THE GPS SINGLE POINT POSITIONING: A DATA PROCESSING PROGRAM FOR TUTORIAL PURPOSES

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ABSTRACT. A data processing program for the GPS single point positioning technique is presented in this paper. The program, which takes its data from the RINEX navigation and observation file, is written in FORTRAN 77 for PC and is divided into two modules. Module 1 computes the satellite's coordinates in the ECEF system and the satellite's clock offset at the measurement epoch; module 2 computes the receiver's antenna coordinates from code pseudorange measurements. The authors have carried out some preliminary testing of the program and have compared the results with those obtained from carrier phase measurements using a commercial software package. From a didactic point of view, the program will be of considerable help to students wishing to understand GPS data processing.

RÉSUMÉ. Cet article présente un programme de traitement de données pour la technique GPS de positionnement par point unique. Ce programme qui recueille ses données du fichier d'observation et de navigation RINEX est écrit en FORTRAN 77 pour PC. Il comporte deux modules. Le module 1 qui calcule les coordonnées du satellite en ECEF et les écarts entre les horloges du satellite et du récepteur à l'époque de mesure, et le module 2 qui calcule les coordonnées de l'antenne du récepteur d'après la pseudodistance. Les auteurs ont effectué des tests préliminaires du programme et ont comparé les résultats à ceux obtenus en calculant la phase de la porteuse avec un progiciel commercialisé. Du point de vue didactique, ce programme sera extrêmement utile aux étudiants souhaitant comprendre comment sont traitées les données GPS.

INTRODUCTION

The GPS technique is by now an established part of university level topography and geodesy courses.
In this context, the development of didactic software packages (software tutorials), designed to give a gradual approach to the complex issues involved in data processing, is of great interest.
This paper presents a data processing program for GPS single point positioning. This program was developed within the context of the topography courses taught by the Engineering Faculty. The results obtained can be easily compared with those of any commercially available package.
The program not only allows direct interaction with the Rinex format data, but also teaches systematic application of the algorithm which underlies the solution of the GPS single point positioning problem.

1. THE MATHEMATICAL MODEL

GPS single point positioning is a technique for positioning the GPS user receiver both statically and dynamically (Figure 1).

1.1. Code pseudorange

With reference to figure (2), the code ranges are determined in the delay look loop by using the code correlation technique (Hoffmann-Wellenhof et al., 1992).

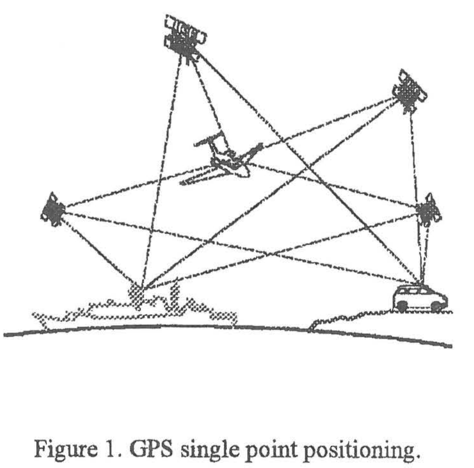


Figure 1. GPS single point positioning.

The difference between the clock readings is equivalent to the time shift Δt which aligns the satellite and reference signal during the code correlation procedure. Thus (see Figure 3):

$$\Delta t = T_R - t^s \tag{1}$$

Let $T_R = T_{GPS} - \tau$ and $t^s = t^{GPS} - \delta$, where τ is the receiver clock offset and δ is the satellite clock offset. We have:

$$\Delta t = T_{GPS} - \tau - \left(t^{GPS} - \delta\right) \tag{2}$$

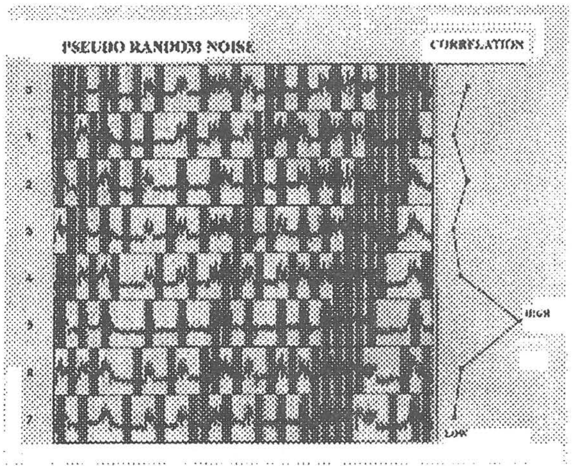


Figure 2. The code correlation technique.

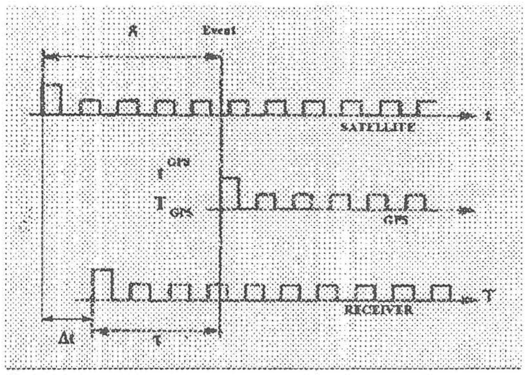


Figure 3. The clock offset.

Multiplying the terms of the equation (2) by the speed of light (299792458 m/sec), we get:

$$c\Delta t = c\left(T_{GPS} - t^{GPS}\right) - c\left(\tau - \delta\right) \qquad (3)$$

where $c\Delta t$ denotes the pseudorange measurement from the RINEX Observation File, $c\left(T_{GPS} - t^{GPS}\right)$ the true distance between the position of the satellite and the receiver's antenna and $c(\tau - \delta)$ the range bias.
Assuming that the δ bias correction is applied by a polynomial model whose coefficients are transmitted by the navigation message at the measurement epochs, we have:

$$\rho = \sqrt{\left(X_J - X^S\right)^2 + \left(Y_J - Y^S\right)^2 + \left(Z_J - Z^S\right)^2} - c\tau \qquad (4)$$

Equation (4) is the mathematical model for single point positioning, where ρ is the pseudorange measurement adjusted for satellite clock bias; $\underline{X} = \left[X_J; Y_J; Z_J; \tau\right]^T$ is the unknown coordinate and receiver clock offset vector; $\underline{X}^s = \left[X^S; Y^S; Z^S\right]^T$ is the known satellite coordinate vector.

1.2 Linear mathematical model

Assuming the approximate vector $\underline{X_0} = \left[X_{J,0}; Y_{J,0}; Z_{J,0}; 1\right]^T$, we can expand expression (4) into a Taylor series in the approximate coordinates:

$$\rho = \rho^0 + \left(\frac{\partial\rho}{\partial X_J}\right)_0 x_J + \left(\frac{\partial\rho}{\partial Y_J}\right)_0 y_J + \left(\frac{\partial\rho}{\partial Z_J}\right)_0 z_J - \left(\frac{\partial\rho}{\partial c\tau}\right)_0 \tau \qquad (5)$$

where $\underline{x} = \left[x_J; y_J; z_J; \tau\right]^T$ is the vector of unknown corrections and ρ^0 is the approximate distance between the satellite and the receiver.

1.3 Least squares principle

The linear observation model resulting from equation (5) can be written in Gauss-Markov notation as:

$$A\,\underline{x} - \underline{f} = \underline{v} \qquad (6)$$

where:
A is the design matrix;
f is the vector of observations;
v is the vector of residuals.
If the number of satellites is greater than four, then the least squares solution is:

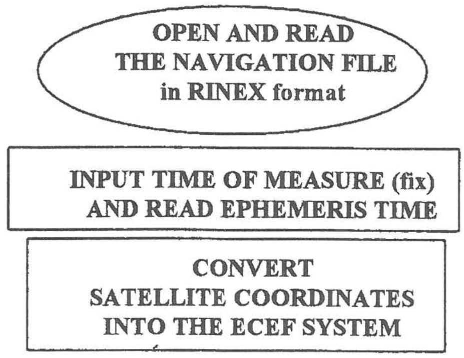
$$\hat{\underline{x}} = \left(A^T A\right)^{-1} A^T \underline{f} \qquad (7)$$

where the matrix weight P is equal to the unitary matrix I (Crocetto et al, 1997).

2. DATA PROCESSING PROGRAM

The data processing program, written in FORTRAN 77 for PC, is made of two principal modules.
a) Module 1: Computes the satellite positions;
b) Module 2. Computes the single point position.

Module 1 reads the data from the RINEX navigation file (Gurtner and Mader, 1990) and computes the instantaneous satellite position at the epoch of the measurements.



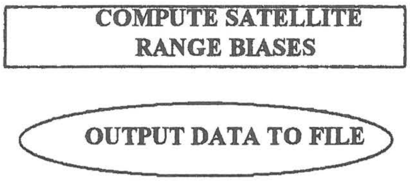


Figure 4. Flow chart of module 1.

The formulas used are the well known satellite orbit formulae (Leick, 1990). We give the structure (table 1) of the input data file for module 1:

PRN Satellite, Data, Time	a0	a1	a2
	Crs	ΔN	M0
Cuc	ecc	Cus	√A
t0	Cic	Ω0	Cis
i0	Crc	Ω	dΩ/dt
di/dt			

Table 1. Structure of module 1 input data file.

The elements of table 1 are defined as follows:

a0, a1, a2	Bias satellite clock coefficients polynomial
Crs	Correction terms to orbital radius
Δn	Mean motion difference
M0	Mean anomaly
Cuc	Latitude argument correction terms
ecc	Eccentricity
Cus	Latitude argument correction terms
√A	√ of semi-major axis
t0	Ephemeris reference time
Cic	Inclination correction terms
Ω0	Right ascension
Cis	Inclination correction terms
i0	Inclination
Crc	Orbital radius correction terms
Ω	Perigee argument
dΩ/dt	Rate of right ascension
di/dt	Rate of inclination

Table 2. Satellite Ephemeris.

Tab. 3 gives an example of the data output by module 1. X, Y and Z and δ are the coordinates of the satellite in ECEF and its clock offset, computed at epoch 12:00:00.

PRN	X(m)	Y(m)	Z(m)	δ (10^{-6}
29	8587925.943	13673631.113	212282271.15	2.777
4	21099881.848	-6950490.502	14718647.435	52.434

Table 3. Satellite coordinates in ECEF and clock offset at 12:00:00.

The module 2 reads the data from the RINEX observation file (Gurtner and Mader, 1990) and solves the normal linear system. It outputs the single point position at the epoch of measurement.

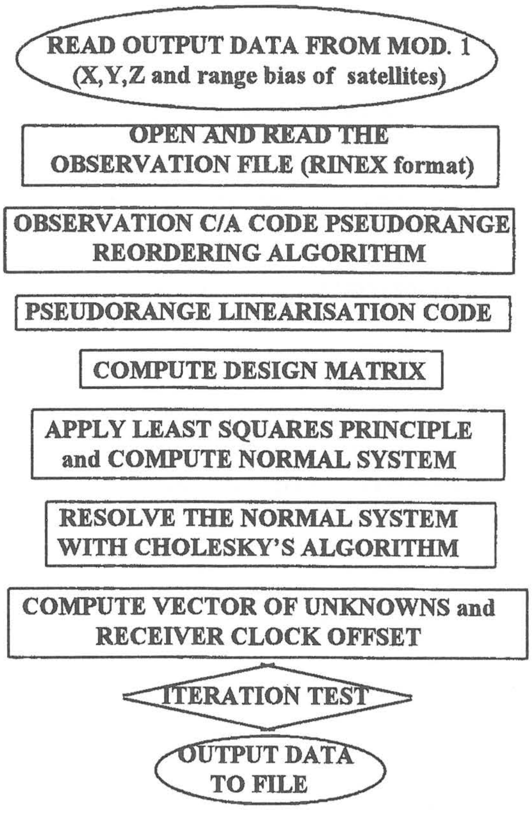


Figure 5. Flow chart of module 2.

Tab. 4 gives the structure of the input data file and Tab.5 shows an example.

DATA, TIME measurement	NUMBERS SATELLITE and PRN	PSEUDORANGE C/A Code (m)

Table 4. Structure of the input data (module 2).

95 11 10 11 45 0.0000000 8 19 18 24 04 29 14 27 22
22377410.77004 -5772225.10104 -4497835.27001
0.00000 22377413.93401
20075560.12808 -876226.09108 -682772.18105
0.00000 20075561.18105
22819260.02303 -4831168.66803 -3764544.69301
0.00000 22819264.36201
21022627.88607 -1104200.94507 -860415.55204
0.00000 21022628.59504
21145483.55307 2514171.97407 1959095.59704
0.00000 21145483.76804
22649551.98204 4116887.63104 3207962.07402
0.00000 22649554.24102
24871073.10403 -335253.26403 -261234.94701
0.00000 24871076.41001

24477661.64412 -0.23712 0.88010
0.00000 24477661.64410

Table 5. Example of input data from the RINEX Observation file (input data in boldface)

3. DATA PROCESSING

As a test, the measurements referring to the Poggiorenatico GPS benchmark were processed.. The computation (geocentric coordinates of receiver's antenna and clock receiver bias) is summarised in tables 6-a, 6-b and 6-c, for different satellite configurations and epochs of measurement.

Number of satellites	X (m)	Y (m)	Z (m)	Bias (cτ) (m)
4	4445208.991	903255.082	4468566.027	559.107
5	4445535.121	903312.547	4468603.141	681.619
6	4445523.753	903324.552	4468664.676	718.951

Table 6-a. Point position at epoch 11:30:00.

Number of satellites	X (m)	Y (m)	Z (m)	Bias (cτ) (m)
4	4445426.597	903301.577	4468531.826	46.038
5	4445493.460	903329.835	4468550.582	98.226
6	4445477.583	903338.244	4468592.006	120.280
7	4445490.482	903347.330	4468620.927	144.193
8	4445520.829	903344.366	4468632.026	166.329

Table 6-b. Point position at epoch 11:45:00.

Number of satellites	X (m)	Y (m)	Z (m)	Bias (cτ) (m)
4	4445208.991	903205.921	4468437.505	-168.504
5	4445444.108	903324.754	4468513.220	24.228
6	4445405.186	903328.364	4468549.574	34.010
7	4445413.435	90335.452	4468569.570	50.311
8	4445522.102	903340.145	4468613.192	133.628

Table 6-c. Point position at epoch 12:00:00.

4. PRELIMINARY TEST

The coordinates of the Poggiorenatico bench-mark obtained with the program have been compared with the values obtained with carrier phase measurements using the GPSurvey program (Gatti, 1996).
The differences (ΔX, ΔY, ΔZ) are listed in tables 7-a, 7-b and 7-c for different satellite configurations and epochs of measurement.

Number of satellites	ΔX (m)	ΔY (m)	ΔZ (m)
4	-296.914	-8.086	-31.093
5	29.216	49.379	6.020
6	17.848	61.384	67.555

Table 7-a. Satellite configuration at epoch 11:30:00.

Number of satellites	ΔX (m)	ΔY (m)	ΔZ (m)
4	-79.3075	38.409	-65.295
5	-12.444	66.667	-46.539
6	-28.321	75.076	-5.115
7	-15.422	84.162	23.806
8	14.924	81.198	34.905

Table 7-b. Satellite configuration at epoch 11:45:00.

Numbers satellite	ΔX (m)	ΔY (m)	ΔZ (m)
4	-296..914	-57.247	-159.626
5	-61.796	61.586	-83.901
6	-100.718	65.196	-47.547
7	-92.469	72.829	-27.551
8	16.197	76.977	16.071

Table 7-c. Satellite configuration at epoch 12:00:00.

CONCLUSION

This purely didactic project had two aims:

- exemplification of the mathematical model;
- the development and structuring of the program itself.

It was thus our aim to tackle the complex issues of GPS data processing not only theoretically but also practically.
The complexity of computational models which combine various algorithms is by now such as to make practical application an essential part of the process of comprehension. From this point of view the project was extremely successful, as confirmed by the teachers and students themselves, and will certainly find its application in the teaching of other data processing techniques.

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