A New Approach for Global DTM Modelling

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ABSTRACT

In Photogrammetry and Geodesy various methods for DTM-modelling exist. A surface, where the height coordinates $z(x, y)$ are a function of the planimetric coordinates x and y, is usually described by an height matrix, but we can also do a Fourier series expansion. Since many processes in environmental modelling and climate research are described on the sphere, we have developed a method which uses spherical coordinates instead of planimetric coordinates. Starting with a set of scattered data the height of a surface point is calculated by analyzing adjacency relationships using locally supported basis functions. Organizing our algorithm in a hierarchical way with different scale functions errors in the data set can be detected. The method is triggered by B-spline interpolation and wavelet techniques. It is tested on data sets of Australia and the asteroid 243 IDA.

1 INTRODUCTION

Topographical information, especially digital terrain models (DTM), are used in various applications, i.e. in Meteorology, climate and environmental modelling and in Geodesy. In these applications often regional and global processes are simulated. One of the main requirements is the fast access of topographic data in different aggregation levels. The same aspects arise in computer graphics, visualization and visual simulation (Gross, 1995). For this purpose efficient algorithms for coding and representation of very large data sets, in particular DTM, are needed.

The variation of the surface elevation over a planar, spherical or ellipsoidal area can be modelled in many ways. DTM can be represented either by mathematically defined surfaces or by point and line methods. For global DTM the point methods often result in coarse regular grids, in which one grid cell represents a large area. Since the height is calculated as an average over this region, terrain features such as peaks, pits, passes, ridge lines, and stream courses cannot be represented using a coarse matrix. The advantage of these grid DTM is the fast data access nevertheless much storage capacity is needed in case of a global DTM.

The mathematical methods of surface modelling rely on continuous 3-dimensional functions that are capable of representing complex forms with a high degree of smoothness. The surface can be split into patches of equal area and the associated heights are calculated from point observations within the patches. Weighting factors are used to ensure that the surface patches match at the edges, though the surface does not always need to be continuous in slope at the borders. For global DTM modelling Fourier series and interpolation with multiquadric polynomials are also used. In the latter case we have to solve a linear system which causes problems if the data amount is too high. The above methods are described in (Burrough, 1986; Gold, 1989) in more detail.

Wavelets have proven their efficiency for use in numerical analysis and signal processing and there are also a few publications (Li, Shao, 1994; Zhou, Dorrer, 1994) in Photogrammetry. Their power lies in the fact that a small number of coefficients can represent general functions and large data sets quite accurately. The method proposed in this paper is similar to Fourier series expansions but governed by ideas of B-spline functions and wavelet theory.

After a short description of the mathematical fundamentals (Chapter 2) the new algorithm is given in Chapter 3. In Chapter 4 we present results of the first tests with a DTM of Australia. In a second example a data set derived from images of the asteroid 243 IDA is processed. Chapter 5 concludes the paper with an outlook on future work.

2 MATHEMATICAL FUNDAMENTALS

Starting with the description of the well known "next neighbour" approximation we introduce a new approach for global DTM modelling on the sphere. In our description we denote as reference points the given primary data, acquired from digitized maps, Photogrammetry (e.g. image matching) or GPS measurements. In the next neigh-

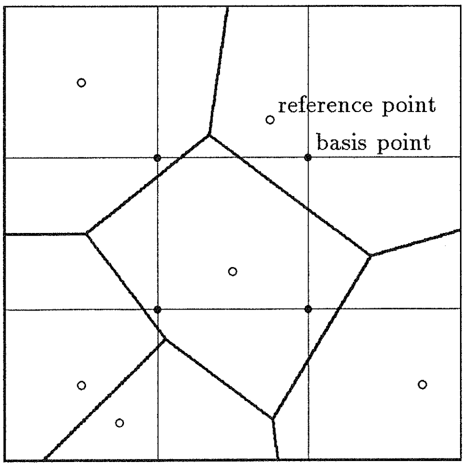

Figure 1: Voronoi diagram of irregular data superimposed with regular grid of basis points

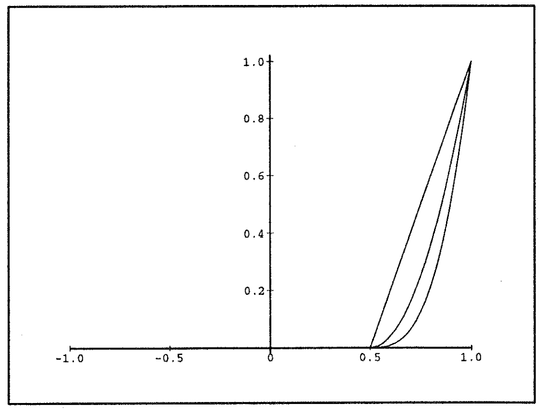

Figure 2: $B_r^{(k)}(t)$ for $k = 1, 2, 3$ and $r = 0.5$

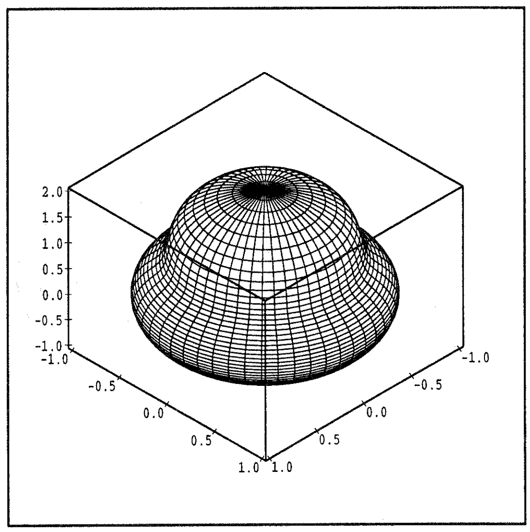

Figure 3: Spherical representation of $B_r^{(k)}$ centered at the North-Pole for $k = 3$ and $r = 0.5$

bour approximation the height on a regular grid, in our words at a basis point, is linked to the height of the next reference point. The region of influence of a reference point can then be plotted in a Voronoi diagram (see Figure 1). Covering our domain with a grid with fixed mesh-width the surface is represented by a height matrix which can be processed very efficiently. The height of a sampling point (e.g. point on a finer grid for displaying the DTM) can be calculated by bilinear interpolation or other methods from the heights of the basis points. This means that the next neighbour approximation involves 3 data sets, the reference points, the basis points and the sampling points. Our approach is characterized by the same property. The transmission of the height information from a reference point to a basis point depends on the spatial distance. In our method we analyze the polar distance between two points on a unit sphere, defined as the cosine of the angle between them. If the points are given in cartesian coordinates $x = (x_1, x_2, x_3)$ it can be calculated by the scalar product $t = (x, y) = x_1y_1 + x_2y_2 + x_3y_3$ very easily. This distance is equal to 1 if the angle between two points is zero and therefore it is suitable to define a weighting function. Our weighting functions are spherical polynomials of degree k, $B_r^{(k)} : [-1, 1] \to \mathbb{R}$, $k = 1, 2, \ldots$, $r \in [0, 1)$ given by

$$B_r^{(k)}(t) := \begin{cases} 0 & \text{for } -1 \le t \le r \\ (\frac{t-r}{1-r})^k & \text{for } \quad r < t \le 1. \end{cases} \tag{1}$$

This function is printed in Figure 2 for different values of k and fixed r. The spherical representation of this functions for a given point y, for example $y = (0, 0, 1)$, is plotted in Figure 3. The function has rotational symmetry about the axis through the point y and compact support. Points lying on the same latitude have the same spherical distance to the North-Pole y. Points with spherical distance greater than $r = 0.5$ (angle > 60 degree) lie outside the defined spherical cap around y and get the weight zero.

The calculation of the height at a basis point can then be formulated as analyzing the distance between the basis points and the reference points and as weighting the heights of the reference points in the spherical cap according to their distance. This can be formulated as summation

$$g(x) = \sum_y h_y \overline{B}_r^{(k)}((x, y)) \tag{2}$$

where x are the basis points, y are the reference points and h_y is the height at the reference points. $\overline{B}_r^{(k)}((x, y))$ denotes a normalization of the function $B_r^{(k)}$ defined in (1) where the argument is the scalar product (x, y). The choice of this normalized function is due to (Cui, et al., 1992) and discussed in (Brand, 1994) in more detail. In contrast to the next neighbour approximation the height of the basis points depends not only on one reference point but also on the reference points in a spherical cap of radius r around them. For the evaluation of the height at a sampling point z formula (2) is used as well, so that our approximation $g(z)$ can be calculated according to

$$g(z) = \sum_x c(x) \overline{B}_r^{(k)}((x, z)) \tag{3}$$

with

$$c(x) = \sum_y h_y \overline{B}_r^{(k)}((x, y)). \tag{4}$$

The functional parameter r depends on the distribution of reference points and basis points. Its choice is similar to the choice of the mesh–width which characterizes the resolution of a DTM given by a height matrix. The coefficients $c(x)$ of the functional expansion (3) in connection with the functional parameter r represent the DTM in our approach. This approximation method is tested on several artificial and practical examples in (Brand, 1994) for different sets of reference points and basis points. It works well but has the following drawbacks :

- A suitable value for the region of influence r is not known in advance. Its choice can be very delicate.
- If the characteristic scale of the reference points varies in space, one has to choose the value of r according to the smallest feature of the scale size in the region. If not doing so, essential information may be lost. A spatially variable value of r would be appropriate in this case.
- The error of the DTM can only be calculated at the end of the process. An improvement requires a completely new run with modified r and/or modification of the given grid.

These drawbacks can be avoided by a hierarchical algorithm.

3 THE NEW ALGORITHM

The new algorithm can be described as follows

(*i*) Choose a relatively small scale parameter r_0 (large spherical caps) and relatively coarse basis grid Γ_0.

(*ii*) Compute an approximation g_0 from (3).

(*iii*) Compute the error E_1 at the given data (reference points).

(*iv*) Decide wether E_1 is sufficiently small in all parts of the domain. If true stop.

(*v*) Increase the scale parameter to r_1 (smaller spherical caps) and refine the basis grid (grid of coefficients) to Γ_1

(*vi*) Compute the approximation of the discrete error E_1 in those parts of the domain where E_1 is above the threshold in (*iv*). Add this contribution to the approximation obtained in (*ii*).

(*vii*) Iterate steps (*iii*) to (*vi*) up to the situation where

  (a) The error E_j is sufficiently small in the whole domain or

  (b) The refinement of the grid Γ_j approaches a cell size where the number of points from Ξ that serve to determine a particular coefficient decreases below a threshold which ensures sufficient averaging

Profiles along the equator of the resulting succesive approximation (first 3 steps) for an artificial example are represented in Figure 4. The height of the reference points are plotted as circles, the basis points as crosses.

This algorithm is characterized by different resolution and the adaptive choice of the basis points. First tests have been conducted with several basis grids described in (Freeden, Schreiner, 1993). In (Brand, et al., 1995) a different

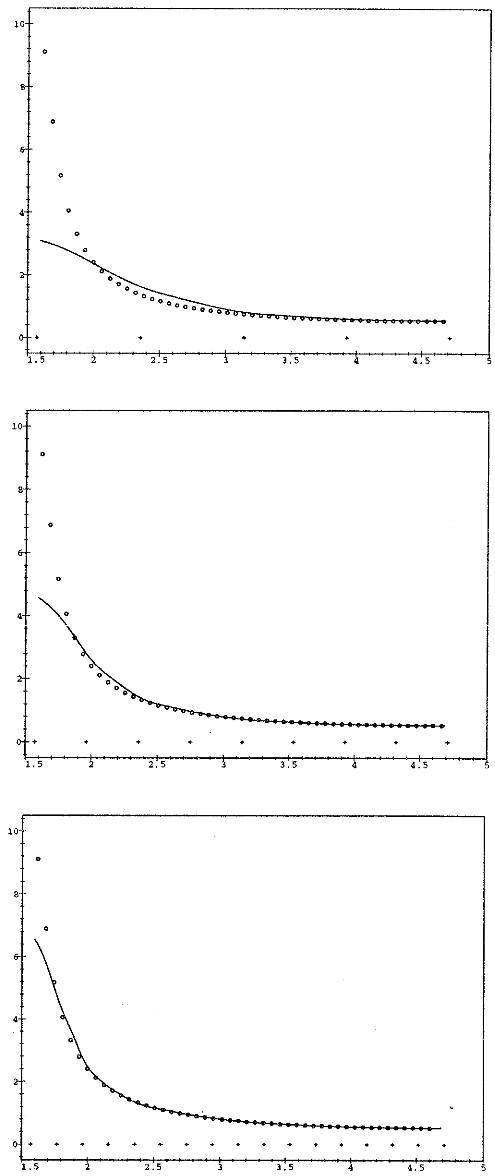

Figure 4: Synthetic example, profiles along the equator for different levels of approximation

hierarchical grid with tree structure has been employed which considerably speeds up the computation. It can be described as follows : each point of a grid Γ_j has n sons located in the neighbourhood of this point. The union makes up Γ_{j+1}. With respect to the unit square, which can be transformed to the sphere, a sequence of basis grids reads $\Gamma_0 = \{(1/2, 1/2)\}$, $\Gamma_1 = \{(1/4, 1/2); (3/4, 1/2)\}$, $\Gamma_2 = \{(1/4, 1/4); (1/4, 3/4); (3/4, 1/4); (3/4, 3/4)\}$ and so on, each of the previous points having two sons alternately in horizontal and vertical direction. This organization allows easy and efficient management of the basis points. Similarly, the hierarchical organization of the data points in form of a quadtree (see e.g. Samet, Weber, 1988) is introduced for very large data sets. By sorting the given reference points as well as the above defined basis grids according to their latitudes the calculation of the sums (3) is simplified. The calculation of the scalar product has only be done in a small region around the sampling point, which can be expressed by differences in latitude and longitude, so that the whole data set has not to be worked through.

The above hierarchical algorithm is triggered by the multilevel approaches (multigrid, hierarchical basis, wavelets) developed in mathematics in the last years. An important component is error control which is done after each step. The surface is built up successively and in every step errors in the given data set can be eliminated. The method is extremely simple in its principle and efficiently implementable. It is also very flexible and by its adaptivity may significantly reduce the computational cost with respect to other approximation methods.

4 EXAMPLES

The new algorithm was tested first with a data set of Australia. The elevation data (5,220 points) were derived from the data set ETOPO5 (Warnken, 1995) and have a maximal resolution of $0.5^\circ \times 0.5^\circ$, i.e. 1 point per 55 km^2. The main aim of this test was the automation of the processing. We had chosen very simple grids (Brand, et al., 1995; Freeden, Schreiner, 1994) and the scale parameter r (region of influence) had been chosen to yield constant overlapping on each level. A regular grid DTM (97 $\times$ 81 points) was obtained in 15s (HP 9000/700) and is shown in Figure 5. The maximum error is 352 m, the mean error 13.5 m. Similar height accuracy figures had been computed using the commercial software HIFI (Ebner, et al., 1988), which is a package covering a wide range of DTM applications.

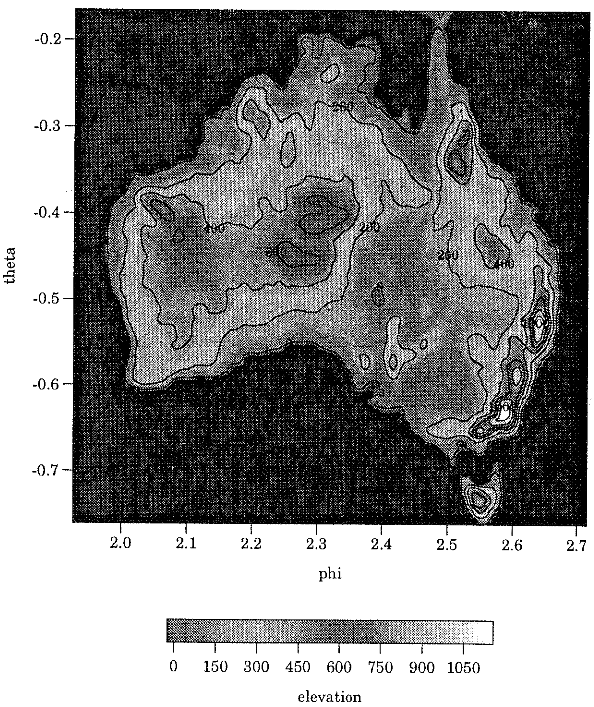

Figure 5: DTM of Australia

In a second example the shape of the asteroid 243 IDA was determined. On its trip to Jupiter the Galileo spacecraft took images of the asteroid IDA and its satellite Dactyl by a SSI-CCD camera. Since the irregular shape of IDA affects physical properties of the asteroid, its determination is of great interest. In (Thomas, et al., 1995) limbs, terminators and shadow data were used in addition to 101 control points to derive a $2^\circ \times 2^\circ$ shape model. In our case more than 30,000 image points were found by digital image matching. From these image points 10,350 ground points were calculated by forward intersections using the exterior orientation parameters, which were determined previously by bundle block adjustment (Ohlhof, et al., 1996). Also points in deep space, which are not located on IDA's surface, were found by the matching algorithm. Single outliers could be detected by the error control feature of our new DTM approach, clusters of outliers were eliminated by analyzing the z-coordinates of the ground points. Using 7,535 reference points with spherical coordinates $0^\circ < \varphi < 230^\circ$ and $45^\circ < \vartheta < 145^\circ$, a $1^\circ \times 1^\circ$ shape model within this region was computed using the new spherical approach.

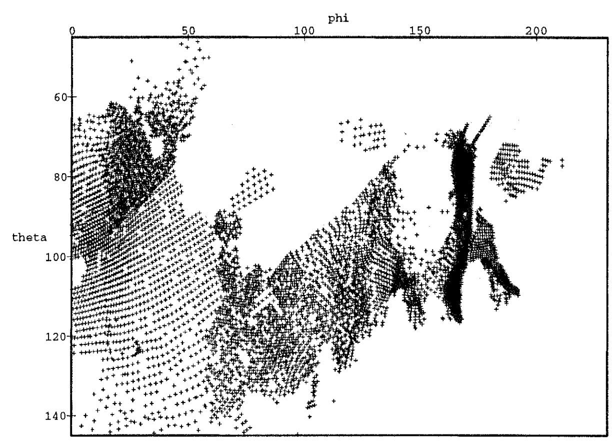

Figure 6: Reference points for IDA example

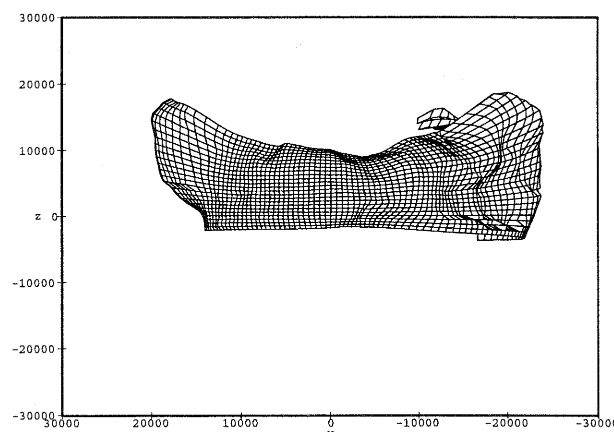

Figure 7: Shape model of asteroid IDA (x-axis: E-W, z-axis: N-S)

The distribution of the reference points is represented in Figure 6, the resulting DTM in Figure 7. The DTM was calculated in 40s (SUN Sparc 20), starting with a 4 $\times$ 4 grid and scale parameter $r = 0.5$ ending after 9 iterations with 5,568 basis points using $r = 1 - 2^{-10}$. The maximum error in the reference points is 900 m, the mean error 91 m. In Figure 8 it can be seen that the maximum errors occur at the boundary of the observed area and in areas

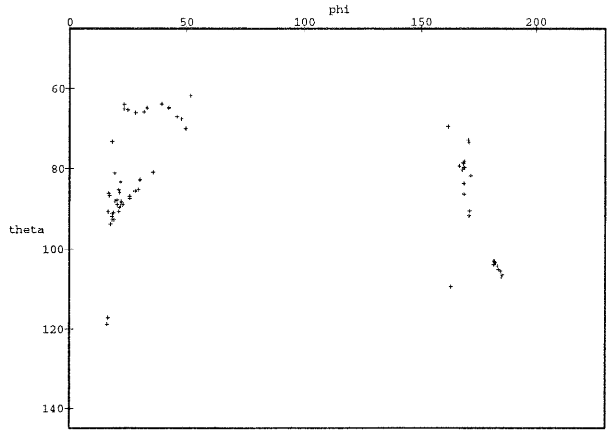

Figure 8: Position of errors > 600 m in the DTM of IDA

where reference points are not available. In these regions it is obvious that the given reference points cannot represent the surface adequately.

5 CONCLUSIONS AND OUTLOOK

In this paper we have presented an efficient hierarchical method for global DTM modelling, which makes use of a sphere as reference surface. It is based on the principle of series expansions and employs locally supported basis functions. The crucial point is the explicit computation of the error on each level which allows the adaption of a regular grid to a given arbitrary set of reference points.

Digital terrain models of Australia and part of the asteroid 243 IDA, demonstrate the power and flexibility of the new approach.

Further extensions concern the usage of reference points with different accuracy, e.g. including control points measured by an operateur. Each reference point can be equipped with an additional weight in the summation (3). In connection with the applications in Geodesy, Geology and Photometry, the calculation of the normal vector to the surface as well as the modelling of local features (e.g. craters) are very interesting (Duxbury, 1991).

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