the orientation parameters. This image transformation cannot be carried out without errors due to both deviations of the approximations and height differences in the terrain. Thus, a correction method of the image transformation errors is required.

In this paper, the orientation problem of two-dimensional affine imagery is extended to the model connection with adjacent stereo models so as to discuss space triangulation of satellite CCD camera imagery based on affine transformation. Then, the transformation of central-perspective images into affine ones is described in detail and the correction of the image transformation errors is discussed, in which they are classified into two types: errors due to deviations of the approximations of the orientation parameters and errors caused by height differences in the terrain. The

methods proposed here are tested with simulated examples so as to explore the difficulties when applying them to practical cases.

GENERAL ORIENTATION THEORY OF TWO-DIMENSIONAL AFFINE IMAGES

ORIENTATION PROBLEM OF A STEREOPAIR OF AFFINE IMAGES

Let a three-dimensional object space  $(X, Y, Z)$  be projected into a plane based on affine transformation (See Figure-1.). The basic equations relating an object

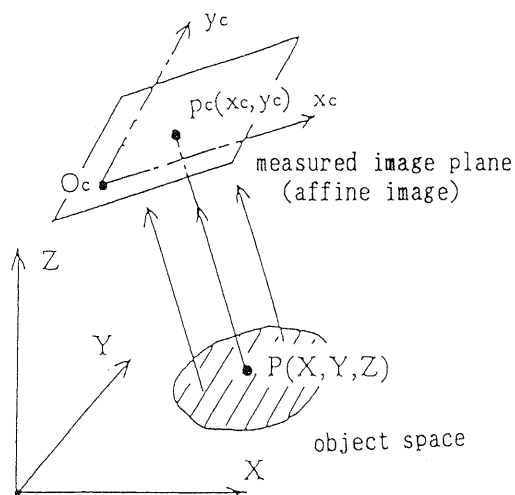


Figure-1 : parallel projection of an object space into the measured plane of the comparator coordinate system

point P and its measured image point  $p_c(x_c, y_c)$  are described as

$$\begin{aligned} x_c &= a_1X + a_2Y + a_3Z + a_4 \\ y_c &= a_5X + a_6Y + a_7Z + a_8 \end{aligned} \quad (1)$$

in which  $a_i (i=1, \dots, 8)$  are independent coefficients. Geometrically, the eight orientation parameters of the affine image are considered to be three rotation parameters  $(\omega, \phi, \kappa)$  of the image, two translation elements  $(X_{oc}, Y_{oc})$  which indicate two of the three-dimensional coordinates of the origin of the measured image coordinate system  $(x_c, y_c)$  with respect to the object space coordinate system  $(X, Y, Z)$ , the image scale  $s$ , and two rotation parameters  $(\alpha, \beta)$  describing the relationship between projected rays and the normal to the image plane. The eight orientation parameters of a single affine image can thus be provided uniquely if four control points are available.

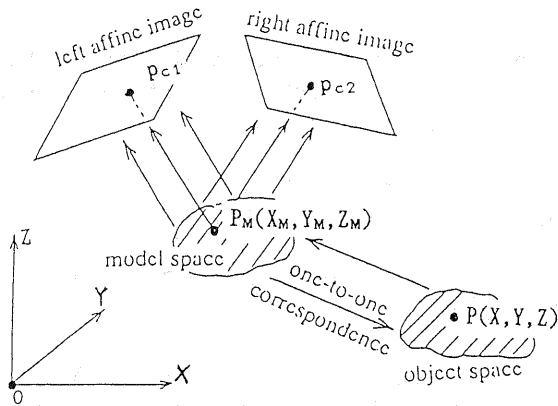


Figure-2 : relative and absolute orientation of a stereopair of affine images

Next, we will consider the orientation problem of a stereopair of two-dimensional affine images (See Figure-2.). The basic equations are written down as

$$\begin{aligned} x_{c1} &= a_{11}X + a_{12}Y + a_{13}Z + a_{14} \\ y_{c1} &= a_{15}X + a_{16}Y + a_{17}Z + a_{18} \end{aligned} \quad (2)$$

for the left image, and in the form

$$\begin{aligned} x_{c2} &= a_{21}X + a_{22}Y + a_{23}Z + a_{24} \\ y_{c2} &= a_{25}X + a_{26}Y + a_{27}Z + a_{28} \end{aligned} \quad (3)$$

for the right one, respectively. The condition that Equations 2 and 3 are valid for all object points photographed in common on the left and right images can be formulated as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14}-x_{c1} \\ a_{15} & a_{16} & a_{17} & a_{18}-y_{c1} \\ a_{21} & a_{22} & a_{23} & a_{24}-x_{c2} \\ a_{25} & a_{26} & a_{27} & a_{28}-y_{c2} \end{vmatrix} = 0 \quad (4)$$

which is equivalent to the coplanarity condition of corresponding rays. Under the condition of Equation 4 we can form a three-dimensional space  $(X_M, Y_M, Z_M)$  which can be transformed into the object space  $(X, Y, Z)$  by a three-dimensional affine transformation having 12 independent coefficients, i.e.,

$$\begin{aligned} X_M &= B_1X + B_2Y + B_3Z + B_4 \\ Y_M &= B_5X + B_6Y + B_7Z + B_8 \\ Z_M &= B_9X + B_{10}Y + B_{11}Z + B_{12} \end{aligned} \quad (5)$$

Also, the space  $(X_M, Y_M, Z_M)$  is equivalent to the model space. From the results obtained above we can find the following characteristics of the orientation problem of overlapped affine images:

- 1) The coplanarity condition of corresponding rays can mathematically provide four orientation parameters among the twelve ones of the stereopair of affine images, and
- 2) The one-to-one correspondence relating the model and object spaces can be uniquely determined, if four control points are given in the object space.

#### MODEL CONNECTION THEORY WITH ADJACENT STEREO MODELS

We will assumed that an object was imaged on four different planes based on parallel projection. The first stereo model is constructed with the first and second images, and the second stereo model with third and fourth images. We will investigate what relationship is valid between the first and second stereo models (See Figure-3.). The general three-dimensional affine

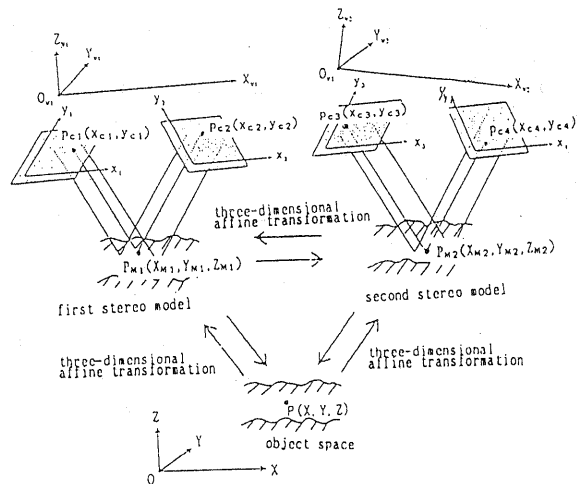


Figure-3 : model connection problem of multiple affine images

transformation (Equation 5) is satisfied between the first stereo model space  $(X_{M1}, Y_{M1}, Z_{M1})$  and the object space  $(X, Y, Z)$  in the form

$$\begin{aligned} X_{M1} &= B_1X + B_2Y + B_3Z + B_4 \\ Y_{M1} &= B_5X + B_6Y + B_7Z + B_8 \\ Z_{M1} &= B_9X + B_{10}Y + B_{11}Z + B_{12} \end{aligned} \quad (6)$$

or inversely as

$$\begin{aligned} X &= C_1X_{M1} + C_2Y_{M1} + C_3Z_{M1} + C_4 \\ Y &= C_5X_{M1} + C_6Y_{M1} + C_7Z_{M1} + C_8 \\ Z &= C_9X_{M1} + C_{10}Y_{M1} + C_{11}Z_{M1} + C_{12} \end{aligned} \quad (7)$$

The same can be described between the second stereo model space  $(X_{M2}, Y_{M2}, Z_{M2})$  and the object space  $(X, Y, Z)$  in the form

$$\begin{aligned} X_{M2} &= D_1X + D_2Y + D_3Z + D_4 \\ Y_{M2} &= D_5X + D_6Y + D_7Z + D_8 \\ Z_{M2} &= D_9X + D_{10}Y + D_{11}Z + D_{12} \end{aligned} \quad (8)$$

or inversely as

$$\begin{aligned} X &= E_1X_{M1} + E_2Y_{M1} + E_3Z_{M1} + E_4 \\ Y &= E_5X_{M1} + E_6Y_{M1} + E_7Z_{M1} + E_8 \\ Z &= E_9X_{M1} + E_{10}Y_{M1} + E_{11}Z_{M1} + E_{12} \end{aligned} \quad (9)$$

By substituting Equation 9 into Equation 6, we get the relationship between the first and second stereo models in the form

$$\begin{aligned} X_{M1} &= F_1X_{M1} + F_2Y_{M1} + F_3Z_{M1} + F_4 \\ Y_{M1} &= F_5X_{M1} + F_6Y_{M1} + F_7Z_{M1} + F_8 \\ Z_{M1} &= F_9X_{M1} + F_{10}Y_{M1} + F_{11}Z_{M1} + F_{12} \end{aligned} \quad (10)$$

which coincides with the general affine one-to-one correspondence (Equation 5) between two three-dimensional spaces. It means that the first and second stereo models can be connected by the general three-dimensional affine transformation.

#### TRANSFORMATION OF CENTRAL-PERSPECTIVE IMAGES INTO AFFINE ONES

The satellite CCD camera conventionally has an extremely narrow field angle. Thus, the conventional orientation approach is not effective due to very high correlations among the orientation parameters. On the other hand, the orientation theory of two-dimensional affine imagery may be effectively applied to the analysis of satellite CCD camera imagery, if the central-perspective images can be transformed into affine ones. Also, this transformation will be performed in a following way.

Let the ground surface be flat and a central-perspective

photograph be taken with the rotation angles  $\omega$  and  $\phi$ . The reference coordinate system  $(X', Y', Z')$  is selected as a right-handed, rectangular Cartesian system with its origin at the projection center of the photograph and with its  $X' - Y'$  plane parallel to the scaled ground surface, as is demonstrated in Figure-4. Further, the

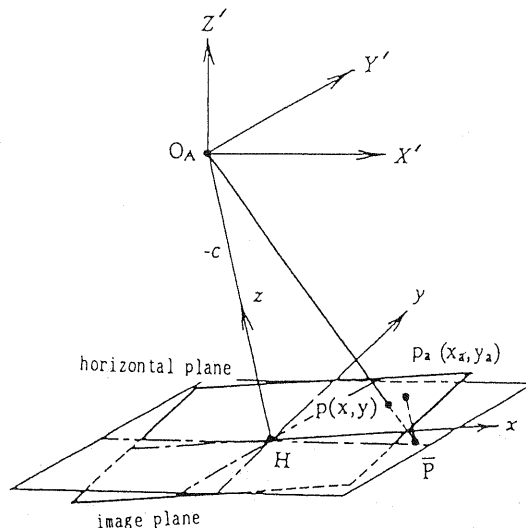


Figure-4 : transformation of a central-perspective image into an affine image

photograph is considered to intersect the scaled ground surface in such a way that its principal point H lies on the surface. The three-dimensional coordinates  $(X'_p, Y'_p, Z'_p)$  of an image point  $p(x, y)$  of the central-perspective photograph are expressed with respect to the reference coordinate system in the form

$$\begin{bmatrix} X'_p \\ Y'_p \\ Z'_p \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & -\sin \omega \\ 0 & \sin \omega & \cos \omega \end{bmatrix} \begin{bmatrix} x \\ y \\ -c \end{bmatrix} \quad (11)$$

in which  $c$  denotes the principal distance of the picture. Also, the principal point H of the central-perspective picture is given with respect to the reference coordinate system as

$$\begin{bmatrix} X'_H \\ Y'_H \\ Z'_H \end{bmatrix} = \begin{bmatrix} -c \cdot \sin \phi \cos \omega \\ c \cdot \sin \omega \\ -c \cdot \cos \phi \cos \omega \end{bmatrix} \quad (12)$$

Further, Let  $\bar{P}(\bar{X}, \bar{Y}, \bar{Z})$  denote the point at which the ray  $\vec{O_A P}$  intersects the scaled ground surface. The three-dimensional coordinates of this point can be

described as

$$\begin{aligned} \overline{X}' &= \frac{X'_P}{Z'_P} Z'_{II} \\ \overline{Y}' &= \frac{Y'_P}{Z'_P} Z'_{II} \\ \overline{Z}' &= Z'_H \end{aligned} \quad (13)$$

Observing the point  $\overline{P}(\overline{X}', \overline{Y}', \overline{Z}')$  on the scaled ground surface with respect to the picture coordinate system  $(X, Y, Z)$  we can find the corresponding affine image point  $p_a(X_a, Y_a)$  in the form

$$\begin{aligned} x_a &= (\overline{X}' - X'_H) \cos \phi \\ y_a &= (\overline{X}' - Y'_H) \sin \omega \sin \phi + (\overline{Y}' - Y'_H) \cos \omega \end{aligned} \quad (14)$$

Next, we will consider image errors which cannot be avoided in this image transformation. The image transformation errors of the first type are caused by the errors of the given orientation parameters and those of the second type by height differences in the photographed terrain. The exterior orientation parameters

$(\omega, \phi)$  and the interior ones  $(X_H, Y_H, c)$  of satellite photographs are usually given with very accurate approximations. Thus, the image transformation errors of the first type are considered to be small. Also, these errors can be modeled in a linear form, if the deviations of the orientation parameters are small. This means that the errors of the first type can be corrected automatically in the orientation calculation using Equation 1, because coefficients describing these errors are absorbed by the orientation parameters  $A_i (i=1, \dots, 8)$ . On the other hand, the image transformation errors of the second type increase with the height difference and distribute almost randomly over the terrain. Thus, these errors must be removed by developing an appropriate correction method. As for the correction technique, an iterative orientation calculation may be the most pertinent one, where the image transformation error of each ground point is corrected by changing the principal distance of the camera, corresponding to its height difference from the average height obtained in the previous iteration step.

#### TESTS WITH A SIMULATED EXAMPLE

The proposed orientation method of satellite CCD camera imagery was tested with a simulated example. For the construction of the simulation model 22 satellite CCD camera images are assumed to be taken consecutively in a convergent manner (See Figure-5.). The image coordinates of 63 object points were

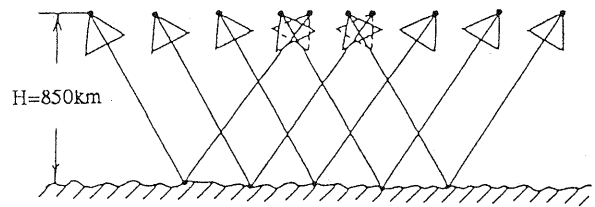


Figure-5 : satellite CCD camera images taken consecutively in a convergent manner

calculated by means of the collinearity equations under the following conditions:

- flying height of the satellite :  $H = 700 \text{ km}$
- focal length of the used CCD camera :  $c = 300 \text{ mm}$
- picture scale :  $1/2,300,000$
- convergent angles :  $\pm 30 \text{ degrees}$
- CCD pixel size :  $5 \mu\text{m} \times 5 \mu\text{m}$
- number of CCD pixels : 4,000,000
- maximum height differences in the photographed terrains : 100m, 2000m, 4,000m

The perturbed image coordinates were provided in which the perturbation consisted of random normal deviates having a standard deviation of 2.0 micrometers. In the orientation calculation, maximum errors of the orientation parameters were assumed to be 1 degree for the rotation parameters, 1000m for the translation parameters, and 1mm for the interior orientation elements. Also, the arrangement of ground control points and check points is shown in Figure-6. The

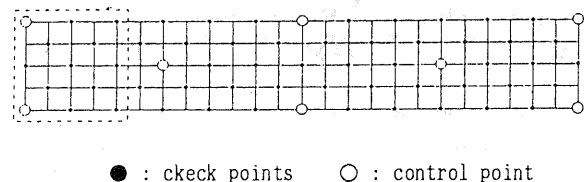


Figure-6 : configuration of check and control points

obtained results regarding the standard error of unit weight, the average internal and external errors are given in Tables-1. From these results we can discuss the characteristics of the proposed orientation approach of satellite CCD camera imagery as follows:

- 1) The field angle of the CCD camera in this case is about 2 degrees. With such a narrow field angle, the image transformation errors due to height differences in the terrain give almost no influences on the external accuracy, even though the maximum height difference amounts to 2,000 meters.
- 2) The external error approaches the theoretical one,

maximum height difference : 100m		
iteration step	I	II
standard error of unit weight( $\mu\text{m}$ )	2.1	2.1
average internal error(m)	6.2	6.2
average external error(m)	4.6	4.6
maximum height difference : 2000m		
iteration step	I	II
standard error of unit weight( $\mu\text{m}$ )	2.1	2.0
average internal error(m)	9.0	8.5
average external error(m)	8.6	8.1
maximum height difference : 4000m		
iteration step	I	II
standard error of unit weight( $\mu\text{m}$ )	2.3	2.0
average internal error(m)	10.7	8.7
average external error(m)	15.4	8.5

Table-1 : obtained results in simulated space triangulation using affine transformation

if the terrain is extremely flat. However, the external error increases with the height differences in the terrain.

- 3) The iterative orientation calculation becomes effective when the maximum height difference in the terrain exceeds 4,000 meters. This means that the proposed correction technique of the image transformation errors is mathematically sound.

#### CONCLUDING DISCUSSIONS

This paper has presented an iterative orientation method of satellite CCD camera imagery based on affine transformation and developed an effective correction approach of the image errors which arise in the transformation of the central-perspective CCD camera images into affine ones. The proposed orientation technique has been tested with a simulated example of 22 satellite CCD camera images taken consecutively and proved to have a high accuracy.

#### REFERENCES

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