# 3D-Surface Modelling with Basic Topologic Elements

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**KEY WORDS:** DEM/DTM, Surface, GIS, Triangulation Algorithms, Three-dimensional Surface Modeling

## ABSTRACT

Systems for modelling of surfaces are an indispensible tool in geodesy, photogrammetry, geo-informatics and many other disciplines. Common systems are designed for applications dealing with the surface of the earth. In general, these systems fail in modelling more complex structures, such as parts of the human body or artificial buildings. In this paper an alternate system to model arbitrary surfaces is presented.
The surface is decomposed into basic topological elements: nodes, edges, triangles - and tetrahedrons for bodies. The structure of the surface is determined by the topological relations between these basic elements. These topologic relations have to be deduced from the scattered data-points, what is done by a triangulation of the measured surface-points. The triangulation uses a local order-criterion which utilizes the surface-normals in the data-points. A major element in modelling of surfaces are lines, such as break-lines, contour-lines or boundary-lines. These lines are topological constraints and are incorporated within the triangulation. To gain a better representation and to filter errors of measurement, these lines are approximated by piecewise cubic polynomials in space. The adjustment is done locally by a fully three-dimensional algorithm working on a sub-network.

## KURZFASSUNG

Systeme zur Modellierung von Oberflächen sind ein wichtiges Hilfsmittel in Geodäsie, Photogrammetrie, im Geo-Informationswesen und vielen anderen Bereichen. Die dabei gebräuchlichen Systeme sind auf die Modellierung der Erdoberfläche zugeschnitten. Komplexere Flächenstrukturen (Kunstbauten, menschliche Körperteile, etc.) können damit i.a. nicht modelliert werden. In dieser Arbeit wird ein alternatives Konzept zur Modellierung beliebiger Oberflächen vorgestellt.
Die Fläche wird in topologische Grundelemente zerlegt: Knoten, Kanten, Dreiecke - und Tetraeder für Körper. Die Gesamtstruktur der Fläche wird durch die Nachbarschaftsbeziehungen zwischen diesen Grundelementen beschrieben. Die gemessenen Oberflächenpunkte liegen meist als unstrukturierte Punktwolke vor. Daraus sind die topologischen Beziehungen abzuleiten. Dies erfolgt durch eine Triangulation (=Dreiecksvermaschung) der Datenpunkte. Die Triangulierung verwendet ein lokales Ordnungskriterium, welches die Flächennormalen in den Punkten ausnutzt. Bei der Modellierung von Oberflächen stellen Linien, wie Bruchlinien, Formlinien oder Randlinien, ein zentrales Element dar. Diese Linien werden als topologische Zwangsbedingungen in die Triangulation aufgenommen. Um eine gute Oberflächenmodellierung zu erreichen, und um zufällige Meßfehler zu filtern, werden die Linien durch kubische Splinekurven approximiert. Die Ausgleichung erfolgt lokal in Subnetzen und arbeitet völlig dreidimensional.

## 1. INTRODUCTION

In many parts of science the necessity of a mathematical representation of surfaces occurs. In the field of geo-science this surface often is the surface of the earth. To perform the various operations with the surface, a model of the surface is to be created first. In geo-sciences the so called 'Digital Terrain Models', in short DTM are commonly used: The surface is represented by a regular grid. The heights in this grid, together with algorithms for interpolation, form the model of the surface (Kraus, 1987). Such kind of surface-models is very efficient for many applications. But sometimes problems occur: e.g. the modelling of vertical walls or overhangs is not possible. These problems can only be solved with a new and different way of modelling. In this paper a different approach to avoid these problems is presented.

## 2. BASIC CONCEPT

### 2.1 Requirements

The main requirements for a new concept of surface-modelling are independence of the coordinate-system, as well as independence of the orientation of the surface in space, and the potentiality to model arbitrary surfaces. Resulting from problems with common systems for terrain-modelling, there arises a set of requirements:

- Usage of the original data as a main element in modelling.
- Possibility of dynamical editing of the data - the aim is 'progressive sampling'.
- Local algorithms instead of global ones.
- Smooth surface representation with filtering of random errors.
- Separate smoothing of lines.
- Modelling of thematic attributes.
- Automatic detection of gross errors.
- Automatic reduction of redundant data.

### 2.2 Theoretical model

Common digital terrain models, as well as Geo-Information-Systems, are based on two-dimensional data-models, i.e. the general data-structure and the used algorithms are essentially 2D. The modelling of the surface happens in the ground plane and the object-height is reduced to an attribute of the twodimensional points. Hence the surface is represented in the form

$$z = f(x,y), \quad \text{with } f \text{ generally the object-height.}$$

This approach is called 2.5D and is condemned to fail for arbitrary surfaces.

The modelling of real three-dimensional surfaces needs a 3D-datamodel for the exact geometric and thematic representation. Whereby the meaning of '3D' not only implies the usage of three coordinates, but furthermore complete independence of position and orientation of the surface in space, i.e. independence from the coordinate-system.

This can be achieved by decomposing the surface into simple pieces, thus obtaining high flexibility concerning the shape of the surface. In our approach the surface is described and modelled only with few basic topological elements. These are the commonly used simplices: node, edge, triangle and tetrahedron (Frank, 1986, Molenaar, 1994). These simplices are the basic geometric entities of the respective dimension, see figure 1.

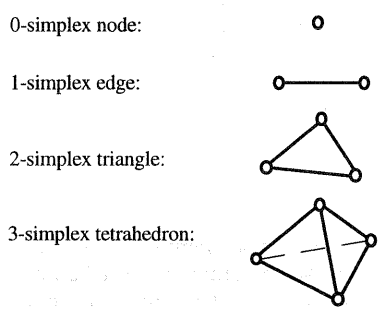


Fig. 1: Simplices of respective dimension.

The adjacency and incidence relations among these basic elements determine the topology of the surface (Neureither, 1992). The simplices together with their topological relations form a skeleton of the surface. But it is not sufficient to describe a surface only with topological relations. The surface also has to be determined geometrically. This geometric determination is done by relating the nodes uniquely to the measured data-points.

The topological structure automatically can be used for mathematical representation of the surface, which can be done, for example, with cubic Bézier-patches - which is not part of this paper (Pfeifer, 1996).

An important element in the modelling of surfaces are lines. These lines are used to model discontinuities of tangent-planes, to exclude regions and for many other tasks. How to include these lines into the basic concept? Lines can be described as an ordered, connected aggregation of nodes and edges, i.e. a formal sum of nodes and edges called 'chain' in Frank, 1986. Lines are topological constraints which have to be kept and preserved in the triangulation.

### 2.3 Discussion

The presented approach has some advantages in comparison to former concepts in surface modelling (refer to the requirements in chapter 2.1):

- The simplices, as well as the topological relations are completely independent of the coordinate-system.
- The topological relations determine the situation of adjacent elements. These neighbourhood-relations can be instantly used for local algorithms.
- The concept is flexible enough for the modelling of arbitray surfaces.
- The concept is general in the sense that it can be used to model lines, surfaces and bodies. Hence different dimensions can be combined.
- The topological relations can be used immediately for the mathematic representation of the surface, e.g. with triangular patches.
- The geometric decomposition of the surface can also be extended to thematic attributes related to the surface, thus to model non-geometric information.

The main disadvantage of the approach is the problem to find the topological relations which are not known a priori. A solution to this problem is presented in the following chapter.

## 3. BUILDING THE TOPOLOGICAL RELATIONS

### 3.1 Triangulation

In general the topological relations between the data-points are not „measured", i.e. no further informations than the coordinates are sampled. The topological relations have to be deduced from an unorganized cloud of points in space. Furthermore they are not uniquely determined, i.e. it depends on someones interpretation which points can be seen as neighbours on the surface and which can not.

A well known method to establish these relations is a triangulation of the data-points (Cline 1984). A triangulation is a partition of the surface into triangles with the data-points as vertices. The triangles mustn´t overlap, nor are holes allowed. There exist many solutions to the triangulation of points in the plane, but only few for triangulation of points of $\Re^3$. Three different approaches for 3D-triangulations can be suggested:

- Tessellation with tetrahedrons and extraction of the desired surface.
- Triangulation of the 3D-points by utilizing additional information or properties of the surface.
- Projection of the problem into $\Re^2$, e.g. triangulation in a plane, such as the ground plane.

A method of the second approach has been presented in Choi, 1988. But this method has a big disadvantage: a distinct point is necessary, from which the whole surface has to be visible. Such an outstanding point can not always be guaranteed. The method presented here is a further development of Choi´s method, without the necessity of the above mentioned point.

The developed method works locally and incrementally - to support progressive sampling. The insertion of a point happens in four steps:

- Locating the triangle the point belongs to.
- Integrating the point within the triangle.
- Optimizing the triangulation.
- Establishing constraints (lines).

### 3.2 Locating the triangle

The first step of inserting a point is to find the correct triangle the point belongs to. This is done by using an order-criterion. Order is a relation, which stands between geometric and topological relation (Schlieder, 1995). In the plane a point lies inside a triangle, if it lies left of every edge of the triangle (assuming the edges are ordered in anticlockwise manner). This left/right-relation is clear in the plane. But in 3D-space no such relation exists between edges and points.

To substitute this order-relation for the edges of the triangulation, Choi uses the given distinct point. In our approach another - more intrinsic - property of the surface is utilized: the surface-normal in the data-points.

With the use of the normal a left/right-criterion can be established: Let $k$ be an edge of the triangulation with the endpoints A and B, with coordinate-vectors $\underline{a}$ and $\underline{b}$. Let $\underline{n}$ be the surface-normal in A, and P, coordinate-vector $\underline{p}$, the point to be inserted. P lies to the right of the edge $\underline{k}$, if

$$d = ((\underline{k} \times \underline{n}), (\underline{p} - \underline{a})) > 0 \text{ (figure 2).}$$

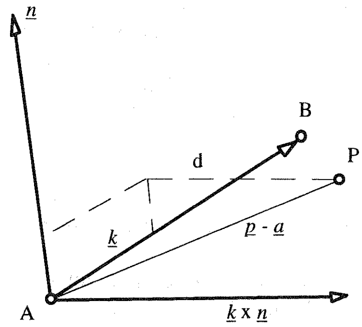


Fig. 2: Order-criterion using the surface-normal.

This order-criterion is repeatedly applied to the edges of the triangulation, until the correct triangle is found.

The surface-normal in a data-point is estimated with the use of the neighbours of the point. A common method is to build the normals on every triangle the point belongs to. The normal in the point results as a weighted sum of these triangle-normals. Whenever the triangulation changes, all concerned normals will be refined. Sometimes the normals are given a priori, in that case these vectors will be used - e.g. in automatic image matching the object-normals easily can be estimated.

Nevertheless, the estimated normals sometimes are only rough approximations of the actual normals what may lead to errors in the locating-step.

If additional information exists - such as break-lines or contour-lines -, they will be utilized for the locating of the triangle, as well as for the estimation of the normals. Finally, it is planned to check the correctness of the integration with regard to blunder detection.

### 3.3 Integration of the point

As soon as the correct triangle has been found, the point will be inserted, i.e. all necessary edges are build. The point can also lie outside the triangulated region. In that case the point has to be connected with the „visible“ part of the boundary.

### 3.4 Optimization

As mentioned above, the topological relations between the data-points are not uniquely determined. Nor is the triangulation of the point-set. One has to formulate a criterion to determine the result of the triangulation. Such criterions are known as optimization-criterions, as the criterion optimizes the triangulation in a specific sense.

Commonly, as in this work, the optimization-criterion is applied edgewise (Cline, 1984 and Choi, 1988): an existing edge of the triangulation will be tested, whether the criterion is hurt or not. If the edge doesn´t match the criterion, it will be swapped in the corresponding quadrilateral - see figure 3.

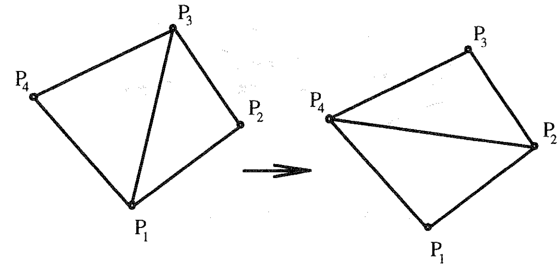


Fig. 3: Swapping of an edge.

The best-known optimizing-criterion is the circle-criterion, which is equivalent to the max-min criterion. This criterion leads to the well-known Delaunay-triangulation. This triangulation is optimal in the sense that it produces regular triangles, near to equilateral triangles. Unfortunately, these algorithms are 2D only.

For our 3D-triangulation several criterions have been implemented and tested:

- '3D-Delaunay' - the minimum angle of two adjacent triangles is maximized.
- 'Smoothness1' - the angle between the planes of two adjacent triangles is maximized, thus to gain a smooth surface.
- 'Smoothness2' - the minimum angle between the planes of the two adjacent triangles and the four neighbour-triangles is maximized. This method was presented in Choi, 1988.
- 'Smoothness3' - a combination of Smoothness1 and Smoothness2.
- 'Greedy' - the sum of the length of the edges is minimized. The greedy-triangulation is a widley used method.
- '2D-Delaunay' - the triangulation is optimized by applying the Delaunay-criterion to the ground-projection of the points. Naturally, all other 2D-criterions could be used also.
- combinations - all above criterions can be combined by a target function to form new criterions.

The optimization-criterions can be - or have to be - extended with further constraints, e.g. conditions to avoid irregular swaps, which would lead to overlapping triangles.

### 3.5 Constraints

For a flexible modelling of surfaces, it should be possible to apply topological constraints on the triangulation. A prominent example are lines, which are included as a chain of edges. These edges must exist within the triangulation. In the last step of the insertion of a point such constraints are introduced.

Figure 4 shows an example for a constraint-edge. The building of this edge works again with the use of the local order-criterion.

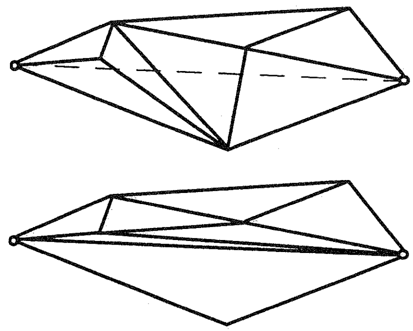

Fig. 4: Building of a constraint edge.

### 3.6 Discussion

The presented method satisfies the requirements noted in chapter 2. The order-criterion works independently of the coordinatesystem and is (nearly) independent of the shape of the surface. The incremental way of building the triangulation supports dynamical editing and progressive sampling. The method is qualified to be implemented only with the use of local algorithms.

Unfortunately there are two problematic aspects of the method, which may lead to errors. The first one is the locality of the order-criterion. This criterion works well in the neighbourhood of an edge and with moderate surfaces. If the surface is bending strongly, the criterion will possibly fail. Figure 5 shows a situation, when the order-criterion says left, but the point should apparently lie right of the edge k.

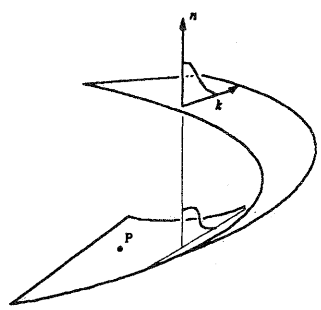


Fig. 5: Wrong result of the order-criterion

Another problem is the estimation of the surface-normals. Especially when only a few points have been inserted already, the surface is badly represented by these points. Hence the normals, estimated with these points, do not correspond sufficiently with the actual normals.

These problems can be solved by applying a verification of the locating-step. This test can be used to detect gross errors of measurement, as well as to expose wrong locatings.

## 4 CORRECTION AND SMOOTHING OF LINE-NETWORKS

Due to the data capturing, points along lines are measured with more or less accuracy. The unfiltered connection of such a sequence of line-points would reproduce the real course of the line just with low or even unacceptable quality. Therefore measured points are to be considered only as noisy ***line-supporting-points (LSPs)***.

By smoothing the course of the LSPs, gross measuring errors can be found and eliminated. The adjustment of LSP-sequences, based on a mathematical well-defined type of curve, enables us to replace the often very extensive measuring data by other suitable data allowing an unique reconstruction of the lines. This circumstance is very advantageous because of the reduction of data-amount.

For the application dealt with, curves consisting of joined cubic polynomials (Spline-curves) are of advantage. In this case the course of the curve is uniquely defined by the chosen type of interpolation (e.g. Osculatory-, Akima-, Spline-interpolation), the type of parametrisation (e.g. chordal, centripetal, equidistant) and the location of the ***(spline-)knots (SKs)*** between the separate polynomials (Forkert, 1994). Very long lines and, furthermore, more or less expanded networks of several lines can occur in practice. Considering manipulation of data and computing time, it is therefore necessary to use such a kind of curve by which only the LSPs within a small surrounding area (around the SK-interval to be calculated) have any influence concerning the course of the curve. This demand also helps to avoid disturbing oscillations of curves due to the position of LSPs laying far away with regard to the part of the curve being calculated at the time. The estimation of the spatial position of the SKs is therefore done based on the Osculatory-interpolation.

The possibility of a correct stochastic interpretation of the smoothed course of the LSPs is guaranteed by applying an adjustment following the method of least squares. The correct adjustment of line networks (free of gaps) is a precondition for modelling surfaces with high quality. Therefore, if there exists a junction or a crossing of several lines, it is not only demanded each line to be smoothed separately, but furthermore that such a ***line-net-knot (LNK)*** itself gets a unique position without any contradiction. (In many cases, a LNK is not captured directly by a measured LSP but has to be calculated by intersection.)

In general, line-networks contain more than one LNK and can get very extensive if there are artificial objects to be reconstructed (like traffic buildings, machine parts or urban areas). Such big line-networks have to be divided - as far as possible - into small 'subnets' in order to be adjusted independently. In consequence, the principle of a strict adjustment of the whole net of lines cannot be followed any longer. This blemish however has nearly no effect if there are arranged overlapping areas wherein the LSPs have to be taken into account for both adjustments whenever two ***subnets (SNs)*** join together. Depending on the type of interpolation used, an overlapping area will occur every time a line had to be cut at one end of a SN. Its extension (number of SK-intervals) depends on the type of interpolation used. Within the overlapping areas the adjusted SKs have to be calculated in a way that no gaps remain in the whole line-network after smoothing is finished. A SN cannot be cut if there exists a further LNK within that overlapping area.

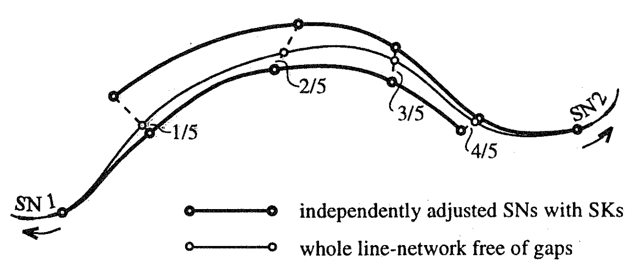


Fig. 6: Overlapping area of two already adjusted SNs to be „sewed" together without gaps.

The adjustment of the SN is done by the program system ORIENT (Kager, 1989), due to its universal possibilities in adjustment matters. The required data (a planar, possibly cyclic SN-graph and the LSP-sequences) have to be extracted (with the help of a 'rover' moved along constraint-edges) from the triangulation for each SN and have to be converted into a structure suitable for ORIENT. Before a SN can be extracted, all those candidates of junctions of constraint-lines have to be detected which are not contained obviously in the captured data and are therefore not triangulated as LNKs until yet (due to gaps stemming from the data capture process).

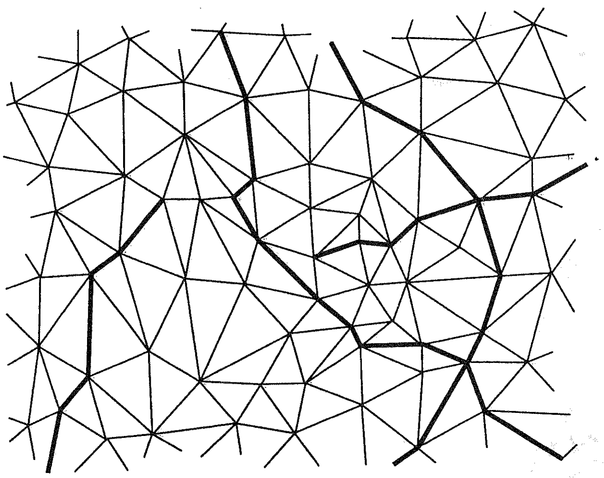

Fig. 7: Lines ending with a junction at another line or crossing another line. Because the junctions have not been captured directly with a LSP, they have to be searched for during the extraction of the actual SN.

Subsequent to each adjustment of a SN with ORIENT, the quality of the adjustment is verified. As criterion of quality the deviations of the LSPs from the corresponding adjusted curves are used. If the deviation from the adjusted curve exeeds three times the root mean square error of the observations at at least one LSP, a further adjustment of the SN with a condensed arrangement of SKs follows. (Until now it is assumed, that blunder-detection of the LSPs has been done before the SNs are adjusted.) This loop of optimisation is repeated until the demanded quality is reached, or a further condensation of SKs becomes impossible.

After the adjustment of a SN, the courses of the original constraint-edges have to be removed from the triangulation and, correspondingly, the adjusted ones have to be triangulated anew. In addition, the user shall have the possibility to judge and edit the results of the adjustment in a graphical way, before a new triangulation of the adjusted SN follows.

## 5 IMPLEMENTATION AND RESULTS

The surface is modelled - as described above - by decomposition into simple objects and the determination of the relations among those. This object-orientated concept imposes an object-orientated implementation. The main attributes of these objects are the adjacency and incidence relations among them. Thereupon bases an important concept of the implementation - the 'rover - concept': Rovers contain references to few data-objects, and perform, under use of the objects relations, local operations on these objects. E.g. a 'triangle-rover' contains references to the three vertices of a triangle and performs operations, such as :

- changing to the neighbouring triangle
- calculating the triangle-normal
- inserting of a point into the triangle
- positioning on the triangle, nearest to a given point

A rover only works locally and always processes few and adjacent data-objects.

The presented concepts were implemented and tested. Especially the various optimization-criterions were examined in regard to their characteristics and properties.

Results of the presented methods are shown in figure 8 and 9.

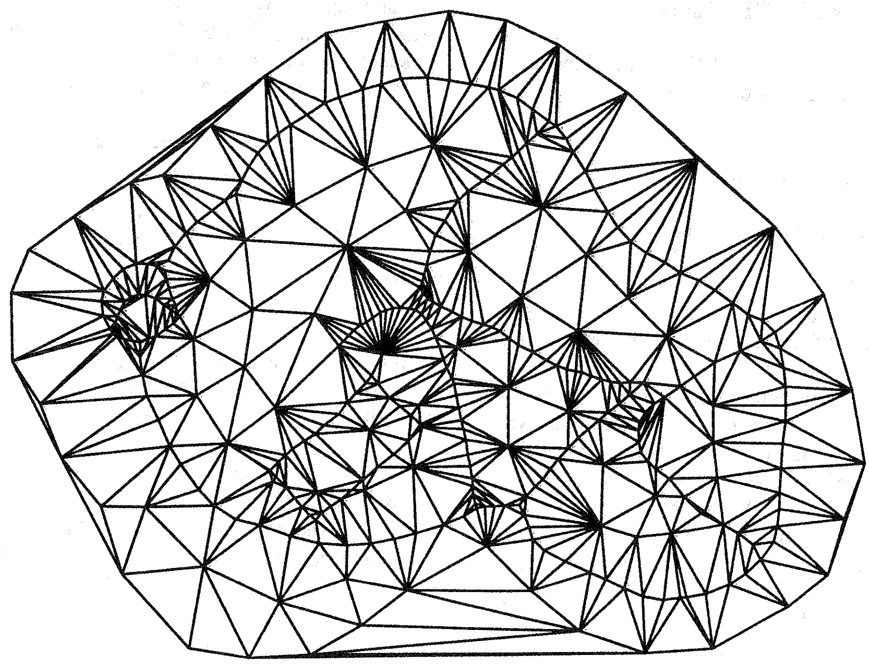

Fig. 8: Triangulation of a sand-pit. About 360 points have been measured. A combination of minimizing the maximum angle and maximizing the angle between two adjacent triangles has been used for optimization.

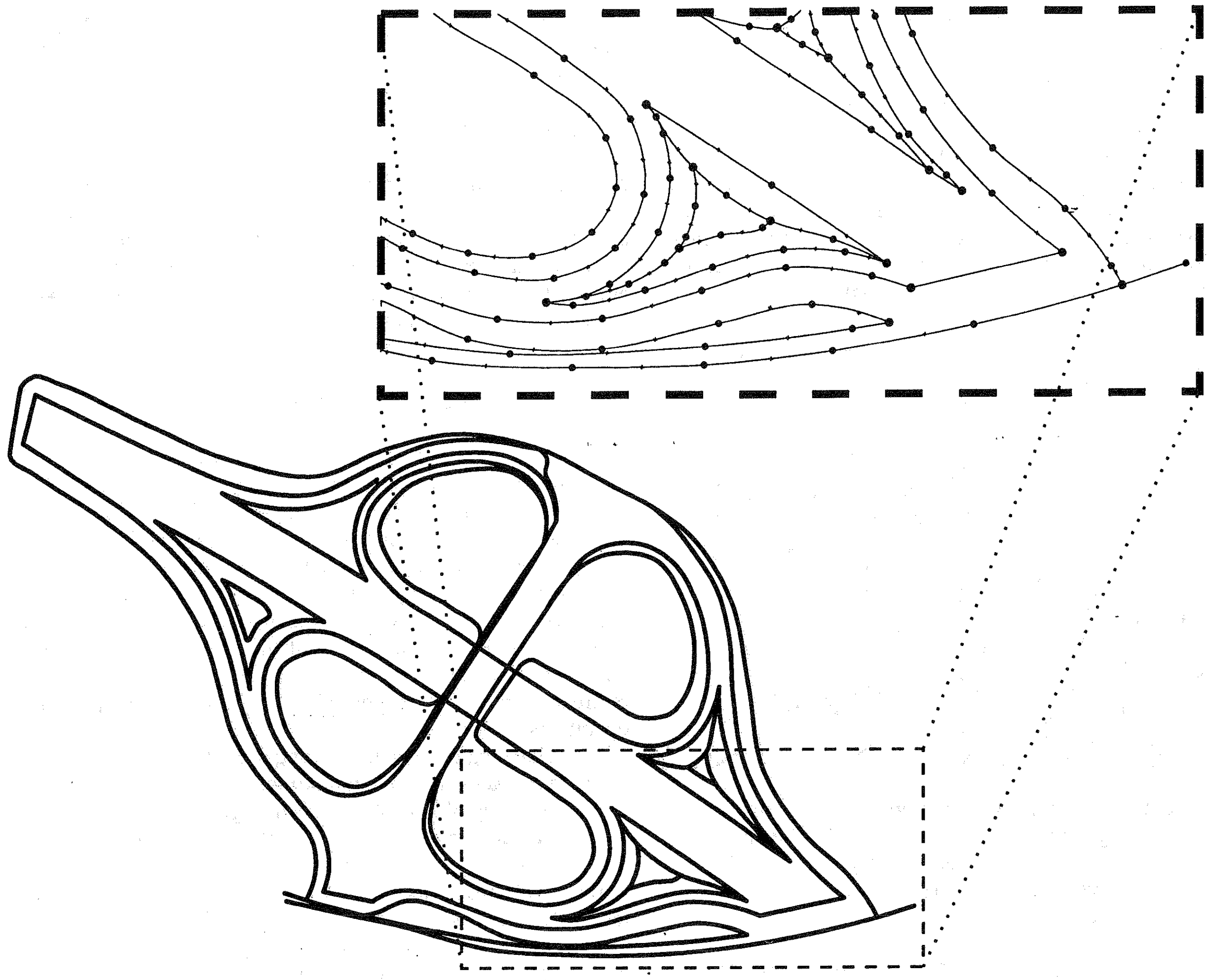

Fig. 9: Drawing of the adjusted line-network of a crossing of two highways with view onto the spread of the LSPs, SKs and LNKs.


## 6 ACKNOWLEDGEMENTS

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## 7 LITERATURE


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