# PROCESSING OF PSEUDORANGE MEASUREMENTS: AN EXACT AND AN ITERATIVE ALGORITHM FOR THE GPS SINGLE POINT POSITIONING

**CROCETTO N. (*) - D'URSO M.G. (*) - GATTI M. (**)**

ISPRS Commission VI, Working Group 3

(*) Second University of Naples, Aversa (Italy)
(**) University of Ferrara (Italy)

**KEY WORDS**: GPS, SP3, Pseudorange, Data Processing Program, Algorithm

**ABSTRACT** We present an "exact" and an "iterative" computational procedure for determining the phase center coordinates of the GPS receiver antenna and its clock offset by using pseudorange measurements and a minimal number of satellites. Both procedures do not need the linearization of the pseudorange equations, thus avoiding the computation of the initial (approximate) values of the unknowns.

**RESUME'** Nous présentons une méthode de calcul "exacte" et itérative" qui permet de déterminer, en utilisant la mesure de la pseudodistance et un nombre minimum de satellites, les coordonnées du centre de la phase relatives au récepteur GPS et au décalage des horloges. Les deux procédures ne nécessitent pas la linéarisation des équations par rapport aux observations de la pseudodistance, ce qui évite de calculer les valeurs initiales (approximatives) des inconnues.

## INTRODUCTION

The data required for computing the GPS receiver position through pseudorange measurements are represented by the satellite istantaneous coordinates and its clock offset, while the unknowns are the coordinates X, Y, Z of the phase center of the receiver antenna and its clock offset.
In this way a set of non linear pseudorange equations is originated. The procedure usually exploited for its solution is the iterative Newton's method:

$$\mathbf{x}_{n+1} = \mathbf{x}_n + J^{-1}(\mathbf{r} - f(\mathbf{x}_n)) \qquad (1)$$

where $\mathbf{x}$ is the vector collecting the receiver's positioning together with clock offset, J is the matrix of partial derivates and $\mathbf{r}$ is the vector of four pseudorange observations. If more than four satellites are available, a least squares procedure can be implemented:

$$\mathbf{x}_{n+1} = \mathbf{x}_n + (J^T W J)^{-1} J W(\mathbf{r} - f(\mathbf{x}_n)) \qquad (2)$$

where W is a positive definite weighting matrix (Crocetto et al.,1997). This procedure is particularly time consuming since it requires not only the elaboration of the parameters relative to the satellite's orbit but also the determination of the approximate coordinates relative to the receiver's antenna, the linearization of the pseudorange equations and, finally, the "iterative" solution. For this reason simplified algorithms for the low cost single point positioning have been presented in literature (Noe et al. 1978). Anyhow the problem of solving a system of at least four nonlinear equations in four unknowns still remains.
A closed-form solution of the nonlinear pseudorange equations has been presented by Bancroft (1986) who uses a direct (non iterative) algebraic solution. This procedure can be implemented also for more than four satellites and pseudoranges, but the corresponding solution is not optimal in the $L_2$ sense.
Further, in the case of four satellites direct solutions have been obtained by Krause (1987), Chauffee and Abel (1991).
Particularly Hoshen (1996) proposes a closed-form solution by introducing spherical coordinates and by using the relationship existing between the ancient Problem of Apollonius and the pseudorange equations.
In this paper we present two new algorithms for computing the GPS single point position by using only four satellites; the former provides a closed-form solution, while the latter is iterative. Both algorithms do not require the linearization of the pseudorange equations and hence the a-priori receiver's approximate coordinates.
Whenever more than four satellites are considered, the receiver's coordinates evaluated through the exact algorithm can be exploited as initial values in the least square procedure.
Such algorithms have been implemented and tested by simulating a typical post processing condition in which the input data are the satellite's istantaneous coordinates (simply deduced from a file relative to the precise ephemerides in the SP3 format) and only the four code measurements. Furthermore the same algorithms have been tested in several real cases.

## 1. THE COMPUTATIONAL PROCEDURES

### 1.1 The exact algorithm for processing four pseudorange measurements

The relations between the observations of pseudoranges $r_i$, the known Cartesian coordinates of four satellites $(x_i, y_i, z_i)$ (i=1,2,3,4) and the four unknown parameters of the receiver $(x, y, z, \Delta)$ (respectively equal to the three Cartesian geocentric coordinates and to the clock offset) are for $i = 1,2,3,4$ :

$$\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}+\Delta = r_i \tag{1.1.1}$$

from which setting $\Delta$ at the right side and by squaring we get for $i = 1,2,3,4$ :

$$(x-x_i)^2+(y-y_i)^2+(z-z_i)^2 = r_i^2+\Delta^2-2r_i\Delta \tag{1.1.2}$$

Hence, manipulating and ordering, subtracting the first equation, the second and the third one from the fourth equation of the set (1.1.2), the quadratic terms in the unknowns are removed and we obtain the following system of linear equations:

$$\begin{cases}(x_4-x_1)x+(y_4-y_1)y+(z_4-z_1)z-(r_4-r_1)\Delta+f_1=0\\(x_4-x_2)x+(y_4-y_2)y+(z_4-z_2)z-(r_4-r_2)\Delta+f_2=0\\(x_4-x_3)x+(y_4-y_3)y+(z_4-z_3)z-(r_4-r_3)\Delta+f_3=0\end{cases} \tag{1.1.3}$$

where we assume for $i = 1,2,3$ :

$$f_i = \frac{-(x_4^2+y_4^2+z_4^2-r_4^2)+(x_i^2+y_i^2+z_i^2-r_i^2)}{2} \tag{1.1.4}$$

The system can be written as:

$$\begin{cases}a_1x+b_1y+c_1z+d_1\Delta+f_1=0\\a_2x+b_2y+c_2z+d_2\Delta+f_2=0\\a_3x+b_3y+c_3z+d_3\Delta+f_3=0\end{cases} \tag{1.1.5}$$

where for $i = 1,2,3$ :

$$\begin{cases}a_i=x_4-x_i\\b_i=y_4-y_i\\c_i=z_4-z_i\\d_i=-(r_4-r_i)\end{cases} \tag{1.1.6}$$

The unknown vector of the Cartesian geocentric coordinates of the receiver can be obtained from the system (1.1.5):

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{bmatrix}^{-1}\cdot\begin{bmatrix}-d_1\Delta-f_1\\-d_2\Delta-f_2\\-d_3\Delta-f_3\end{bmatrix}= \\ =\begin{bmatrix}\alpha_1&\beta_1&\gamma_1\\\alpha_2&\beta_2&\gamma_2\\\alpha_3&\beta_3&\gamma_3\end{bmatrix}\cdot\begin{bmatrix}-d_1\Delta-f_1\\-d_2\Delta-f_2\\-d_3\Delta-f_3\end{bmatrix}=\begin{bmatrix}x_o+l\Delta\\y_o+m\Delta\\z_o+n\Delta\end{bmatrix} \tag{1.1.7}$$

where:

$$\begin{cases}x_o=-\alpha_1f_1-\beta_1f_2-\gamma_1f_3\\y_o=-\alpha_2f_1-\beta_2f_2-\gamma_2f_3\\z_o=-\alpha_3f_1-\beta_3f_2-\gamma_3f_3\end{cases}$$

$$\begin{cases}l=-(\alpha_1d_1+\beta_1d_2+\gamma_1d_3)\\m=-(\alpha_2d_1+\beta_2d_2+\gamma_2d_3)\\n=-(\alpha_3d_1+\beta_3d_2+\gamma_3d_3)\end{cases} \tag{1.1.8}$$

Substituting the equations (1.1.7) in the first relation of the (1.1.2) and setting the coefficients equal to:

$$\begin{aligned}e_1&=l^2+m^2+n^2-1\\e_2&=(x_o-x_1)l+(y_o-y_1)m+(z_o-z_1)n+r_1\\e_3&=(x_o-x_1)^2+(y_o-y_1)^2+(z_o-z_1)^2-r_1^2\end{aligned} \tag{1.1.9}$$

a quadratic equation in the unknown $\Delta$ is derived :

$$e_1\Delta^2+2e_2\Delta+e_3=0 \tag{1.1.10}$$

Substituting each possible solution of the previous in the relationships (1.1.7) provides, in general, a double set of four parameters $(x, y, z, \Delta)$.

Differently from Bancroft (pag.56) and Krause (pag. 226), Abel and Chauffee (1991) prove that in the case of four satellites, a fix may not exist and, if it exists, it is not guaranteed to be unique. Besides, the existence of the fix depends on the receiver-satellite geometry and pseudorange errors while the uniqueness of the fix depends only on the receiver-satellite geometry and not on the receiver clock bias.

According to Abel and Chauffee (1991) the existence of the solutions of the quadratic equation (1.1.10) depends on the receiver-satellite geometry (upon which affect the values of $\alpha_i, \beta_i, \gamma_i, f_i$) and on the pseudorange errors (occurring in the expressions of $d_i, f_i$).

When the equation (1.1.10) admits two solutions, the ambiguity is removed by checking the associated residual obtained substituting their values in the basic primary system (1.1.1).

If both residuals are negligible, then the solution of the problem at hand will be the one according to which the distance of the receiver from the earth's center equals the terrestrial radius.

## 1.2 The iterative algorithm for processing four pseudorange measurements

Let us consider the relationships between four pseudorange measurements $r_i$, the known Cartesian geocentric coordinates of four satellites $(x_i, y_i, z_i)$ (i=1,2,3,4) and the four unknown parameters of the receiver $(x, y, z, \Delta)$ :

$$\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}+\Delta = r_1 \qquad (1.2.1)$$

$$\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}+\Delta = r_2 \qquad (1.2.2)$$

$$\sqrt{(x-x_3)^2+(y-y_3)^2+(z-z_3)^2}+\Delta = r_3 \qquad (1.2.3)$$

$$\sqrt{(x-x_4)^2+(y-y_4)^2+(z-z_4)^2}+\Delta = r_4 \qquad (1.2.4)$$

A first procedure of iterative nature consists of the following phases:

a) assume one equation as " pivot "
b) assume the value zero of the clock offset $\Delta$ in the other three equations
c) deduce from these three equations a trial value for the Cartesian coordinates $(x, y, z)$ of the receiver by using the exact procedure reported in the paragraph 1.3
d) get a trial value of the clock offset $\Delta^{(1)}$ from the " pivot equation"
e) introduce this value $\Delta^{(1)}$ in the four equations (1.2.1)-(1.2.4), whose righthand sides $(r_i - \Delta^{(1)})$ take account of the clock offset
f) compute again the Cartesian geocentric coordinates of the receiver by using the three equations of the phase c) whose righthand side are equal to $(r_i - \Delta^{(1)})$
g) by using the updated coordinates , the correction $\delta^{(1)}$ from the k-th " pivot " equation associated with the right-hand side $(r_k - \Delta^{(1)})$ is computed. The clock offset is updated via $\delta^{(1)}$ in the form $\Delta^{(2)} = \Delta^{(1)} + \delta^{(1)}$ and the process is repeated till when the values of unknown parameters do not change significantly.

This procedure could not converge for every equation assumed as "pivot". Thus it is possible to exploit a second iterative procedure in which a different "pivot equation" is adopted at each iteration, i.e. the first one, then the second, the third and the fourth one.

## 1.3 The exact procedure for processing three pseudorange measurements

The algorithm for finding the intersection of three spheres used in the determination of the receiver's coordinates, when its clock offset is slighted, is conceptually similar to the one reported in the exact procedure for the four pseudoranges. The relations between the pseudorange measurements, the known geocentric Cartesian coordinates of the three satellites $(x_i, y_i, z_i)$ and the three unknown parameters $(x, y, z)$ are for i=1,2,3 :

$$\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2} = r_i \qquad (1.3.1)$$

upon squaring the system is thus obtained for i=1,2,3 :

$$x^2+y^2+z^2-2x_ix-2y_iy-2z_iz+(x_i^2+y_i^2+z_i^2-r_i^2)=0 \qquad (1.3.2)$$

Subtracting the third equation from the first one and the second one we get the equation of one straight line:

$$\begin{cases} a_1x+b_1y+c_1z+d_1=0 \\ a_2x+b_2y+c_2z+d_2=0 \end{cases} \qquad (1.3.3)$$

where for i=1,2

$$\begin{cases} a_i=(x_3-x_i) \\ b_i=(y_3-y_i) \\ c_i=(z_3-z_i) \\ d_i=\dfrac{-(x_3^2+y_3^2+z_3^2-r_3^2)+(x_i^2+y_i^2+z_2^2-r_i^2)}{2} \end{cases} \qquad (1.3.4)$$

From the system (1.3.3) we have:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}^{-1} \begin{bmatrix} -d_1 & -c_1z \\ -d_2 & -c_2z \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} -d_1 & -c_1z \\ -d_2 & -c_2z \end{bmatrix} \qquad (1.3.5)$$

from which the parametric equation of the same straight line is obtained:

$$\begin{cases} x=x_0+\lambda t \\ y=y_0+\mu t \\ z=t \end{cases} \qquad (1.3.6)$$

where

$$\begin{cases} x_0 = -\alpha\, d_1 - \beta\, d_2 \\ y_0 = -\gamma\, d_1 - \delta\, d_2 \\ \lambda = -(\alpha\, c_1 + \beta\, c_2) \\ \mu = -(\gamma\, c_1 + \delta\, c_2) \end{cases} \qquad (1.3.7)$$

Substituting the (1.3.7) in the first equation of the system (1.3.2) and performing some algebraic manipulations a second degree equation in the unknown t is obtained:

$$g_1\, t^2 + 2\, g_2\, t + g_3 = 0 \qquad (1.3.8)$$

where

$$\begin{cases} g_1 = \lambda^2 + \mu^2 + 1 \\ g_2 = -\left[(x_1 - x_0)\lambda + (y_1 - y_0)\mu + z_1\right] \\ g_3 = (x_1 - x_0)^2 + (y_1 - y_0)^2 + z_1^2 - r_1^2 \end{cases} \qquad (1.3.9)$$

The receiver's coordinates associated with such value of t are supplied by the system (1.3.6). Also this algorithm can be characterized by a double solution and in this evenience the ambiguity is removed as it is explained at the end of paragraph 1.1

## 2. THE DATA PROCESSING PROGRAM

The data processing program, written in FORTRAN 77 for PC, consists of two principal sections:
1. reading the input data;
2. computing the GPS single point position.

The input data regard the satellite positions, its clock offset and pseudorange measurements made by the receiver.
The satellite's positions and its clock offset are assumed to be given from a SP3 ASCII format data file at pseudorange measurement epochs. This format is available by National Geodetic Survey (NGS) or other agencies (for example International GPS Service for Geodinamics (IGS) or National Imagery and Mapping Agency NIMA) and it is particularly convenient for post-processing of the single-receiver in the GPS applications. Some observations relevant to the SP3 orbital format are given in Appendix I.
The pseudorange measurements are inputed from LST Ashtech ASCII format data file. This file provides, in particular, the epochs of measurements (seconds of week) and the pseudoranges (in meters). For further information about the Ashtech ASCII format LST see Appendix II.
The flowchart of section 1 is shown in the Figure 1 .

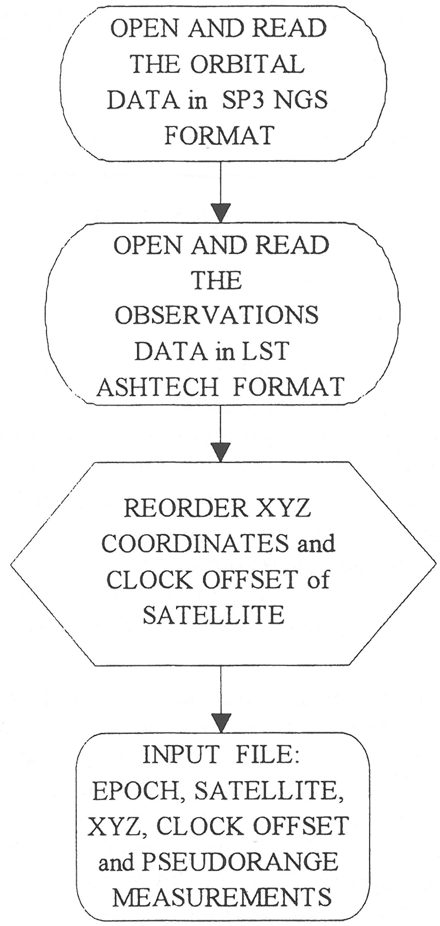


Fig. 1 : Flowchart of section 1.

In the table 1 we present an example of input data file at epoch 16:37:35 of $31^{st}$ /7/97.

| SV | X , Y , Z (meters) | Clock offset (10**-9 sec) | Pseudorange (meters) |
|---|---|---|---|
| 4 | 17793439.324<br>-8176464.484<br>18108291.173 | 39645.437 | 21170050.406 |
| 16 | 15756822.963<br>11394605.265<br>18140255.817 | 21834.677 | 20655099.547 |
| 18 | 18115313.847<br>479663.739<br>19207135.164 | 3052.907 | 20153311.596 |
| 19 | 25777488.288<br>6395349.493<br>2144500.015 | 138186.014 | 22156354.849 |

Tab. 1 : Input data at epoch 16:37:35 of $31^{st}$ /7/97.

In the table 2 we present an example of output data at the same epoch

| X (meters) | Y (meters) | Z (meters) | Clock Offset (meters) |
|---|---|---|---|
| 4445679.278 | 903260.440 | 4468732.869 | 48037.598 |

Tab. 2 : Output data at epoch 16:37:35 of 31st /7/97.

## 3. PRELIMINARY TEST

The data of two different sessions obtained via dual frequency receivers have been elaborated.

In the first session the ephemerides data of four satellites PRN 4,16,18,19 have been elaborated with 116 measurement epochs at sampling rate of 1 second in 31st/7/97.

In the second session the numerical test have affected the elaboration of the ephemerides' data relative to the four satellites PRN 4,18,24,29 and in relation to measurement epochs of 15 minutes between 11.30 and 12.30 in 10th /7/95.

In the first session the exact procedure proposed by the authors, the Newton's method, the least square procedure, the iterative procedure proposed by authors and the Bancroft's exact method provide the same estimate of the receiver's coordinates and of its clock offset.

In the first epoch of the second session it has been verified that the system of four nonlinear pseudorange equations admits a double solution (the same two solutions have been obtained with the exact procedure proposed by the authors and with the Bancroft's exact method). In the table 3 are reported the two solutions, the corresponding maximum residuals and the distances from the origin of the coordinate's system.

| X<br>Y<br>Z<br>(meters) | Maximum Residual (meters) | Distance from the Origin (Km) |
|---|---|---|
| 10082783.666<br>-1383713.873<br>26041905.007 | 7.45E-009 | 27959.935 |
| 4445043.205<br>903667.958<br>4466072.366 | 5.73E-009 | 6365.597 |

Tab. 3 : The two solutions of the first epoch of the second session

Therefore it was necessary to adopt the criterion of the distance from the earth's center for avoiding the ambiguity of the solution.

The acceptable solution (the second one) is identical to those obtained with the Newton's method and least square solution, provided that the initial approximate values are assumed sufficiently close to the real ones.

Conversely if the initial values are assumed close to those relative to the unacceptable solution, the iterative methods, available in the literature (Newton and least-squares) converge indeed to the unacceptable solution.

This fact shows the limits of the traditional iterative methods in the presence of a double solution: the solution to which such methods converge to depends upon the initial approximate values. The iterative procedure proposed by the authors does not turn out to be convergent, whatever is the chosen "pivot equation".

The application of the previous methods yielded the same results also for the other four epochs.

The computer time of the different algorithms have been considered for measuring their rate of convergence. The computations have been repeated 1000,10000 and finally 100000 times for each method and for each epoch of the second session.

Several hardware platforms have been employed: 486 type computers with clock frequency 66 MHz and 100 MHz, Pentium to 100 MHz, 133 MHz and 200 MHz.

The computer times refer only to elaboration of the data: the time for reading the input files is not taken into account.

For the iterative methods reported in the literature the computer time has been estimated for only two iterations; conversely the computer time in the iterative method proposed by the authors has been valued for 20 iterations.

The results are reported in the table 4 .

| | **486 66 MHz** (msec) | **486 100 MHz** (msec) | **PENT. 100 MHz** (msec) | **PENT. 133 MHz** (msec) | **PENT. 200 MHz** (msec) |
|---|---|---|---|---|---|
| **EXACT** | 1.3 | 0.82 | 0.66 | 0.31 | 0.20 |
| **NEWTON** | 1.9 | 1.2 | 0.94 | 0.49 | 0.30 |
| **LEAST-SQUARES** | 4.3 | 2.6 | 1.6 | 1.1 | 0.61 |
| **ITERATIVE** | 11 | 7.1 | 5.8 | 2.6 | 1.7 |
| **BANCROFT** | 1.69 | 1.03 | 0.85 | 0.40 | 0.25 |

Tab. 4 : Computing time of the different methods.

Times in the order of 0.001 sec. are deduced from the table. This value is reduced for the Pentium computers. Using the same hardware the exact method is faster 30 per cent than the Bancroft's one, 50 per cent faster than the Newton's one and about three times faster than least square's one.

Conversely the longest computer time is referred to the iterative method proposed by the authors.

From the pratical point of view the computing times present unsignificant differences: however such differences can be not negligible in the applications of the Kinematic GPS, where the processing requires the elaboration of a large amount of data.

## 4. CONCLUSIONS

The numerical simulations have shown that the exact algorithm is particularly efficient and numerically stable from the computational point of view.

The proposed iterative procedures do not necessarily converge when the system of four nonlinear pseudorange equations admits a double solution.

Further, in the case of the double solution also the traditional iterative methods (Newton, least squares) turn out to be not convergent or converge to the wrong solution. The convergence to the correct solution depends upon the choice of the initial point of iteration, which must be sufficiently close to the true receiver position.

Finally in the case of the double solution and in the presence of blunders, the iterative procedures (Newton, least - squares procedure) turn out to be not convergent.


### Acknowledgments

The authors wish to express their deep gratitude to prof. Paolo Russo for his valuable suggestions during the preparation of the manuscript and the careful reading of the final version of the paper.


### References


Abel J., Chauffe J. (1991): *Existence and uniqueness of GPS solutions*; IEEE Transactions on aerospace and electronic systems Vol. 27, No 6, pp.952-95.

Ashtech (1995): *User's Manual*; Ashtech Document: Sunnyvale (USA).

Bancroft S., (1985): *An algebraic solution of the GPS equations*; IEEE Transactions on aerospace and electronic systems Vol. AES-21, No 7, pp.56-59.

Chauffee J., Abel J. (1994): *On the exact solutions of pseudorange equations*; IEEE Transactions on aerospace and electronic systems Vol. 30, No 4, pp. 1021-1030.

Crocetto N., Gatti M., Perfetti N. (1997): *The GPS single point positioning: a data processing program for tutorial purposes;* Proceedings of ISPRS Commission VI, Working Group 3, Padua (Italy), pp. 131-135.

Hoshen J. (1996): *The GPS equations and the problem of Apollonius*; IEEE Transactions on aerospace and electronic systems Vol. 32, No 3, pp.1116-1124.

Krause O., (1987): *A direct solution to GPS-type navigation equations*; IEEE Transactions on aerospace and electronic systems Vol. AES 23, No 2, pp.225-232.

Noe P.S., Myersand K.A., Wu T. K. (1978): *A Navigation Algorithm for the Low-Cost GPS Receiver;* Navigation Journal of the Institute of Navigation, Vol. 25, No. 2, pp. 258-265.

Spofford P.R., Remondi B.W. (1997): *The National Geodetic Survey Standard GPS format SP3*; National Geodetic Survey; Silver Spring - USA.


## APPENDIX I - SP3 GPS Orbital Format

The SP3 Orbital Format is a precise format GPS relative to the ephemerides fixed by National Geodetic Survey (Spofford and Remondi, 1997) and consists of a binary and an ASCII format file. In the basic format, identified by the P (Position) record flag, are reported the coordinates (in km) X, Y, Z for each GPS satellite and the satellite's clock offset (in microsec) from GPS time.

The second optional record reported in the V flag (Velocity) contains the velocities (in dm/sec) of each satellite relative to datum XYZ and the clock rate of changes (in $10^{-4}$ microsecond/second).

The orbits have been computed with reference to the International Earth Rotation Service Terrestrial Reference Frame 1994 (ITRF 94) or to the WGS84 system and are currently expected to be accurate to less than 0.1 parts of million (2.6 m).

The clock offset is computed via the GPS Navigation message and it has an accuracy of 1 picosecond. The data storage into the files has been computed for full day, for all GPS satellites of the constellation and with a sampling rate of 15 minute epochs.

Below we report an example of SP3 ASCII V mode format file and we show in tab. 5 an SP3 ASCII format file.

```
#a V 1997  7 31 12  0  0.00000000      2 ORBIT ITR94
HLM  IGS
## 916 388800.00000000   900.00000000 50660
0.5000000000000
+   1   18 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
++       6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
%c cc cc ccc ccc cccc cccc cccc cccc ccccc ccccc ccccc
ccccc%c cc cc ccc ccc cccc cccc cccc cccc ccccc ccccc
ccccc ccccc%f 0.0000000 0.000000000 0.00000000000
0.000000000000000
/* RAPID ORBIT COMBINATION FROM WEIGHTED
AVERAGE OF:
/* cod emr esa gfz ngs sio
/* REFERENCED TO GPS CLOCK AND TO
WEIGHTED MEAN POLE:
*  1997  7 31 12  0  0.00000000
P 18  11858.770600 -11675.915350 -20920.849700
3.033300
V 18  24100.533241  10123.680781   8074.045892
0.734118
*  1997  7 31 12 15  0.00000000
P 18  14005.368000 -10834.558500 -20019.661400
3.019900
V 18  23517.171233   8566.515372  11924.579061
0.507707
EOF
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Line n. 1** | P or V | Year, Month, Day | Hour, Minute, Second | Ephocs | Coord. System | Agency | |
| **Line n. 2** | GPS week | Second of week | Epoc interval | Mod Julian Day | Fractional Day | | |
| **Line n. 3** | Number of SVs | Identif. SV1 | Identif. SV2 | Identif. SV3 | ............... | Identif. SVn | |
| **Line n. 4** | X Y Z SVs accuracy | Accuracy SV1 | Accuracy SV2 | Accuracy SV3 | ............... | Accuracy SVn | |
| **Line from n. 5 to n. 10** | Characters and Comment | | | | | | |
| **Line n. 11** | Symbols | Year | Month | Day | Hour | Minute | Second |
| **Line n. 12** | P | SV1 | X (km) | Y (km) | Z (km) | Clock (microsecond) | |
| **Line n. 13** | V | SV1 | VX (dm/s) | VY (dm/s) | VZ (dm/s) | Clock rate ($10^{-4}$ $\mu$s/s) | |
| **Line n. 14** | P | SV2 | X (km) | Y (km) | Z (km) | Clock (microsecond) | |
| **Line n. 15** | V | SV2 | VX (dm/s) | VY (dm/s) | VZ (dm/s) | Clock rate ($10^{-4}$ $\mu$s/s) | |
| **Line n. 16** | ............... | ............... | ............... | ............... | ............... | ................................. | |
| **Line n. 17** | ............... | ............... | ............... | ............... | ............... | ................................. | |
| **Line n. 18** | P | SVn | X (km) | Y (km) | Z (km) | Clock (microsecond) | |
| **Line n. 19** | V | SVn | VX (dm/s) | VY (dm/s) | VZ (dm/s) | Clock rate ($10^{-4}$ $\mu$s/s) | |
| **Line n. 20** | Repeat lines from line n. 11 until to line n. 19 | | | | | | |
| **End Line** | EOF | | | | | | |

Tab. 5 : SP3 ASCII format.

### APPENDIX II - LST GPS Ashtech data format

The LST GPS Ashtech format is an efficient way to convert a raw receiver data file, downloaded from a receiver at the end of the measurements' session into an ASCII data file. We report in tab. 6 an example of LST GPS Ashtech format file at epoch 16:37:35 of $31^{st}$ /7/97. The file contains raw data, the carrier phase and information about the code phase.
In particular at the first line it is recorded the number of epochs collected from receiver and the beginning and ending session times.

The second line contains the record number, the receiver number, the type, channel and nav boards firmware versions and CA/L1/L2 capabilities.
In the third line are reported the receive time (second of week), the satellite PRN, the number of channels, the code pseudorange (in meters), the doppler effect (in Hertz), the carrier phase (in cycles), the satellite elevation, the azimuth, the ratio signal to noise (in dbw) and the observation type (CA/L1P/L2P). This record is repeated as many times as the number of satellites.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| VERSION | | RV | RCVR_ TYPE | CHAN_ VER | NAV_ VER | CAPABILITY | | |
| Version: | | 3 | Z-XII P3 | 1D02 | 1I00 | L1CP_L2P | | |
| RECORD = 1 | | | RECEIVE TIME = 405455.000000 | | | | | |
| SV | CH | CDPHASE | DOPPL | CARRIER_PH | EL | AZ | S/N | DTYPE |
| 4 | 4 | 21170050 | -8636960 | -706488.199 | 56 | 278 | 191 | L1 |
| | | 21170051 | -8636960 | -706488.199 | | | 156 | L1P |
| | | 21170056 | -6730098 | -545945.724 | | | 159 | L2P |
| 19 | 6 | 22156354 | -33143550 | -2216476.682 | 39 | 176 | 179 | L1 |
| | | 22156355 | -33143550 | -2216476.683 | | | 138 | L1P |
| | | 22156364 | -25826142 | -1714775.369 | | | 144 | L2P |
| 18 | 8 | ............... | ............... | ..................... | ...... | ...... | ...... | ...... |
| RECORD = 2 | | | RECEIVE TIME = 405456.000000 | | | | | |

Tab. 6 : Example of LST GPS Ashtech format file at epoch 16:37:35 of $31^{st}$ /7/97.