SPATIAL DECISION MAKING BASED ON FUZZY SET METHODOLOGIES

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ABSTRACT

At present, Geographic Information Systems (GIS) though powerful toolboxes, most with hundreds of functions, they suffer from several limitations which render them inefficient tools for spatial decision-making. This paper focuses on the inappropriate logical foundation incorporated into them and examines the necessity of adopting fuzzy logic methodologies in GIS operations.

1 INTRODUCTION

Geographic Information Systems (GIS) are computer-based systems designed to support the capture, management, manipulation, analysis, modeling and display of spatially referenced data at different points in time [Aronoff, 1989]. Today, GISs are widely used in many government, business and private activities; which fall into three major categories [Maguire et al., 1991]: a) socio-economic applications (e.g., urban and regional planning, cadastrial registration, archaeology, natural resources, etc.); b) environmental applications (e.g., forestry, fire and epidemic control, etc.); and c) management applications (e.g., organization of pipeline networks and other services, such as electricity and telephones, real-time navigation for vessels, planes and cars, etc.). The role of GISs in these applications is to provide the users and decision-makers with effective tools for solving the complex and usually ill- or semi-structured spatial problems, while providing an adequate level of performance.

At present, GISs though powerful toolboxes, most with hundreds of functions, they suffer from several limitations which render them inefficient tools for decision-making. It has been widely recognized [Fischer, 1994, Leung et al., 1993] that current commercial systems: a) are based on an inappropriate logical foundation; b) they provide a limited number of built-in analytical and modeling functionalities; and c) their level of intelligence is inadequate.

The discussion in this paper focuses on the first feature. Current GISs are predominantly based on *boolean logic*. The employment of the two-valued mathematical system imposes an artificial precision on inherently imprecise spatial data and processes, and consequently leads to fundamental problems in the representation, manipulation and analysis of spatial information [Fischer, 1994, Leung et al., 1993]. *Fuzzy logic* [Zadeh, 1988] is an alternative logical foundation coming from artificial intelligence (AI) technology with several useful implications for spatial data handling. Contrary to traditional logic, fuzzy logic accommodates the imprecision in information, human cognition, perception and thought. This feature renders it more suitable for dealing with real world problems, because most human reasoning is imprecise. Several researchers dealing with geographic phenomena have examined the new logical foundation in the past and show its advantages over standard logic in spatial data handling [Aubert et al., 1994, Burrough, 1989, Davidson et al., 1994, Kollias et al., 1991, Leung et al., 1993].

Fuzzy logic appears to be instrumental in the design of efficient tools for spatial decision making. This paper examines the necessity of incorporating fuzzy set methodologies into operations available in GIS packages. The discussion is organized as follows. Section 2 provides a brief classification of GIS operations, focusing on data-interpretation operations. Section 3 gives a simplified, though representative, example of a sequence of GIS operations to support the decision making process on a real world problem. The drawbacks of standard logic adopted in these operations and how they may be overcome by adopting the new logical foundation are presented in Sections 4 and 5 respectively. Finally, Section 6 concludes the discussion.

2 CLASSIFICATION OF GIS OPERATIONS

The operations available in geographic information systems (GIS) vary from one system to another. However, their fundamental capabilities can be expressed in terms of four types of operations [Tomlin, 1990]:

- *Programming operations:* They consist of a number of routines in the operating system level, such as supervise and direct the system operations and control the communication with peripheral devices connected to the computer.
- *Data preparation operations:* They encompass a variety of methods for capturing data from different sources (e.g., digital or paper maps, land measurements), processing and storing them appropriately in the database.
- *Data presentation operations:* They encompass a variety of methods for presentation of data, such as drawing maps, drafting charts, generating reports, and so on.
- *Data interpretation operations:* These operations transform data into information and as such they comprise the heart of any geographic information system. Consequently, the discussion that follows focuses on them.

Operations for data interpretation can be viewed as dealing with a hierarchy of data [Samet et al., 1995, Tomlin, 1990]. At the highest level, there is a library of *maps* (more commonly referred to as *layers*), all of which are in registration

(i.e., they have a common coordinate system). Each layer is partitioned into *zones* (regions), where the zones are sets of locations with a common *attribute value*. Examples of layers are the land-use layer, which is divided into land-use zones (e.g., wetland, river, desert, city, park and agricultural zones) and the road network layer, which contains the roads that pass through the portion of space that is covered by the layer.

Data interpretation operations available in GISs characterize [Aronoff, 1989, Berry, 1987, Samet et al., 1995, Tomlin, 1990]:

- *individual locations*,
- *locations within neighborhoods*, and
- *locations within zones*,

and constitute respectively the three classes of operations, i.e., *local*, *focal* and *zonal* operations. Notice that all data interpretation is done in a layer-by-layer basis. That is, each operation accepts one or more existing layers as input (the operants) and generates a new layer as output (the product).

The first class of data-interpreting operations (local operations) includes those that compute a new value for each location on a layer as a function of existing data explicitly associated with that location. The data to be processed by these operations may include the zonal values associated with each location on one or more layers. *Local operations* include:

- *Search operations*, i.e., retrieval of information associated with individual locations on a layer.
- *Classification and recoding operations*, i.e., assignment of new attribute values to individual locations on a layer.
- *Generalization operations*, i.e., reduction of detail associated with individual locations on a layer.
- *Overlay operations*, i.e., assignment of new attribute values to individual locations resulting from the combination of two or more layers.

Focal operations compute new values for every location as a function of its *neighborhood*. A neighborhood is defined as any set of one or more locations that bear a specified distance and/or directional relationship to a particular location, the *neighborhood focus*. *Focal operations* include:

- *Search operations*, i.e., retrieval of information characterizing the immediate or extended vicinity (the region of interest) of individual locations on a layer.
- *Proximity operations*, i.e., assignment of new attribute values to individual locations on a layer, which depict their distance or direction in a neighborhood with respect to the neighborhood focus.
- *Interpolation operations*, i.e., assignment of new attribute values to individual locations on a layer derived by averaging sets of two or more target values associated to selected locations in their immediate or extended vicinity.
- *Surfacial operations*, i.e., assignment of new attribute values to individual locations on a layer indicating their surfacial characteristics (slope, aspect, volume, etc.).
- *Connectivity operations*, i.e., assignment of new attribute values to individual locations on a layer derived from a running total of the results being retained in a quantitative or qualitative step-by-step fashion and considering the values associated to locations in the immediate or extended vicinity (optimum path finding, etc.).

The third and final class of data-interpreting operations (zonal operations) includes those that compute a new value for each location as a function of existing values associated with a zone containing that location. *Zonal operations* include:

- *Search operations*, i.e., retrieval of information characterizing individual locations on a layer that coincide with the zones of another layer.
- *Measurement operations*, i.e., assignment of new attribute values to individual locations on a layer that correspond to a measurement (e.g., area, length) characterizing their zones.

3 SITE SELECTION BASED ON A SEQUENCE OF GIS OPERATIONS

The purpose of this section is to present a sequence of data-interpretation operations which may compose one or more procedures[1] to accomplish the task of *site selection* for a *residential housing development*. The basic approach to this is to create a set of *constraints*, which restrict the planned activity, and a set of *opportunities*, which are conducive to the activity. The combination of the two is considered in order to find the best locations.

In the simplified situation that follows the set of constraints and opportunities consists of[2]:

- vacant area (i.e., no development),
- dry land,
- level and smooth site (e.g., slope $< 10\%$),
- nearness to the existing road network, and
- south-facing slope.

In addition all candidate sites should have an adequate size to satisfy the needs of the planning activity (e.g., between 1 and 1.5 sq km).

The whole task requires as input three layers of the region under examination:

- *hypsography* layer: the three-dimensional surface of the region (altitude values),
- *development* layer: it depicts the existing infrastructure of the region (e.g., roads, buildings, etc.), and
- *moisture* layer: it depicts the soil moisture of the region (e.g., lakes, wet-lands, dry-lands, etc.).

The procedure of site selection, based on the sets of constraints and opportunities determined above, may consist of the following sequence of operations[3]:

[1] A *procedure* is any finite sequence of one or more operations that are applied to meaningful data with deliberate intent [Tomlin, 1990].

[2] a wider set could be taken into account, but this subset is enough to illustrate some basic data-interpreting operations available in GISs.

[3] the syntax adopted for the operations is:
new-layer = Operation-class(operation-subclass) of *existing-layer* and ...

1. *Vacant areas:* A new layer of vacant areas is produced from the layer of development by classifying, generalizing and finally performing a selective search on the result.
    - *development-classes* = Local(classification) of *development*
    - *vacant-developed* = Local(generalization) of *development-classes*
    - *vacant* = Local(search) of *vacant-developed*
2. *Dry lands:* A new layer of dry lands is produced from the layer of moisture by classifying, reducing detail and performing a selective search on the result.
    - *moisture-classes* = Local(classification) of *moisture*
    - *dry-wet* = Local(generalization) of *moisture-classes*
    - *dry* = Local(search) of *dry-wet*
3. *Level sites:* A new layer of level and smooth sites is produced from the layer of hypsography by computing, classifying, generalizing and finally performing a selective search on the result.
    - *slope* = Focal(surfacial) of *hypsography*
    - *slope-classes* = Local(classification) of *slope*
    - *level-steep* = Local(generalization) of *slope-classes*
    - *level* = Local(search) of *level-steep*
4. *Accessible areas:* A new layer of accessible sites by the existing road network is produced implicitly from the layer of development by highlighting the road network, computing, classifying, generalizing and finally performing a selective search on the proximities.
    - *roads* = Local(search) of *development*
    - *road-proximity* = Focal(Proximity) of *roads*
    - *road-proximity-classes* = Local(classification) of *road-proximity*
    - *accessible-inaccessible*= Local(generalization) of *road-proximity-classes*
    - *accessible* = Local(search) of *accessible-inaccessible*
5. *South-facing areas:* A new layer of south-facing areas is produced from the layer of hypsography by computing, classifying, generalizing and finally performing a selective search on the aspects.
    - *aspect* = Focal(surfacial) of *hypsography*
    - *aspect-classes* = Local(classification) of *aspect*
    - *south-north* = Local(generalization) of *aspect-classes*
    - *south* = Local(search) of *south-north*
6. *Good-sites:* A new layer of sites that satisfy the set of constraints and opportunities is produced by the successive overlay of layers produced in the previous steps. Finally, good sites are highlighted by performing a selective search on the result.
    - *vacant-dry* = Local(overlay) of *vacant* and *dry*
    - *vacant-dry-level* = Local(overlay) of *level* and *vacant-dry*
    - *vacant-dry-level-accessible* = Local(overlay) of *accessible* and *vacant-dry-level*
    - *vacant-dry-level-accessible-south* = Local(overlay) of *south* and *vacant-dry-level-accessible*
    - *good-sites* = Local(search) of *vacant-dry-level-accessible-south*
7. *Candidate sites:* A new layer of sites that satisfy the set of constraints and opportunities and have adequate size is produced from the layer of good sites by measuring the sizes of zones and highlighting those that are within the predefined size interval.
    - *good-sites-size* = Zonal(measurement) of *good-sites*
    - *candidate-sites* = Local(search) of *good-sites-size*

4 SPATIAL DECISION MAKING AND BOOLEAN LOGIC

The potential of operations available in current GIS packages is heavily restrained from the standard logical foundation incorporated into them. It is argued [Leung et al., 1993] that the employment of *boolean logic* (the all-or-nothing system) in GIS design causes the following problems: a) it imposes artificial precision on intrinsically imprecise information, graded spatial phenomena and processes, b) it fails to determine and communicate to users the extent of imprecision and error, c) it is inappropriate to model human cognition, perception and thought processes, which are generally embedded with imprecision, d) it is inadequate to model natural languages, which are imprecise in nature.

Several of these impediments are originated from the standard logical foundation incorporated into data-interpreting operations available in GIS packages. Following the classification of Section 2, GIS operations fall into three categories according to their intent:

- *Computational operations:* they compute and assign new attribute values to individual locations based on a mathematical model (i.e., overlay, proximity, connectivity and measurement operations).
- *Retrieval operations:* they perform a selective search on analyzed data (i.e., search operations).
- *Auxiliary operations:* they process further analyzed data in order to facilitate the retrieval of desired information (i.e., classification and generalization operations).

The logical foundation adopted in the design of a GIS package is tightly interwoven with the last two categories of operations (i.e., retrieval and auxiliary operations). Currently, the linkages between the spatial entities (i.e., individual locations on a layer) and their non-spatial attributes are based on the membership concept of classical set theory, that is, an entity either has an attribute entirely or does not have it at all. No third situation is allowed. Hence, the selective search is intended to provide as a result the set of individual locations

whose attribute values satisfy absolutely a constraint posed by decision makers. In addition, decision makers are obliged to express their constraints through arithmetical terms and mathematical symbols (e.g., slope $< 10\%$), since they are not allowed to use natural language lexical terms (e.g., level land). Finally, there in no factor available for the ordering of qualified locations derived from a sequence of GIS operations.

These problems caused by the standard logical foundation can also be distinguished by examining the simplified, though representative, approach (process) to spatial decision making in a real world problem, as given in the previous section. Specifically, the employment of a sequence of standard GIS operations to support the residential site selection is accompanied with all problems of an "early and sharp classification". First, the overall decision is made in steps which *drastically* and *sharply* reduce the intermediate results. Any constraint is accompanied with an absolute *threshold value* and no exception is allowed. For instance, if the threshold for a level land is slope $= 10\%$, a location with slope equal to 9.9% is characterized as level, while a second location with slope equal to 10.1% is characterized as steep. Moreover, for decisions based on multiple criteria, it is usually the case an entity (i.e., an individual location) which satisfies quite well the majority of constraints and is marginally rejected in one of them to be selected as valid by decision makers. However, based on boolean logic, a location with slope 10.1% will be rejected, even if it satisfies quite well all other constraints posed by decision makers. Finally, the effect of classical set theory is that the selection result is *flat* in the sense that there is no overall ordering of the valid entities as regard to the degree they fulfill the set of constraints. For instance, *dry-level* layer (Section 3) highlights all locations which satisfy the constraints: dry land and level ground, however there is no distinction between a location with moisture $= 10\%$ and slope $= 3\%$ and another with moisture $= 20\%$ and slope $= 7\%$.

These impediments, and all those stated in literature [Aubert et al., 1994, Burrough, 1989, Davidson et al., 1994, Kollias et al., 1991, Leung et al., 1993], call for a more general and sound logical foundation for GISs as offered by the concept of *fuzzy logic*.

5 SPATIAL DECISION MAKING AND FUZZY LOGIC

Fuzzy logic methodologies [Zadeh, 1988] may provide a scheme for the representation and manipulation of the uncertainty which is related to the classification of individual locations according to their attribute values. Instead of numerical values real world entities and measurements are assigned *lexical values*. For instance, "the site is *far away* from the highway". This statement has *uncertainty* features. The uncertainty is related to the perception of distance between the site and the road network. The perception of distance may be formed by the objective distance measurement to the nearest highway (e.g., 20 km) and the perceptual and cognitive background of the observer. The concept of the uncertainty represents the degree to which an object belongs to a set. This measure is referred to as *degree of belief*[4] (d.o.b.) [Gupta et al., 1988, Zadeh, 1968]. The d.o.b. is usually normalized in the interval [0,1], termed as *fuzzy domain*.

Lexical values assigned to physical entities correspond to a range of physical values (e.g., *far away* $\Rightarrow$ distance $\in$ [15 km, ∞)). The transformation of physical values to *fuzzy values* (i.e., values in the fuzzy domain) is accomplished through the employment of *transformation functions* of the form $f : R \rightarrow [0, 1]$.

This procedure is called *fuzzification* and fuzzy values are measures of the d.o.b. that the corresponding physical value belongs to the set denoted by the lexical value.

An important issue for decision making is reasoning based on lexical values assigned to physical entities. According to the scheme proposed in [Vazirgiannis et al., 1994] a set of lexical values should be assumed to classify entities and measurements in categories. Each lexical value corresponds to a range of physical values, while transformation functions are provided to map physical values to fuzzy values. There is one transformation function associated to each lexical value. Hence the number of transformation functions is equal to the number of lexical values assumed. Several transformation functions are exploited [Gupta et al., 1988]. For the purposes of this study, the following simple linear transformation functions are assumed:

- *Linear increasing:* It is used in the cases where a straight-forward mapping of physical values to the fuzzy domain is needed. The linear increasing function is represented by the equation:

$$LI(x) = \frac{x - c_0}{c_1 - c_0} \quad , \quad \forall x \in [c_0, c_1]$$

- *Linear decreasing:* It is represented by the equation:

$$LD(x) = \frac{x - c_0}{c_0 - c_1} + 1 \quad , \quad \forall x \in [c_0, c_1]$$

- *Triangle:* The set of physical values is divided into k parts: $[c_0, c_1]$, $[c_1, c_2]$, $\ldots$, $[c_{k-1}, c_k]$. The transformation function of the physical values to the fuzzy domain are:

$$TR_1(x) = \frac{x - c_0}{c_0 - c_1} + 1 \quad , \quad \forall x \in [c_0, c_1]$$

$$TR_2(x) = \frac{2(x - c_i)}{c_{i+1} - c_i} \quad , \quad \forall x \in [c_i, \frac{c_i + c_{i+1}}{2}]$$

$$TR_2(x) = \frac{2(c_i - x)}{c_{i+1} - c_i} + 1 \quad , \quad \forall x \in [\frac{c_i + c_{i+1}}{2}, c_{i+1}]$$

$$TR_3(x) = \frac{x - c_0}{c_1 - c_0} \quad , \quad \forall x \in [c_{k-1}, c_k]$$

Consider the classification of individual locations on a layer based on the slope values of the ground (physical values). Four lexical values are used: [*level, gentle, moderate, steep*]. The transformation functions are linear decreasing and increasing for the first and last lexical values respectively, and triangle for the rest of them. Figure 1 illustrates the conventional (Fig.1a) and fuzzy classification (Fig.1b) of slope values. Notice that the conventional way to classify slope involves discrete classes with specific ranges, while fuzzy classification captures the gradual transition between classes (lexical values), providing a better way to categorize imprecise concepts such as gentle and steep land. Based on the fuzzy

[4] also referred to as *grade of membership*

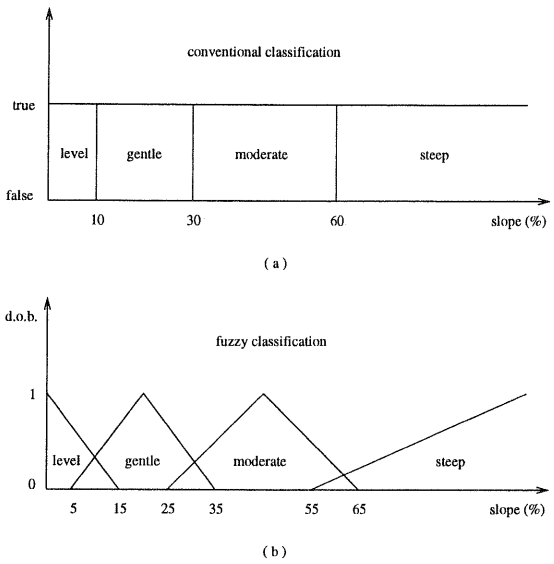

Figure 1: An example of the conventional and fuzzy classification of ground slope.

classification a location with slope 6% is assigned a d.o.b. of 0.6 for level, 0.1 for gentle, 0 for moderate, and 0 for steep.

Individual locations of the region under study may by classified in a similar way based on the rest of criteria posed by decision makers. For the constraints of the residential site selection example (Section 3), the following lexical values could be considered:

- *development*: [vacant, semi-developed, developed]
- *soil moisture*: [dry, moderate, wet, water]
- *ground slope*: [level, gentle, moderate, steep]
- *nearness to highways*: [close, near, moderate, far, far away]
- *aspect*: [north, east, south, west]

For decision criteria which combine more than one layer and lexical value (e.g., *level* ground and *dry* land) an overall measure should be computed and assigned to individual locations. This measure is derived from the consideration of d.o.bs on two or more layers. For a fuzzy set $A \in X$ with d.o.b. $\mu_A(x) \in [0,1]$, the overall measure can be provided by an *energy function*, which is given by the following formula [Gupta et al., 1988]:

$$e(A) = \sum_{x \in X} E[\mu_A(x)] \ , \quad where \ \ E : \mu_A[0,1] \rightarrow [0,1]$$

One such function commonly used is:

$$e(A) = \sum_{x \in X} \mu_A^q(x)$$

where q a positive integer[5].

For instance, if there is a requirement to highlight the most level and dry locations of the region under study the overall measure (energy function) is given by:

$$e(level\_and\_dry) = \mu_{dry}^2(x) + \mu_{level}^2(x)$$

for each individual location x.

Reasoning based on lexical values involves the local operations of *classification*, *overlay* and *search* and fuzzy logic methodologies should be incorporated into them as follows:

- *Fuzzy classification operations*, i.e., assignment of the d.o.b. for a lexical value to individual locations on a layer. The d.o.b. is derived by applying the appropriate transformation function.
- *Fuzzy overlay operations*, i.e., computation and assignment of an overall measure[6] to each individual location, which is derived from the consideration of d.o.bs on two or more layers. The overall measure is also expressed in the fuzzy domain [0,1].
- *Fuzzy search operations*, i.e., retrieval of information based on a pre-defined threshold value for the overall measure (d.o.bs) assigned to individual locations on a layer.

The procedure of residential site selection, based on a set of constraints expressed in lexical terms, i.e., *vacant* area, *dry* soil, *level* land, *near* to highway, and *south* aspect, may consist of the following set of operations[7]:

- $vacant_{(d.o.bs)}$ = Local(fuzzy classification) of *development*
- $dry_{(d.o.bs)}$ = Local(fuzzy classification) of *moisture*
- $level_{(d.o.bs)}$ = Local(fuzzy classification) of *slope*
- $near_{(d.o.bs)}$ = Local(fuzzy classification) of *road-proximity*
- $south_{(d.o.bs)}$ = Local(fuzzy classification) of *aspect*
- $good\text{-}sites_{(d.o.bs)}$ = Local(fuzzy overlay) of $vacant_{(d.o.bs)}$ and $dry_{(d.o.bs)}$ and $level_{(d.o.bs)}$ and $near_{(d.o.bs)}$ and $south_{(d.o.bs)}$
- *best-sites* = Local(fuzzy search) of $good\text{-}sites_{(d.o.bs)}$

Obviously, in this scheme, contrary to traditional logic, reasoning is based on a "late and flexible classification", and consequently the problems presented in the previous section are overcome.

6 CONCLUSION

Fuzzy logic methodologies appear to be instrumental in the design of efficient tools for spatial decision-making. The contribution of the paper can be summarized as follows:

- After a brief classification of operations available in current GIS packages, it is shown how a sequence of them may compose one or more procedures to support the spatial decision-making process.

[5] using this equation (e.g., for $q = 2$: *quadratic measure*), big weight values (d.o.bs) are amplified, while small values are nearly eliminated.

[6] The notion of the measure of information is a well-established concept in communication theory and is based on the probabilistic approach. Two metrics that are used extensively for measuring the ambiguity in cognitive information are: the *energy metric* and the *entropy metric* [Gupta et al., 1988].

[7] the pointer *d.o.bs* characterizes layers whose individual locations are assigned the d.o.bs for a lexical value; this lexical value is identical to the name of the layer.

- The drawbacks of standard logic adopted in GIS operations are highlighted.
- The advantages of fuzzy set theory over classical set theory and the necessity of incorporating fuzzy logic methodologies into GIS operations are presented.

Future research includes:

- The design of a prototype spatial decision support system based on fuzzy logic methodologies.
- The selection of appropriate transformation functions and overall measures (information metrics) for the set of constraints posed by decision-makers and problem specific features in a real-world situation (e.g., residential site selection).

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