ADVANCED KALMAN FILTER TECHNIQUES FOR THE DYNAMIC SURVEY IN DIGITAL PHOTOGRAMMETRY

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ABSTRACT

This paper deals with the problem of external orientation of a mobile imaging sensor system. To this purpose an original method is suggested, using exclusively the points of an already existing numerical cartography seen on the images. The method is an alternative to that currently applied, based on the integration of GPS and INS facilities.
To solve the image orientation in a dynamic way, the so-called "wave algorithm" (Wang, Schaffrin and Salychev, 1995), an advanced Kalman filter technique is applied; it belongs to the class of the extended dynamic linear models, the so-called "look ahead filters".
The analytical model used considers the coplanarity equations, to solve the relative orientation of the sequential images, and the collinearity equations, to solve the external orientation of the sensor system. The motion of the vehicle is described by state equations defined by means of cubic spline functions applied to a set of points touched by the ruote and acquired from the digital mapping. It is then possible to orient externally the images using exclusively photogrammetric observations and a wheel counter that measures the length of the course in function of image time acquisition.
Using the wave algorithm makes it possible, in the space-time interval between two series of collinearity observations, to compensate systematic deviations locally defined by the state equation model.

1. STATIC VERSUS DYNAMIC MODELS

Any measurement process sequentially performed in some time intervals can be treated and adjusted either by a static or by a dynamic analytical model.
For instance, a photogrammetric survey is an image acquisition sequence where a lot of images are taken, in general, in different places and at different times. Since a biunivocal relationship exists between any image and its time of acquisition, the global orientation problem can be solved in a:
- static way: when the images are treated together without considering acquisition time;
- dynamic way: when the acquisition time is considered in the adjustment process by applying, for instance, the Kalman filter technique, widely used for the prediction of unknown quantities varying in time.

1.1. Static Models

In photogrammetry, two well-known linear models in the unknown image orientation parameters $\mathbf{x}$ are applied:

1) GAUSS-MARKOV LINEAR MODEL
- for observation equations: $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{v}$

2) MIXED LINEAR MODEL
- for observation equations: $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{s} + \mathbf{v}$
- stochastic prior information for $\mathbf{s}$: $\tilde{\mathbf{s}} = \boldsymbol{\mu}_s + \mathbf{e}_s$

In any case two kinds of solution can be obtained:

1) POST PROCESSING SOLUTION
By a Global Least Squares Adjustment (Dermanis, 1990c), leading to:
- the Best Linear Estimation BLUUE (Dermanis, 1990b) $\hat{\mathbf{x}}$ of $\mathbf{x}$;
- the Best Linear Prediction BLUUP (Dermanis, 1990a) $\tilde{\mathbf{s}}$ of $\mathbf{s}$ for the mixed linear model.

Considering the complete sequence of images together, a huge quantity of data must be handled.

2) "REAL TIME" PROCESSING SOLUTION
By a Sequential Least Squares Adjustment (Crosilla, 1989), leading to:
- a real time solution, when only new observations are introduced;
- an "almost real time" solution, when new unknown parameters are introduced.

Unfortunately the latter is typical of a dynamic survey where new images introduce new unknown orientation parameters. Also this kind of solution is not efficient from the computational point of view because of the huge quantity of data to be handled.

1.2. Dynamic Models

Three different linear models can be considered, with increasing level of complexity, where the unknown $\mathbf{x}_t$ varies in time.

1) DYNAMIC LINEAR MODEL

Is defined by three kinds of equations:
- observation equations: $\mathbf{b}_t = \mathbf{A}_t\mathbf{x}_t + \mathbf{v}_t$
- stochastic constraints, the so-called "state equations": $\mathbf{x}_t = \Phi_{t-1}\mathbf{x}_{t-1} + \boldsymbol{\mu}_t$
- stochastic prior information for $\mathbf{x}_{t-1}$: $\tilde{\mathbf{x}}_{t-1} = \mathbf{x}_{t-1} + \mathbf{e}^0_{t-1}$

With this model a real time processing solution can be obtained by a Least Squares Adjustment, in particular:
- the Best Linear Prediction BLUUP $\tilde{\mathbf{v}}_t, \tilde{\boldsymbol{\mu}}_t, \tilde{\mathbf{e}}^0_{t-1}$ of $\mathbf{v}_t, \boldsymbol{\mu}_t, \mathbf{e}^0_{t-1}$;

- the Kalman Filter Prediction $\tilde{x}_t$ of x_t.

2) EXTENDED DYNAMIC LINEAR MODEL
(so-called "look ahead filters", Schaffrin, 1994)

Unlike the dynamic linear model, two sequential observation equations are concerned; the model is given for the times t=1 and t=2:

- observation equations: $b_1 = A_1 x_1 + v_1$, $b_2 = A_2 x_2 + v_2$
- state equations: $x_1 = \Phi_0 x_0 + \mu_1$, $x_2 = \Phi_1 x_1 + \mu_2$
- stochastic prior information: $\tilde{x}_0 = x_0 + e_0^0$

An "almost real time" processing solution can be obtained by a Least Squares Adjustment:

- the Best Linear Prediction BLUUP $\tilde{v}_1, \tilde{v}_2, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{e}_0^0$ of $v_1, v_2, \mu_1, \mu_2, e_0^0$;
- the Kalman Filter Prediction $\tilde{\tilde{x}}_1, \tilde{x}_2$ of x_1, x_2.

The following condition is always satisfied (Schaffrin, 1994):

$$\mathrm{MSPE}\{\tilde{\tilde{x}}_1\} = Q_{\tilde{\tilde{x}}_1} \le \mathrm{MSPE}\{\tilde{x}_1\} = Q_{\tilde{x}_1}$$

3) WAVE ALGORITHM (Wang, Schaffrin and Salychev, 1995)

This model belongs to the previous class of look ahead filters where a time cycle of measurements is considered together, with the peculiarity that the noise term μ_t in the state equation is always equal to zero, except for the last time. In this way μ_t is not stochastic but deterministic and it must be estimated not predicted.

The great advantage of a dynamic linear model is that the computational effort is much smaller than the static solution since only "few" observations are processed.
These models will be treated in detail in chapters 4, 6, 7; the observation equations (ch. 2) and the state equations (ch. 3) for a dynamic photogrammetric survey by a "mobile mapping system" (MMS) will be now considered.

2. DYNAMIC SENSOR ORIENTATION FOR A PHOTOGRAMMETRIC SURVEY BY DIGITAL MAP POINTS

The dynamic terrestrial survey is a methodology performed with a mobile unit equipped with suitable sensors which allow 3D-positioning of the object. Its peculiarity is that the unit motion is determined exclusively by INS systems and/or GPS receivers set on the mobile unit.
An MMS is constituted by a van equipped with:

- a differential kinematic GPS receiver (rover station);
- a INS strapdown system (3 accelerometers and 3 ring laser gyros, RLG);
- a cluster of 2÷8 CCD cameras (and VHS cameras);
- a workstation for the data control/processing.

The analytical model used is the following:

1. Application of a Kalman filter technique for the MMS motion definition;
2. Determination of the position and the attitude of the digital sensor system stiffly fixed to the mobile unit;
3. 3D ablosolute positioning of the objects acquired by the oriented imaging sensors.

From the geometrical point of view, to solve the 3D object positioning, the following model (El-Sheimy, Schwarz and Gravel, 1995) can be applied (see fig. 1):

$$r_i = r_{GPS/INS}(t) + R_{GPS/INS}(t)\left[S_i R^{GPS/INS}_{camera} r_i^{camera} + a^{GPS/INS}_{camera}\right] \quad (1)$$

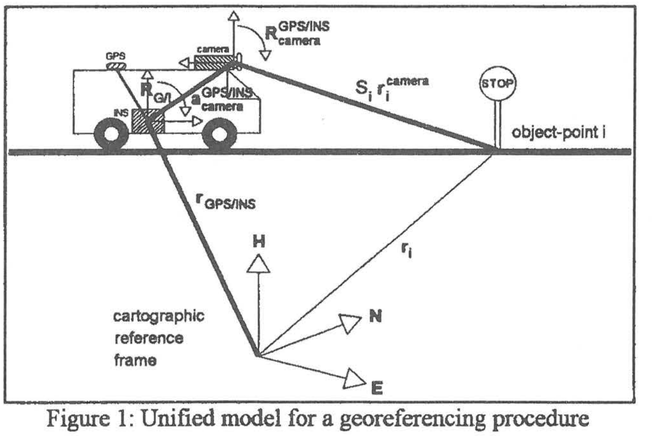


Figure 1: Unified model for a georeferencing procedure

where:

r_i is the coordinate vector of the i-th object-point in the cartographic reference frame;

$r_{GPS/INS}(t)$ is the coordinate vector of the GPS receiver/INS system in the cartographic frame (measured by GPS/INS);

$R_{GPS/INS}(t)$ is the rotation matrix between the GPS/INS body frame and the cartographic frame (measured by the RLG);

S_i is the scale factor for the i-th object-point;

$R^{GPS/INS}_{camera}$ is the rotation matrix between the camera reference frame and the GPS/INS frame (known by the MMS calibration);

r_i^{camera} is the image coordinate vector of the i-th object-point;

$a^{GPS/INS}_{camera}$ is the offset vector between the camera frame and the GPS/INS frame (known by the MMS calibration).

This paper proposes the alternative model, given by formula (2), to solve the object positioning by a photogrammetric procedure (see fig. 2):

$$r_i = r_{known\ point} + r^{known\ point}_{camera} + S_i R_{camera} r_i^{camera} \quad (2)$$

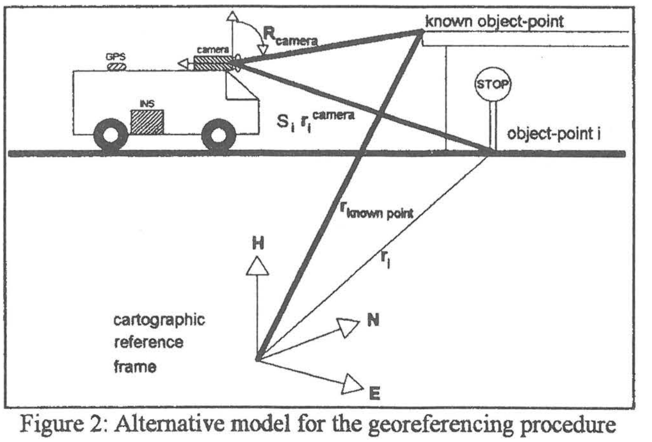
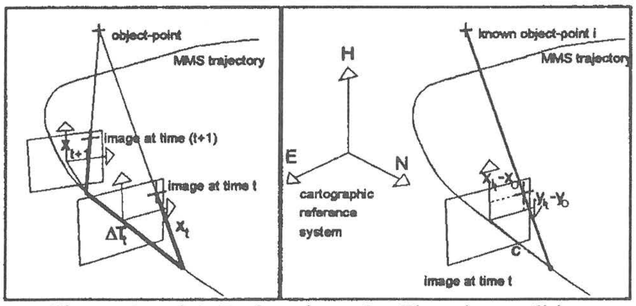


Figure 2: Alternative model for the georeferencing procedure

The coordinates $\mathbf{r}_{\text{known point}}$ of some points, extracted from a digital map, are used to compute the external orientation of the cameras ($\mathbf{r}_{\text{camera}}$ and $\mathbf{R}_{\text{camera}}$). The coordinates $\mathbf{r}_i$ of the i-th object-point, as for the model (1), are computed by photogrammetric intersection.

2.1. Photogrammetric coplanarity condition



Figures 3 and 4: Coplanarity and collinearity condition

Considering the vectors (see fig. 3):

$\mathbf{x}_t = [x_t \quad y_t \quad 1]^T$ normalized image coordinates at time t,

$\mathbf{x}_{t+1} = [x_{t+1} \quad y_{t+1} \quad 1]^T$ normalized image coordinates at time (t+1),

$\Delta\mathbf{T}_t = [E_{t+1} - E_t \quad N_{t+1} - N_t \quad H_{t+1} - H_t]^T$ relative translation between t and (t+1),

the coplanarity condition:

$$\mathbf{x}_{t+1}^T [\Delta\mathbf{T}_t \wedge (\Delta\mathbf{R}_t \mathbf{x}_t)] = 0$$

exists for these vectors and must be satisfied for the relative orientation of two images. After some positions and passages (Crosilla and Visintini, 1997), this condition can be written, in compact form as:

$$\chi \mathbf{q} = 0 \tag{3}$$

where:

χ is a Nx9 matrix whose rows type are:

$$\chi_i = [x_{i_{t+1}} x_{i_t} \quad x_{i_{t+1}} y_{i_t} \quad x_{i_{t+1}} \quad y_{i_{t+1}} x_{i_t} \quad y_{i_{t+1}} y_{i_t} \quad y_{i_{t+1}} \quad x_{i_t} \quad y_{i_t} \quad 1]$$

$\mathbf{q}$ is a 9-row vector obtained by stacking the columns of $\mathbf{Q}_t$ on top of each other, where:

$$\mathbf{Q}_t = \begin{bmatrix} 0 & -\Delta H_t & \Delta N_t \\ \Delta H_t & 0 & -\Delta E_t \\ -\Delta N_t & \Delta E_t & 0 \end{bmatrix} \begin{bmatrix} r_{11_{t+1}} & r_{12_{t+1}} & r_{13_{t+1}} \\ r_{21_{t+1}} & r_{22_{t+1}} & r_{23_{t+1}} \\ r_{31_{t+1}} & r_{32_{t+1}} & r_{33_{t+1}} \end{bmatrix} \begin{bmatrix} r_{11_t} & r_{21_t} & r_{31_t} \\ r_{12_t} & r_{22_t} & r_{32_t} \\ r_{13_t} & r_{23_t} & r_{33_t} \end{bmatrix}$$

Vector $\mathbf{q}$ contains the external orientation parameters of the images at different times.

2.2. Photogrammetric collinearity condition

Considering fig. 4, the collinearity equations for any time t can be written as:

$$x_{i_t} - x_0 = -c \frac{r_{11_t}(E_i - E_t) + r_{12_t}(N_i - N_t) + r_{13_t}(H_i - H_t)}{r_{31_t}(E_i - E_t) + r_{32_t}(N_i - N_t) + r_{33_t}(H_i - H_t)} + \Delta x_i$$

$$(4)$$

$$y_{i_t} - y_0 = -c \frac{r_{21_t}(E_i - E_t) + r_{22_t}(N_i - N_t) + r_{23_t}(H_i - H_t)}{r_{31_t}(E_i - E_t) + r_{32_t}(N_i - N_t) + r_{33_t}(H_i - H_t)} + \Delta y_i$$

with the well-known meaning of the different terms.

In photogrammetry, the coplanarity condition (3) is applied to create the stereo models while the collinearity condition (4) is used to define a unique datum by fixing the coordinates of some points (so-called "control points").

Both analytical problems are solved in static way by using a linear estimation process of the external orientation parameters and a linear prediction process of the cartographic control point coordinates (Crosilla and Visintini, 1996). Furthermore, the coordinates of the control points are obtained by different survey methods (e.g. by GPS measurements).

This paper proposes a method to determine absolute orientation of the image in a dynamic way by a Kalman filter and by using digital map point coordinates as for formula (2).

3. DEFINITION OF THE STATE EQUATIONS BY USING CUBIC SPLINE FUNCTIONS

In general the transition matrix $\mathbf{\Phi}_t$ for the state equations of a dynamic model is defined by physical-mechanical relationships (e.g. the law of motion in kinematic problems).

In the case of dynamic orientation, the unknown quantities are assumed as a 6-row vector $\mathbf{x}_t$ of image external orientation parameters:

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{T}_t \\ \mathbf{\Omega}_t \end{bmatrix} \tag{5}$$

where:

$\mathbf{T}_t = [E_t \quad N_t \quad H_t]^T$ is the image position vector at time t;

$\mathbf{\Omega}_t = [\omega_t \quad \phi_t \quad k_t]^T$ is the image direction vector at time t.

The meaning of the time t is not intended in the physical sense, but instead like an ordinal succession of image acquisition epochs, independently from the time effectively passed for two sequential image acquisitions.

The quantity $\mathbf{x}_t$ is a geometric quantity and, consequently, the transition matrix $\mathbf{\Phi}_t$ can be defined by geometric relationships extracted from a digital map. If a digital map is not available, a traditional map can also be used after scanning and georeferencing processes.

In a first step, n point coordinates are acquired with a mouse from a digital map, along the route covered by the MMS.

The interpolation of these points by using a cubic spline function can be performed; in this way the equations of the "approximate trajectory" are obtained (see fig. 5).

A number (n-1) of polynomials of kind (6) can be written (Rogers and Adams, 1990):

$$\mathbf{P}_k(\tau) = [F_{1k}(\tau) \quad F_{2k}(\tau) \quad F_{3k}(\tau) \quad F_{4k}(\tau)] \begin{bmatrix} \mathbf{P}_k \\ \mathbf{P}_{k+1} \\ \mathbf{P}_k' \\ \mathbf{P}_{k+1}' \end{bmatrix} \tag{6}$$

with $1 \le k \le n-1$ and $\mathbf{P}_k(\tau) \in C^2$

where:

$\tau = s/s_{k+1}$ is the normalized abscissa ($0 \le \tau \le 1$);

s is the curvilinear abscissa measured with a wheel counter;

s_{k+1} is the chord approximation among the (k+1) points;

$F_{1k}(\tau) = 2\tau^3 - 3\tau^2 + 1$ $\qquad$ $F_{2k}(\tau) = -2\tau^3 + 3\tau^2$

$F_{3k}(\tau) = \tau(\tau^2 - 2\tau + 1)s_{k+1}$ $\qquad$ $F_{4k}(\tau) = \tau(\tau^2 - \tau)s_{k+1}$

P_k, P_{k+1} are the position vector of the k-th and (k+1)-th acquired points;

P_k', P_{k+1}' are the angular tangent vector to the trajectory at the same points.

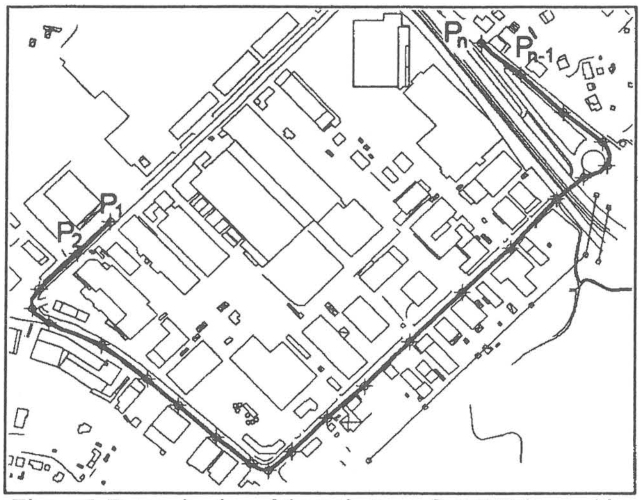


Figure 5: Determination of the trajectory of a MMS by a cubic spline applied to points acquired from a digital map

The values of P_{k+1} are computed by using the continuity tangent condition at any point k common to two following segments. In this case only the values of P_1' and P_n' are required (Crosilla and Visintini, 1997).
Since the digital sensor is fixed to the vehicle, the angular tangent P_k' corresponds to the shot direction for frontal image acquisition. Otherwise for other geometric configurations of acquisition, three known angles $\Theta_\omega, \Theta_\phi, \Theta_k$ of the matrix $R_{camera}^{GPS/INS}$ reported in formula (1) exist between the shot direction and P_k'.
As important result, P_k' is always related to the image direction Ω_t for any value of k. Consequently, to determine P_1' and P_n' it is sufficient that P_1 and P_n correspond to shot points of images oriented by digital map points; that is the first and the last image of the sequence must be externally oriented.
The state equations for E_t, N_t, H_t of (5) can be immediately computed considering the components of $P_k(\tau)$ in (6) along the East, North and Height directions: in a first step, for any segment k, in a second step, for any image acquired at different times t within the segment k, that is, according to the relation $\tau = s/s_{k+1}$, for any measured abscissa s.
The state equations are:

$$E_{t+1} = E_t + \Delta E_t + \mu_{E_{t+1}} = \left(1 + \frac{\Delta E_t}{E_t}\right) E_t + \mu_{E_{t+1}}$$

$$N_{t+1} = N_t + \Delta N_t + \mu_{N_{t+1}} = \left(1 + \frac{\Delta N_t}{N_t}\right) N_t + \mu_{N_{t+1}}$$

$$H_{t+1} = H_t + \Delta H_t + \mu_{H_{t+1}} = \left(1 + \frac{\Delta H_t}{H_t}\right) H_t + \mu_{H_{t+1}}$$

More complex is the computation of the state equations for ω_t, ϕ_t, k_t of (5) since the angles α_t (azimuth) and ψ_t (of height) contained in P_k' are defined with respect to the cartographic reference system (Euler angles).

Since a geometric relationship exists between these angles and the Cardano rotation angles ω_t, ϕ_t, used in photogrammetry, these angles can be indirectly derived from the definition of $P_k'(s) = B_{2k} + 2B_{3k}s + 3B_{4k}s^2$, obtaining:

$$\omega_t = \operatorname{arctg}\left(\frac{P_{N_t}'}{P_{H_t}'}\right) + \Theta_\omega$$

$$\phi_t = \operatorname{arctg}\left(\frac{P_{E_t}'}{\sqrt{(P_{N_t}')^2 + (P_{H_t}')^2}}\right) + \Theta_\phi$$

The increments $\Delta\omega_t, \Delta\phi_t$ and the resulting terms N_{ω_t}, N_{ϕ_t} in the matrix Φ_t are numerically derived, because they cannot be expressed with trigonometric relations.
For the angle k_t, considering a negligible rolling of the MMS:

$$k_{t+1} = k_t + \mu_{k_{t+1}}$$

Summarizing, the state equations for $E_t, N_t, H_t, \omega_t, \phi_t, k_t$ are:

$$x_{t+1} = \Phi_t x_t + \mu_{t+1} \tag{7}$$

where:

$$\Phi_t = \begin{bmatrix} 1+\frac{\Delta E_t}{E_t} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\frac{\Delta N_t}{N_t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{\Delta H_t}{H_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{\omega_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{\phi_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mu_{t+1} = \begin{bmatrix} \mu_{E_{t+1}} \\ \mu_{N_{t+1}} \\ \mu_{H_{t+1}} \\ \mu_{\omega_{t+1}} \\ \mu_{\phi_{t+1}} \\ \mu_{k_{t+1}} \end{bmatrix}$$

4. DYNAMIC LINEAR MODEL

As already mentioned, the model consists of:

- observation equations: $b_1 = A_1 x_1 + v_1 \qquad v_1 \sim (0, \Sigma_1)$
- state equations: $x_1 = \Phi_0 x_0 + \mu_1 \qquad \mu_1 \sim (0, \Theta_1)$
- prior information: $\tilde{x}_0 = x_0 + e_0^0 \qquad e_0^0 \sim (0, \Sigma_0^0)$

Combining the equations, it follows that:

$$b_1 = A_1 \Phi_0 \tilde{x}_0 - A_1 \Phi_0 e_0^0 + A_1 \mu_1 + v_1$$

The constrained least squares adjustment leads to the homBLIP of v_1, μ_1, e_0^0:

$$\begin{bmatrix} \tilde{v}_1 \\ \tilde{\mu}_1 \\ \tilde{e}_0^0 \end{bmatrix} = \begin{bmatrix} \Sigma_1 \\ \Theta_1 \\ -\Sigma_0^0 \Phi_0^T A_1^T \end{bmatrix} \left[\Sigma_1 + A_1\left(\Theta_1 + \Phi_0 \Sigma_0^0 \Phi_0^T\right) A_1^T\right]^{-1} \left(b_1 - A_1 \Phi_0 \tilde{x}_0\right)$$

Replacing these values in the state equations, the inhomBLIP $\tilde{x}_1$ of x_1 can be obatained:

$$\tilde{x}_1 = \hat{x}_1 + K_1 z_1 \tag{8}$$

where:

$\hat{x}_1 = \Phi_0 \tilde{x}_0$ is the homBLIP based on prior information only;

$Q_{\hat{x}_1} = \Theta_1 + \Phi_0 \Sigma_0^0 \Phi_0^T$ is the covarance matrix of $\hat{x}_1$;

$K_1 = Q_{\hat{x}_1} A_1^T \left[\Sigma_1 + A_1 Q_{\hat{x}_1} A_1^T\right]^{-1}$ is the Kalman gain matrix;

$z_1 = b_1 - A_1 \hat{x}_1$ is the pseudoinnovations vector.

The covariance matrix of $\tilde{x}_1$ is:

$$\mathrm{MSPE}\{\tilde{x}_1\} = Q_{\tilde{x}_1} = \left[I - K_1 A_1\right] Q_{\hat{x}_1} = \Sigma_1^0$$

5. DYNAMIC LINEAR MODEL FOR THE SENSOR ORIENTATION

The model consists of:

- coplanarity equations (3) (relative orientation):
$$\chi_t q_t = 0$$
- collinearity equations (4) (absolute orientation):
$$y_t = -c \frac{g_1(E_i, N_i, H_i, E_t, N_t, H_t, \omega_t, \phi_t, k_t)}{g_2(E_i, N_i, H_i, E_t, N_t, H_t, \omega_t, \phi_t, k_t)}$$
- state equations (7) (by cubic spline functions):
$$x_{t+1} = \Phi_t x_t + \mu_{t+1}$$
- prior information (best prediction obtained by the process until time t):
$$\bar{x}_t = \tilde{x}_t$$

1) For small displacements the state equations can be substituted by their differentials (Soatto, Frezza, Perona, 1996):

$$\Delta x_{t+1} = x_{t+1} - \bar{x}_{t+1} = F(\bar{x}_t)\Delta x_t + \mu_{t+1}$$

where $F(\bar{x}_t)$ is the jacobian matrix.
Since the state equations are linear in x_t, it follows that:

$$F(\bar{x}_t) = \left(\frac{\partial}{\partial x_t} \Phi_t x_t\right)_{|\bar{x}_t} = \Phi_t$$

2) The observation equations are substituted by their linearized forms:

$$\chi_{t+1} q_{t+1} = h\{x_{t+1}, y_{1_{t+1}}\} \cong h\{\bar{x}_{t+1}, b_{1_{t+1}}\} + C_{1_{t+1}} \Delta x_{t+1} + D_{1_{t+1}} v_{1_{t+1}} = 0$$

$$y_{2_{t+1}} = -c\frac{g_1(\dots)}{g_2(\dots)} = g\{x_{t+1}, X_{t+1}\} \cong g\{\bar{x}_{t+1}, Y_{t+1}\} + C_{2_{t+1}} \Delta x_{t+1} + E_{2_{t+1}} s_{t+1} + v_{2_{t+1}}$$

where:

$b_{1_{t+1}} = y_{1_{t+1}} + v_{1_{t+1}}$ coplanarity observation vector at time t;
$b_{2_{t+1}} = y_{2_{t+1}} + v_{2_{t+1}}$ coplanarity observation vector at time (t+1);
$Y_{t+1} = X_{t+1} + s_{t+1}$ vector of cartographic point coordinates;

$$C_{1_{t+1}} = \left(\frac{\partial h}{\partial x}\right)_{|\bar{x}_{t+1}, b_{1_{t+1}}} \qquad D_{1_{t+1}} = \left(\frac{\partial h}{\partial b_1}\right)_{|\bar{x}_{t+1}, b_{1_{t+1}}}$$

$$C_{2_{t+1}} = \left(\frac{\partial g}{\partial x}\right)_{|\bar{x}_{t+1}, Y_{t+1}} \qquad E_{2_{t+1}} = \left(\frac{\partial g}{\partial Y}\right)_{|\bar{x}_{t+1}, Y_{t+1}}$$

The linearized model is:

$$\begin{aligned}
&\Delta x_{t+1} = \Phi_0 \Delta x_t + \mu_{t+1} \\
&h\{\bar{x}_{t+1}, b_{1_{t+1}}\} = -C_{1_{t+1}} \Delta x_{t+1} - D_{1_{t+1}} v_{1_{t+1}} \\
&y_{2_{t+1}} - g\{\bar{x}_{t+1}, Y_{t+1}\} = C_{2_{t+1}} \Delta x_{t+1} + E_{2_{t+1}} s_{t+1} + v_{2_{t+1}}
\end{aligned} \tag{9}$$

Setting:

$$b_{t+1} = \begin{bmatrix} b_{1_{t+1}} \\ b_{2_{t+1}} \end{bmatrix} = \begin{bmatrix} h\{\bar{x}_{t+1}, y_{1_{t+1}}\} \\ y_{2_{t+1}} - g\{\bar{x}_{t+1}, Y_{t+1}\} \end{bmatrix} \qquad v_{t+1} = \begin{bmatrix} v_{1_{t+1}} \\ v_{2_{t+1}} \end{bmatrix}$$

$$C_{t+1} = \begin{bmatrix} -C_{1_{t+1}} \\ C_{2_{t+1}} \end{bmatrix} \qquad E_{t+1} = \begin{bmatrix} 0 \\ E_{2_{t+1}} \end{bmatrix} \qquad D_{t+1} = \begin{bmatrix} -D_{1_{t+1}} & 0 \\ 0 & I \end{bmatrix}$$

the observation equations can be written as:

$$b_{t+1} = C_{t+1} \Delta x_{t+1} + E_{t+1} s_{t+1} + D_{t+1} v_{t+1} \tag{10}$$

According to the equation (8) the generic inhomBLIP of x_1 now becomes:

$$\Delta \tilde{x}_{t+1} = \Delta \hat{x}_{t+1} + K_{t+1} z_{t+1} \tag{11}$$

where:

$\Delta \hat{x}_{t+1} = \Phi_t \Delta \tilde{x}_t = \Phi_t(\tilde{x}_t - x_t) = 0$ since $\tilde{x}_t = x_t$;

$Q_{\Delta \hat{x}_{t+1}} = \Theta_{t+1} + \Phi_t \Sigma_t^0 \Phi_t^T$

$K_{t+1} = Q_{\hat{x}_{t+1}} C_{t+1}^T \left[D_1 \Sigma_1 D_1^T + E_1 Q_{XX} E_1^T + C_{t+1} Q_{\Delta \hat{x}_{t+1}} C_{t+1}^T\right]^{-1}$ is the Kalman gain;

$z_{t+1} = b_{t+1} - C_{t+1} \Delta \hat{x}_{t+1} = b_{t+1}$ are the pseudoinnovations.

Since $\tilde{x}_{t+1} = \hat{x}_{t+1} + \Delta \tilde{x}_{t+1}$, from (11) it results that:

$$\tilde{x}_{t+1} = \Phi_t \tilde{x}_t + K_{t+1} b_{t+1} \tag{12}$$

with a covariance matrix:

$$Q_{\tilde{x}_{t+1}} = \left[I - K_{t+1} C_{t+1}\right] Q_{\Delta \hat{x}_{t+1}}$$

6. EXTENDED DYNAMIC LINEAR MODEL (LOOK AHEAD FILTER)

The model considers simultaneously:

- two subsequent observation equations:
$$b_1 = A_1 x_1 + v_1 \qquad v_1 \sim (0, \Sigma_1)$$
$$b_2 = A_2 x_2 + v_2 \qquad v_2 \sim (0, \Sigma_2)$$
- two state equations:
$$x_1 = \Phi_0 x_0 + \mu_1 \qquad \mu_1 \sim (0, \Theta_1)$$
$$x_2 = \Phi_1 x_1 + \mu_2 \qquad \mu_2 \sim (0, \Theta_2)$$
- one prior information:
$$\tilde{x}_0 = x_0 + e_0^0 \qquad e_0^0 \sim (0, \Sigma_0^0)$$

Combining the equations, it results:

$$b_1 = A_1 \Phi_0 \tilde{x}_0 - A_1 \Phi_0 e_0^0 + A_1 \mu_1 + v_1$$
$$b_2 = A_2\left[\Phi_1\left(\Phi_0 \tilde{x}_0 + \mu_1 - \Phi_0 e_0^0\right) + \mu_2\right] + v_2$$

Using these relationships and minimizing the Lagrangean function the least squares adjustment is obtained, from which, after some algebraic passages, the inhomBLIP $\tilde{\tilde{\mathbf{x}}}_1$ of $\mathbf{x}_1$ and $\tilde{\tilde{\mathbf{x}}}_2$ of $\mathbf{x}_2$ are obtained:

$$\tilde{\tilde{\mathbf{x}}}_1 = \tilde{\mathbf{x}}_1 + \mathbf{K}_2\mathbf{z}_2$$

$$\tilde{\tilde{\mathbf{x}}}_2 = \boldsymbol{\Phi}_1\tilde{\tilde{\mathbf{x}}}_1 + \tilde{\tilde{\boldsymbol{\mu}}}_2 = \boldsymbol{\Phi}_1\tilde{\mathbf{x}}_1 + \mathbf{K}_2\mathbf{z}_2 = \tilde{\mathbf{x}}_2$$

where:

$\tilde{\mathbf{x}}_1 = \hat{\mathbf{x}}_1 + \mathbf{K}_1\mathbf{z}_1$ is the inhomBLIP of $\mathbf{x}_1$ for a classical dynamic linear model;

$\mathbf{Q}_{\tilde{\mathbf{x}}_1} = \Sigma_1^0$

$\hat{\mathbf{x}}_2 = \boldsymbol{\Phi}_1\tilde{\mathbf{x}}_1$ is the inhomBLIP of $\mathbf{x}_2$ based on prior information only;

$\mathbf{Q}_{\hat{\mathbf{x}}_2} = \Theta_2 + \boldsymbol{\Phi}_1\Sigma_1^0\boldsymbol{\Phi}_1^T$

$\mathbf{K}_2 = \mathbf{Q}_{\tilde{\mathbf{x}}_1}\boldsymbol{\Phi}_1^T\mathbf{A}_2^T\left[\Sigma_2 + \mathbf{A}_2\mathbf{Q}_{\hat{\mathbf{x}}_2}\mathbf{A}_2^T\right]^{-1}$ is the extended Kalman gain;

$\mathbf{z}_2 = \mathbf{b}_2 - \mathbf{A}_2\hat{\mathbf{x}}_2$ are the extended pseudoinnovations.

The covariance matrices are:

$$\mathrm{MSPE}\left\{\tilde{\tilde{\mathbf{x}}}_1\right\} = \mathbf{Q}_{\tilde{\tilde{\mathbf{x}}}_1} = \left[\mathbf{I} - \mathbf{K}_2\mathbf{A}_2\boldsymbol{\Phi}_1\right]\mathbf{Q}_{\tilde{\mathbf{x}}_1} = \bar{\bar{\Sigma}}_1^0$$

$$\mathrm{MSPE}\left\{\tilde{\tilde{\mathbf{x}}}_2\right\} = \mathbf{Q}_{\tilde{\tilde{\mathbf{x}}}_2} = \left[\mathbf{I} - \mathbf{K}_2\mathbf{A}_2\right]\mathbf{Q}_{\tilde{\mathbf{x}}_1} = \Sigma_2^0$$

7. WAVE ALGORITHM

A possible extension of the model reported in chapter 6 is the so-called "wave algorithm" (Wang, Schaffrin and Salychev, 1995). The model consists of a series of N_1 (with $N_1>2$) subsequent observation equations and state equations:

- observation equations for $t=1,\ldots,N_1$:

$$\mathbf{b}_t = \mathbf{A}_t\mathbf{x}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim (0, \Sigma_t)$$

- state equations for $t=1,\ldots,N_1$:

$$\mathbf{x}_t = \boldsymbol{\Phi}_{t-1}\mathbf{x}_{t-1} + \boldsymbol{\delta}_{t-1} \qquad \boldsymbol{\delta}_{t-1} \sim (0, \Theta_{t-1})$$

- prior information for $t=0$:

$$\tilde{\mathbf{x}}_0 = \mathbf{x}_0 + \mathbf{e}_0^0 \qquad \mathbf{e}_0^0 \sim (0, \Sigma_0^0)$$

In the wave algorithm the term $\boldsymbol{\delta}_{t-1}$ is different from zero only for a given t_k, e.g. $t_k=1$; from which: $\mathbf{x}_1 = \boldsymbol{\Phi}_0\mathbf{x}_0 + \boldsymbol{\delta}_0$.

In this way, a set of N_1 observation equations and one pseudo-observation equation can be written:

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \ldots \\ \mathbf{b}_{N_1} \\ \hat{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2\boldsymbol{\Phi}_1 \\ \ldots\ldots \\ \mathbf{A}_{N_1}\boldsymbol{\Phi}_{N_1-1}\cdots\boldsymbol{\Phi}_1 \\ \mathbf{I} \end{bmatrix}\mathbf{x}_1 - \begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \\ \mathbf{I} \end{bmatrix}\boldsymbol{\delta}_0 + \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \ldots \\ \mathbf{v}_{N_1} \\ \boldsymbol{\Phi}_0\mathbf{e}_0^0 \end{bmatrix}$$

with $\hat{\mathbf{x}}_1 = \boldsymbol{\Phi}_0\tilde{\mathbf{x}}_0$. In compact form, it becomes:

$$\begin{bmatrix} \mathbf{b} \\ \hat{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \varnothing \\ \mathbf{I} & -\mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \boldsymbol{\delta}_0 \end{bmatrix} + \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\Phi}_0\mathbf{e}_0^0 \end{bmatrix}$$

where:

$$\mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \ldots \\ \mathbf{b}_{N_1} \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2\boldsymbol{\Phi}_1 \\ \ldots \\ \mathbf{A}_{N_1}\boldsymbol{\Phi}_{N_1-1}\cdots\boldsymbol{\Phi}_1 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \ldots \\ \mathbf{v}_{N_1} \end{bmatrix} \sim (\mathbf{0}, \Sigma) = \left(\begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_1 & 0 & \ldots & 0 \\ 0 & \Sigma_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \Sigma_{N_1} \end{bmatrix}\right)$$

Applying a least squares adjustment, the solution for $\tilde{\mathbf{x}}_1$ and $\hat{\boldsymbol{\delta}}_0$ can be obtained:

$$\begin{bmatrix} \tilde{\mathbf{x}}_1 \\ \hat{\boldsymbol{\delta}}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{10} \\ \mathbf{N}_{01} & \mathbf{N}_{00} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_0 \end{bmatrix} \tag{13}$$

where:

$$\mathbf{N}_{11} = \mathbf{A}^T\Sigma^{-1}\mathbf{A} + \left(\boldsymbol{\Phi}_0\Sigma_0^0\boldsymbol{\Phi}_0^T\right)^{-1}$$

$$\mathbf{N}_{01} = -\left(\boldsymbol{\Phi}_0\Sigma_0^0\boldsymbol{\Phi}_0^T\right)^{-1} = \mathbf{N}_{10}^T \qquad \mathbf{N}_{00} = \left(\boldsymbol{\Phi}_0\Sigma_0^0\boldsymbol{\Phi}_0^T\right)^{-1}$$

$$\mathbf{c}_1 = \mathbf{A}^T\Sigma^{-1}\mathbf{b} + \left(\boldsymbol{\Phi}_0\Sigma_0^0\boldsymbol{\Phi}_0^T\right)^{-1}\hat{\mathbf{x}}_1 \qquad \mathbf{c}_0 = -\left(\boldsymbol{\Phi}_0\Sigma_0^0\boldsymbol{\Phi}_0^T\right)^{-1}\hat{\mathbf{x}}_1$$

8. WAVE ALGORITHM FOR THE SENSOR ORIENTATION

The photogrammetric model consists of:

- linearized observation equations [from (10)] (relative/absolute orientation) for $t=1,\ldots,N_1-1$:

$$\mathbf{b}_t = \mathbf{C}_t\Delta\mathbf{x}_t + \mathbf{E}_t\mathbf{s}_t + \mathbf{D}_t\mathbf{v}_t$$

- differential state equations [from (9.1)] (by cubic spline functions) for $t=1,\ldots,N_1-1$:

$$\Delta\mathbf{x}_t = \boldsymbol{\Phi}_{t-1}\Delta\mathbf{x}_{t-1} + \boldsymbol{\delta}_{t-1}$$

- prior information (best prediction obtained by the process until time t):

$$\tilde{\mathbf{x}}_t = \bar{\mathbf{x}}_t$$

$\boldsymbol{\delta}_{t-1} \neq 0$ only for a given t_k, e.g. $t_k=1$; from which: $\Delta\mathbf{x}_1 = \boldsymbol{\Phi}_0\Delta\mathbf{x}_0 + \boldsymbol{\delta}_0$.

Making the assumption that t=1 is the epoch when the first image is oriented by digital map points, while $t=N_1$ is the epoch when another image is oriented by digital map points, the equation set for the cycle $t=1,\ldots,N_1-1$ contains a first image absolutely oriented by collinearity equations and (N_1-2) images relatively oriented by coplanarity equations.

The following system can be written:

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \ldots \\ \mathbf{b}_{N_1-1} \\ \Delta\hat{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2\boldsymbol{\Phi}_1 \\ \ldots\ldots \\ \mathbf{C}_{N_1-1}\boldsymbol{\Phi}_{N_1-2}\cdots\boldsymbol{\Phi}_1 \\ \mathbf{I} \end{bmatrix}\Delta\mathbf{x}_1 - \begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \\ \mathbf{I} \end{bmatrix}\boldsymbol{\delta}_0 + \begin{bmatrix} \mathbf{E}_1 \\ 0 \\ \ldots \\ 0 \\ 0 \end{bmatrix}\mathbf{s}_1 + \begin{bmatrix} \mathbf{D}_1 & 0 & \ldots & 0 & 0 \\ 0 & \mathbf{D}_2 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mathbf{D}_{N_1-1} & 0 \\ 0 & 0 & \ldots & 0 & \mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \ldots \\ \mathbf{v}_{N_1-1} \\ 0 \end{bmatrix}$$

with $\Delta\hat{\mathbf{x}}_1 = \boldsymbol{\Phi}_0\Delta\mathbf{x}_0$. In compact form, it becomes:

$$\begin{bmatrix} \mathbf{b} \\ \Delta\tilde{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \varnothing \\ \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x}_1 \\ \delta_0 \end{bmatrix} + \begin{bmatrix} \mathbf{G} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \gamma \\ 0 \end{bmatrix}$$

where:

$$\mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \dots \\ \mathbf{b}_{N_1-1} \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2\Phi_1 \\ \dots \\ \mathbf{C}_{N_1-1}\Phi_{N_1-2}\dots\Phi_1 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{E}_1 & \mathbf{D}_1 & 0 & \dots & 0 \\ 0 & 0 & \mathbf{D}_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \mathbf{D}_{N_1-1} \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ \mathbf{I} \end{bmatrix} \quad \mathbf{s}_1 \sim (0, \mathbf{Q}_{s_1})$$

$$\gamma = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{v}_1 \\ \mathbf{v}_2 \\ \dots \\ \mathbf{v}_{N_1-1} \end{bmatrix} \sim (\mathbf{0}, \Gamma) = \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{Q}_{s_1} & 0 & 0 & \dots & 0 \\ 0 & \Sigma_1 & 0 & \dots & 0 \\ 0 & 0 & \Sigma_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \Sigma_{N_1-1} \end{bmatrix} \right)$$

As for the relationship (13), using a least squares adjustment, the solutions for $\Delta\tilde{\mathbf{x}}_1$ and $\hat{\delta}_0$ can be obtained:

$$\begin{bmatrix} \Delta\tilde{\mathbf{x}}_1 \\ \hat{\delta}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{10} \\ \mathbf{N}_{01} & \mathbf{N}_{00} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_0 \end{bmatrix} \tag{14}$$

where:

$$\mathbf{N}_{11} = \mathbf{C}^T\Gamma^{-1}\mathbf{C} + \mathbf{I} \qquad \mathbf{N}_{01} = \mathbf{N}_{01} = \mathbf{N}_{00} = \mathbf{I}$$
$$\mathbf{c}_1 = \mathbf{C}^T\Gamma^{-1}\mathbf{b} + \Delta\tilde{\mathbf{x}}_1 \qquad \mathbf{c}_0 = -\Delta\tilde{\mathbf{x}}_1$$

while, according to Dermanis (1990c), the inhomBLIP $\tilde{\mathbf{s}}_1$ follows from:

$$\tilde{\mathbf{s}}_1 = \mathbf{Q}_{s_1}\mathbf{E}_1^T\left[\mathbf{E}_1\mathbf{Q}_{s_1}\mathbf{E}_1^T + \mathbf{D}_1\Sigma_1\mathbf{D}_1^T + \dots + \mathbf{D}_{N_1-1}\Sigma_{N_1-1}\mathbf{D}_{N_1-1}^T\right]\mathbf{f}$$

where:

$$\mathbf{f} = \begin{bmatrix} \mathbf{b} \\ \Phi_0\Delta\mathbf{x}_0 \end{bmatrix} - \begin{bmatrix} \mathbf{C} & \varnothing \\ \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \Delta\tilde{\mathbf{x}}_1 \\ \hat{\delta}_0 \end{bmatrix}$$

From this model, $\Delta\tilde{\mathbf{x}}_2, \dots, \Delta\tilde{\mathbf{x}}_{N_1-1}$ can be obtained in recursive way from:

$$\Delta\tilde{\mathbf{x}}_2 = \Phi_1\Delta\tilde{\mathbf{x}}_1$$
$$\dots$$
$$\Delta\tilde{\mathbf{x}}_{N_1-1} = \Phi_{N_1-2}\Delta\tilde{\mathbf{x}}_{N_1-2}$$

The same process can be applied to any cycle of measurements $(N_1, \dots, N_2 - 1)$, $(N_2, \dots, N_3 - 1)$, ... obtaining the estimations $\hat{\delta}_{N_1-1}, \hat{\delta}_{N_2-1}, \dots$.

Since the state equations are simply derived from cubic spline functions, they can be characterized by systematic errors. The estimation of δ_{t_k} makes it possible to compensate for such errors in the state equations by introducing an "impulse vector" δ_{t_k} at some instants t_k.

9. CONCLUSIONS

The use of an advanced Kalman filter technique, as for the "wave algorithm" with state equations defined by spline functions, seems to be promising for the digital sensor orientation of an MMS. This is the case when GPS measurements cannot be executed like, for instance, in cities centers.

Furthermore, the dynamic model proposed in this paper satisfies the necessity for efficient tools for handling and processing large amounts of data.

Finally the algorithm seems to be capable of integrating different sensor observations well (GPS, INS, images, digital points, ...), which is the challenge of the coming surveying era.

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