NM3digit: the DLT algorithm in a digital photogrammetrical software for the survey teaching

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Commission VI, Working Group 3

Abstract
Often in the photogrammetrical survey teaching there are some difficulties in the organization of the exercises of students. This is connect to the insufficient number of instruments used in the restitution compared to the high number of students. Starting from this consideration the authors have realized a software in win95 environment that allows the orientation an the restitution of non-metric photograms digitalized using the D.L.T. algorithms. The paper explains the reasons of the choice of the algorithm, of the digital envirorment, of the operative system, of the programming language, etc. It is proposed to create a Web site for the distribution of free-softwares for photogrammetry and topography. The software presented in this paper is given as compiled and source code.

Introduction

Within the field of the activities done by the Photogrammetrical Laboratory of IUAV, the work presented here is born from a simple idea: it is impossible to place the instruments of photogrammetrical restitution at the disposition of the students of the Survey courses and the graduands who need to use them for their theses. This is due to the large number of students compared with the limited number of restitution systems. In the past, the problem was overcome by the organization of the technical personnel who afforded students the opportunity to take part in demonstrations of the instruments without actual first hand experience in the operations that they have studied on a theoretical basis.
A real understanding of the relative problems of the "instrumental survey" involves not only dealing with the theoretical aspects and methodologies but practical and operative ones as well, which can be acquired only through a real application.
This new need for a direct experience in the teaching of operative survey stems probably from two factors: firstly, from the more and more precise definition of the architectonic survey discipline which has brought about, among other things, the individuation of the methods of transmission of the disciplinary knowledge (of which this symposium is proof). Secondly, the understanding of the importance of survey within the field of conservation on the understanding and knowledge of an architecture, which brings about a rigour in the methods that can be guaranteed only by operators with a complete and deep rooted training.
The objective placed therefore, has been ploting the photogrammetrical restitution accessible to a large number of students, taking advantage of the possibilities offered by present day computers.
This end is reached through the realization of a photogrammetrical system which utilizes low cost hardware such as personal computers and non metrical cameras, tools easily accessible and useable by all students.
The software must allow for the use of non-stereoscopic and non-metrical images, maintaining the rigorousness of the operation of orientation and restitution.

The software realized cannot be considered a digital photogrammetrical system, in so far as it doesn't have any of the fundamental technical characteristics, but it is nearly revolutionary in the production and the utilization of the photogrammetrical products, which will be shown herein. The diffusion of soft-photogrammetry will bring the end-user to be not only the consumer and user of the maps produced by photogrammetry, but he will also be able to produce for himself maps containing all the specific data necessary to his particular project.
In the same way the student can perform the operations of orientation, restitution, evaluation of the estimates of the parameters obtained, and uncertainty of the points surveyed himself.
With the software NM3-Digit, it will not only be possible to change the way in which the exercises in the Survey and Photogrammetrical courses are carried out, but also change the service offered by the Photogrammetrical Laboratory with respect to the student. Up until now, the student received a final product from the Laboratory: a survey or a demonstration of how to execute a survey; with software instruments such as NM3-Digit, available with the freeware formula in so far as they are self produced,

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it will be possible to hand over to the student the software, the photographs and the control points relative to the photographs (and eventually also the orientation). In this way, the student can perform the orientation and the restitution, personally appraising what to plot and how to plot it.

Basic Decisions
In relationship to what is described, the NM3-Digit software must have precise characteristics:
- to allow for the restitution with non-metrical camera;
- to work with PC s and under the Windows operating system.

The choice of the algorithm of the calculation has been decided to be the D.L.T. for its characteristics of the treatment of non-metrical images. Moreover, the D.L.T. is an expression of the projective relationships between two-dimensional space and three-dimensional space, which, being considered the basis of photogrammetry, have great teaching value.
The monoscopy has been another obligatory choice in so far as it eliminates any optical hardware components.
It has been decided not to supply the product with a graphic output, but rather only a numerical output, interfacing it *on line* with CAD.
Another fundamental decision is that of showing all the calculated parameters and the imposing the obligation to manually update the results of every repetition; this slows down the operations but allows the student to evaluate the results and to make the consequential decisions.

What it does and doesn't do
The photogrammetric workstation is characterized by notable services, above all in relation to the automation of certain photogrammetrical procedures;
These essentially allow for the following:
- the visualization of images
- the normalization of images
- the stereoscopic vision
- a high level of automation of the orientation procedures
- the traditional stereoscopic or the monoscopic restitution
- the matching of images
- the automatic generation of D.T.M. and contour lines
- the monoscopic o three-dimensional superimposition
- orthophotos

A system of this kind does not work with a *Personal Computer*, but rather with a computer dedicated, developed and designed by the companies that produce photogrammetrical *Hardware*.

The NM3-Digit software permits some of the aforementioned operations to be carried out, which constitute the minimum operative basis and the starting point for successive developments.
In particular, it permits:
- the reading and visualization of images;
- the reading from the monitor of the coordinate images (pixel);
- the spatial resection:
  - with direct solution (11 parameters of the D.L.T.)
  - with iterative solution (11 parameters of the D.L.T. + 1 parameter that models the radial distortion);
- the spatial intersection for multi-image restitution;
- the evaluation of statistical parameters;
- the saving of the parameters for successive restitutions

Therefore the digital photogrammetric system cannot be defined according to currently used definitions but rather it is simply a photogrammetric system on the PC that utilizes digital images.

D.L.T.
The Direct Linear Transformation (D.L.T.) is an alternative method for facing the problem of analytical orientation that finds an optimal application in photogrammetry with non metrical images.
The main advantages are represented by the fact that with this method a stable interior orientation nor fiducial marks are necessary. In the initial phase of the calculation moreover, it is not necessary to know the approximate values for the unknown parameters.
The solution with D.L.T. is based on the concept of the direct transformation of coordinates in the instrumental system of the comparator of the coordinate object. The intermediate passage from comparator coordinate to image coordinate is latent in the D.L.T. equations and is not calculated explicitly.
The functional tie between the object coordinates and the image coordinates is given by 11 parameters. The geometric significance of the parameters of the D.L.T. is clear: they are combinations of the three parameters of interior orientation (x_0, y_0, c) and of the 6 of external orientation (X_0, Y_0, Z_0, ω, ϕ, κ) of a photogram.
The other two parameters of the D.L.T. compared with the 9 parameters of the photogram can be interpreted as:
- a different scale in x and y axes (m);
- a shear (=orthogonality error) between x and y axes (β).

These measures can be interpreted as a affine transformation applied to the image.
These two conditions can be utilized for the writing of the two equations between the parameters of the D.L.T. if the values of m and β are known (for example, utilizing numbered metrical images with a

photogrammetric scanner for which m=1 and β=0) and can also be used for modeling the errors of the comparator. In the case of the NM3-Digit, with which one must use low cost harware tools, the two extra parameters are used to model the deformations introduced by the scanner which constitutes the comparator. The deformations are simplified into:

- difference of scale along the x and y axes due to the fact that one direction of the scanner the coordinates are determined by the steps of the movement of the motor of the optical group;
- the non-orthogonality of the axes due to the placement of the sensors in respect to the direction of the movement.

It seems that in utilizing non-metrical images and a low cost scanner the presence of errors in the geometry of the image necessitates the use of all 11 parameters (including, therefore, the affine transformation) to avoid running the risk of excessive numbers of parameters.

The projective relationship between the two dimensional space of the image and the three-dimensional space is described by the formula:

$$u = AU$$

where

u represents the vector $\begin{vmatrix} u & v & t \end{vmatrix}^t$, A the matrix coefficient e U the vector $\begin{vmatrix} U & V & W & T \end{vmatrix}^t$.

$$\begin{vmatrix} u \\ v \\ t \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{vmatrix} \cdot \begin{vmatrix} U \\ V \\ W \\ T \end{vmatrix}$$

With reference to the two systems of Cartesian coordinates (x, y) e (X, Y, Z) the above written relationships become

$$x = kBX$$

$$\begin{vmatrix} x \\ y \\ 1 \end{vmatrix} = k \cdot \begin{vmatrix} b_1 & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 \\ b_9 & b_{10} & b_{11} & 1 \end{vmatrix} \cdot \begin{vmatrix} X \\ Y \\ Z \\ 1 \end{vmatrix} \quad [1]$$

having placed:

$$\frac{u}{t} = x; \quad \frac{v}{t} = y; \quad b_j = \frac{a_j}{a_{12}} \quad \left(b_{12} = \frac{a_{12}}{a_{12}} = 1 \right);$$

$$k = \frac{T}{t} a_{12}; \quad X = \frac{U}{T}; \quad Y = \frac{V}{T}: \quad Z = \frac{W}{T}$$

Each of the vectors is assigned an index of identification *i* to the *nth* of the image or object point

$$x_i = k_i B X_i$$

Developing the [1] achieves

$$\begin{vmatrix} x_i \\ y_i \\ 1 \end{vmatrix} = \begin{vmatrix} k_i b_1 X_i + k_i b_2 Y_i + k_i b_3 Z_i + k_i b_4 \\ k_i b_5 X_i + k_i b_6 Y_i + k_i b_7 Z_i + k_i b_8 \\ k_i b_9 X_i + k_i b_{10} Y_i + k_i b_{11} Z_i + k_i \end{vmatrix}$$

Dividing the first two equations by the third achieves:

$$\frac{x_i}{1} = \frac{b_1 X_i + b_2 Y_i + b_3 Z_i + b_4}{b_9 X_i + b_{10} Y_i + b_{11} Z_i + 1}$$
$$\frac{y_i}{1} = \frac{b_5 X_i + b_6 Y_i + b_7 Z_i + b_8}{b_9 X_i + b_{10} Y_i + b_{11} Z_i + 1} \quad [2]$$

These relationships put in relation the image coordinates with the object coordinates through the b_j.

Orientation

Now considering simultaneously the implicit transformation of the system coordinates of the comparator to that of the image and the correction of the systematic errors, the collinear equations can be expressed as follows:

$$x - d_x = x_0 + c_x \frac{m_{11}(X - Xc) + m_{12}(Y - Yc) + m_{13}(Z - Zc)}{m_{31}(X - Xc) + m_{32}(Y - Yc) + m_{33}(Z - Zc)}$$

$$y - d_y = y_0 + c_y \frac{m_{21}(X - Xc) + m_{22}(Y - Yc) + m_{23}(Z - Zc)}{m_{31}(X - Xc) + m_{32}(Y - Yc) + m_{33}(Z - Zc)}$$

where the x and y are image coordinates of a point in the system defined by the comparator, d_x e d_y are the systematic errors of the image coordinates, x_0 e y_0 are the coordinates of the principle points in the comparator system, and c_x e c_y are the principle distances along the x and y directions. The difference between the major distance along the x and y directions is given by the different variation of the scale assumed in the affine transformation of the coordinates from the comparator system to the object one.

Having placed:

$$L_1 = \frac{x_0 m_{31} + c_x m_{11}}{L}$$

$$L_2 = \frac{x_0 m_{32} + c_x m_{12}}{L}$$

$$L_3 = \frac{x_0 m_{33} + c_x m_{13}}{L}$$

$$L_4 = x_0 + \frac{m_{11} X_c + m_{12} Y_c + m_{13} Z_c}{L}$$

$$L_5 = \frac{y_0 m_{31} + c_y m_{21}}{L}$$

$$L_6 = \frac{y_0 m_{32} + c_y m_{22}}{L}$$

$$L_7 = \frac{y_0 m_{33} + c_y m_{23}}{L}$$

$$L_8 = y_0 + \frac{m_{21} X_c + m_{22} Y_c + m_{23} Z_c}{L}$$

$$L_9 = \frac{m_{31}}{L}$$

$$L_{10} = \frac{m_{32}}{L}$$

$$L_{11} = \frac{m_{33}}{L}$$

$$L = -(m_{31} X_c + m_{22} Y_c + m_{13} Z_c)$$

with some algebraic passages the following equations are obtained:

$$x - d_x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y - d_y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

Subtracting d_x e d_y e and substituting the symbol "b" with "L" the equations [3] are identical as those of [2].

The d_x e d_y terms can be written explicitly, the [3] equations become:

$$x + (x - x_0)(K_1 r^2 + K_2 r^4 + K_3 r^6 + \ldots) + [r^2 + 2(x - x_0)^2] P_1 + +2(y - y_0)(x - x_0) P_2 = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1} ;$$

$$y + (y - y_0)(K_1 r^2 + K_2 r^4 + K_3 r^6 + \ldots) + 2(y - y_0)(x - x_0) P_1 + [r^2 + +2(y - y_0)^2] P_2 = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1})$$

where $r^2 = (x^2 + y^2)$; K_1, K_2, K_3, P_1 e P_2 are the coefficients of distortion of the objective system; X, Y, Z are the object-coordinates of the represented point; $L_1 \ldots L_{11}$ are the 11 unknown coefficients of transformation.

On the basis of experimental research, it has been concluded that, as a practical end, in order to significantly correct the distortion of the lenses, only the K_1 term need be taken into consideration.

Therefore:

$$x + x' K_1 r^2 = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y + y' K_1 r^2 = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

Direct solution

The equations of the D.L.T. become the following when the terms of the distortion and the residuals of the observed points (the image coordinates) are not considered:

$$x = \frac{L_1 \cdot X + L_2 \cdot Y + L_3 \cdot Z + L_4}{L_9 \cdot X + L_{10} \cdot Y + L_{11} \cdot Z + 1}$$

$$y = \frac{L_5 \cdot X + L_6 \cdot Y + L_7 \cdot Z + L_8}{L_9 \cdot X + L_{10} \cdot Y + L_{11} \cdot Z + 1}$$

Writing the equations for unknown n points obtains a system of 2n equations in 11 unknowns; if the n > 6 then the parameters of the D.L.T. can be estimated applying the method of the least squares.

In fact, for every point to the *i-nth* degree, the equations of the D.L.T. can be written in the form: [13]

$$L_1 X_i + L_2 Y_i + L_3 Z_i + L_4 - x_i X_i L_9 - x_i Y_i L_{10} - x_i Z_i L_{11} = x_i$$
$$L_5 X_i + L_6 Y_i + L_7 Z_i + L_8 - y_i X_i L_9 - y_i Y_i L_{10} - y_i Z_i L_{11} = y_i$$

or in matrix form

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -x_i\cdot X_i & -x_i\cdot Y_i & -x_i\cdot Z_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -y_i\cdot X_i & -y_i\cdot X_i & -y_i\cdot X_i \end{bmatrix}\cdot\begin{bmatrix} L_1\\ L_2\\ L_3\\ L_4\\ L_5\\ L_6\\ L_7\\ L_8\\ L_9\\ L_{10}\\ L_{11}\end{bmatrix}=\begin{bmatrix} x_i\\ y_i\end{bmatrix}$$

If the 6 points of the known coordinates are considered, therefore, errors not withstanding, it is then possible to determine with only one repeat the 11 unknown parameters of the D.L.T.

The direct solution is not very precise because the hypothesis is made that the residuals of the observation equations (image coordinates) can be neglected; it is, however, an very important solution because it allows for obtaining approximate parameters of orientation, to use after which in the *linearization* of the collinear equations.

This solution is calculated in the program and is then defined as an approximate solution.

Iterative solution

Considering v_x e v_y, random errors in the image coordinates measured the following expressions can be derived from the equation [4] neglecting the effect of v_x e v_y on the parameters of the distortion of the lens, K_1.

$$v_x - \frac{X}{A}L_1 - \frac{Y}{A}L_2 - \frac{Z}{A}L_3 - \frac{1}{A}L_4 + \frac{xX}{A}L_9 + \frac{xY}{A}L_{10} + \frac{xZ}{A}L_{11} + x'r^2K_1 + \frac{x}{A} = 0;$$

$$v_y - \frac{X}{A}L_5 - \frac{Y}{A}L_6 - \frac{Z}{A}L_7 - \frac{1}{A}L_8 + \frac{yX}{A}L_9 + \frac{yY}{A}L_{10} + \frac{yZ}{A}L_{11} + y'r^2K_1 + \frac{y}{A} = 0$$

in which $A = L_9X + L_{10}Y + L_{11}Z + 1$.

From the point in which the equations [5] are not linear, it is necessary in the first place to linearize them with using the Taylor's series expansion in and around the approximate values, and consequentially the solution to the least squares will have to have recourse to a iterative procedure. The initial values for the eleven transformation coefficients $L_1 \ldots L_{11}$ in this iterative procedure, can be calculated disregarding the terms $x'K_1r^2$ e $y'K_1r^2$ and resolving the remaining eleven linear equations according to what was previously described. The initial value for K_1 can be considered to be zero.

The traditional parameters of the interior and external orientation are included in the coefficient D.L.T. $L_1 \ldots L_{11}$ and can be expressed as follows:

$$L = \frac{-1}{\sqrt{(L_9^2 + L_{10}^2 + L_{11}^2)}}$$

$$x_0 = (L_1L_9 + L_2L_{10} + L_3L_{11})L^2$$

$$y_0 = (L_5L_9 + L_6L_{10} + L_7L_{11})L^2$$

$$c_x = \sqrt{[(L_1^2 + L_2^2 + L_3^2)L^2 - x_0^2]}$$

$$c_y = \sqrt{[(L_5^2 + L_6^2 + L_7^2)L^2 - y_0^2]}$$

$$\phi = \sin^{-1}(L_9L)$$

$$\omega = \tan^{-1}\frac{-L_{10}}{L_{11}}$$

$$m_{11} = \frac{L(x_p L_9 - L_1)}{c_x}$$

$$\kappa = \frac{\cos^{-1}m_{11}}{\cos f}$$

$$LL = \begin{bmatrix} L_1 & L_2 & L_3\\ L_5 & L_6 & L_7\\ L_9 & L_{10} & L_{11}\end{bmatrix}$$

$$\begin{bmatrix} X_c\\ Y_c\\ Z_c\end{bmatrix} = LL^{-1}\begin{bmatrix} L_4\\ L_8\\ 1\end{bmatrix}$$

Once the transformation coefficients and the error correction for all of the photographs has been obtained, the coordinates can then be estimated in the object-space in any other point that is represented in two or more photographs.

Linearization of the equations

The equations [5] are not linear and cannot be used within the program of calculation; therefore that have been linearized with using the Taylor's series expansion within and around the approximate values. Assuming:

$$f_1 = x + x' K_1 r^2 = \frac{X}{A} L_1 + \frac{Y}{A} L_2 + \frac{Z}{A} L_3 + \frac{1}{A} L_4 - \frac{xX}{A} L_9 - \frac{xY}{A} L_{10} - \frac{xZ}{A} L_{11}$$

$$f_2 = y + y' K_1 r^2 = \frac{X}{A} L_5 + \frac{Y}{A} L_6 + \frac{Z}{A} L_7 + \frac{1}{A} L_8 - \frac{yX}{A} L_9 - \frac{yY}{A} L_{10} - \frac{yZ}{A} L_{11}$$

where

$$x' = x - x_0$$

$$y' = y - y_0$$

$$r^2 = x'^2 + y'^2$$

$$A = L_9 X + L_{10} Y + L_{11} Z + 1$$

and assuming further

$$B = L_1 X + L_2 Y + L_3 Z + L_4$$

$$C = L_5 X + L_6 Y + L_7 Z + L_8$$

the equations can be written

$$-f_1 l_0 = x + x' K_1 r^2 - \frac{B}{A}$$

$$-f_2 l_0 = y + y' K_1 r^2 - \frac{C}{A}$$

and the coefficients of the Δ increments:

$$\frac{\partial f_1}{\partial L_1} = \frac{X}{A}$$

$$\frac{\partial f_1}{\partial L_2} = \frac{Y}{A}$$

$$\frac{\partial f_1}{\partial L_3} = \frac{Z}{A}$$

$$\frac{\partial f_1}{\partial L_4} = \frac{1}{A}$$

$$\frac{\partial f_1}{\partial L_5} = \frac{\partial f_1}{\partial L_6} = \frac{\partial f_1}{\partial L_7} = \frac{\partial f_1}{\partial L_8} = 0$$

$$\frac{\partial f_1}{\partial L_9} = -\frac{L_1 XX}{A^2} - \frac{L_2 YX}{A^2} - \frac{L_3 ZX}{A^2} + - \frac{L_4 X}{A^2} + \frac{xXA - xXL_9 X}{A^2} + + \frac{yXL_{10} X}{A^2} + \frac{zXL_{11} X}{A^2} =$$

$$= - \frac{X}{A^2} (L_1 X + L_2 Y + L_3 Z + L_4) + + \frac{xXA - xXL_9 X}{A^2} + \frac{xX}{A^2} (YL_{10} + ZL_{11} + 1) =$$

$$= - \frac{XB}{A^2} - \frac{xX}{A} + + \frac{xX}{A^2} (L_9 X + L_{10} Y + L_{11} Z + 1) =$$

$$= - \frac{XB}{A^2}$$

$$\frac{\partial f_1}{\partial L_{10}} = \dots = - \frac{YB}{A^2}$$

$$\frac{\partial f_1}{\partial L_{11}} = \dots = - \frac{ZB}{A^2}$$

$$\frac{\partial f_1}{\partial K_1} = - x' r^2$$

analogously for f_2

$$\frac{\partial f_2}{\partial L_1} = \frac{\partial f_2}{\partial L_2} = \frac{\partial f_2}{\partial L_3} = \frac{\partial f_2}{\partial L_4} = 0$$

$$\frac{\partial f_2}{\partial L_5} = \frac{X}{A}$$

$$\frac{\partial f_2}{\partial L_6} = \frac{Y}{A}$$

$$\frac{\partial f_2}{\partial L_7} = \frac{Z}{A}$$

$$\frac{\partial f_2}{\partial L_8} = \frac{1}{A}$$

$$\frac{\partial f_2}{\partial L_9} = -\frac{XC}{A^2}$$

$$\frac{\partial f_2}{\partial L_{10}} = -\frac{YC}{A^2}$$

$$\frac{\partial f_2}{\partial L_{11}} = -\frac{ZC}{A^2}$$

$$\frac{\partial f_2}{\partial K_1} = -y' r^2$$

For every control point it is possible to write two equations of the kind we have just seen; the system formatted by such equations, expressed in matrix form, for n points, is:

$$A\,\Delta L = l$$

where:

$$A = \begin{bmatrix} \frac{X_1}{A} & \frac{Y_1}{A} & \frac{Z_1}{A} & \frac{1}{A} & 0 & 0 & 0 & 0 & -\frac{X_1 B}{A^2} & -\frac{Y_1 B}{A^2} & -\frac{Z_1 B}{A^2} & -x_1' r_1^2 \\ 0 & 0 & 0 & 0 & \frac{X_1}{A} & \frac{Y_1}{A} & \frac{Z_1}{A} & \frac{1}{A} & -\frac{X_1 C}{A^2} & -\frac{Y_1 C}{A^2} & -\frac{ZC_1}{A^2} & -y_1' r_1^2 \\ \dots & \dots & \dots & \dots & & \dots & \dots & & & & & \\ \frac{X_n}{A} & \frac{Y_n}{A} & \frac{Z_n}{A} & \frac{1}{A} & 0 & 0 & 0 & 0 & -\frac{X_n B}{A^2} & -\frac{Y_n B}{A^2} & -\frac{Z_n B}{A^2} & -x_n' r^2 \\ 0 & 0 & 0 & 0 & \frac{X_n}{A} & \frac{Y_n}{A} & \frac{Z_n}{A} & \frac{1}{A} & -\frac{X_n C}{A^2} & -\frac{Y_n C}{A^2} & -\frac{ZC_n}{A^2} & -y_n' r_n^2 \end{bmatrix}$$

$$l = \begin{bmatrix} x_1 + x_1' K_1 r_1{}^2 - \frac{B}{A} \\ y_1 + y_1' K_1 r_1{}^2 - \frac{C}{A} \\ \dots \\ \dots \\ x_n + x_n' K_1 r_n{}^2 - \frac{B}{A} \\ y_n + y_n' K_1 r_n{}^2 - \frac{C}{A} \end{bmatrix} \qquad \Delta L = \begin{bmatrix} DL_1 \\ DL_2 \\ DL_3 \\ DL_4 \\ DL_5 \\ DL_6 \\ DL_7 \\ DL_8 \\ DL_9 \\ DL_{10} \\ DL_{11} \\ DK_1 \end{bmatrix}$$

with n> 6.

As it has already been said, it is essential to know the approximate initial values of the unknown $L_1 \dots L_{11}$, K_1 for the solution of the system; these values can be arrived at assuming $K_1=0$ and using the direct solution. At the end of the calculation procedure the program visualizes the value of σ_0^2 and those of the residuals of the equations; it asks the user if the values of L must be updated adding the values of ΔL or if the values of the preceding repetition will be used as definitive values.

The values of $L_1..L_{11}$, the calculated value of K_1 and their median squared remainders are shown visually on the monitor; also shown are the traditional parameters of orientation, x_c, y_c, f, ω, ϕ, κ, X_0, Y_0, Z_0 .

Condition equation on the parameters of the D.L.T.

It is possible to insert into the basic equations:

$$x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

the link that lies between the 11 D.L.T. parameters and the 9 independent parameters necessary for the transformation. That has been realized introducing two condition equations for the 11 parameters. The equations are:

$$(L_1^2 + L_2^2 + L_3^2) - (L_5^2 + L_6^2 + L_7^2) - (C^2 - B^2)/D = 0$$

where

$$A = (L_1 L_5 + L_2 L_6 + L_3 L_7)$$
$$B = (L_1 L_9 + L_2 L_{10} + L_3 L_{11})$$
$$C = (L_5 L_9 + L_6 L_{10} + L_7 L_{11})$$
$$D = (L_9^2 + L_{10}^2 + L_{11}^2)$$

These equations can be used in case the difference in scale should not be modeled and when there is a lack of orthogonality between the axes; in the software such equations have not been utilized in order to allow for the deformations introduced in the scanner that can be corrected with the similar transformation introduced utilizing all 11 parameters.

The multi-image restitution

After having calculated the 11 parameters of the D.L.T. and the K_1 term, isolating X,Y,Z, the pair of [4] equations can be written as follows:

$$x + x' K_1 r^2 - L_4 = X\,[L_1 - (x + x' K_1 r^2)\, L_9] + Y\,[L_2 - (x + x' K_1 r^2)\, L_{10}] + Z\,[L_3 - (x + x' K_1 r^2)\, L_{11}]$$

$$y + y' K_1 r^2 - L_8 = X\,[L_5 - (x + x' K_1 r^2)\, L_9] + Y\,[L_6 - (x + x' K_1 r^2)\, L_{10}] + Z\,[L_7 - (x + x' K_1 r^2)\, L_{11}]$$

Writing these equations for the n images in which the point to plot does appear, the following system is obtained:

$$A\,x = l$$

introducing the following terms in order to simplify the notation:

$$R_x = x + x' K_1 r^2$$

$$R_y = y + y' K_1 r^2$$

achieves:

$$A = \begin{bmatrix} L_{1_1} - R_{X_1} L_{9_1} & L_{2_1} - R_{X_1} L_{10_1} & L_{3_1} - R_{X_1} L_{11_1} \\ L_{5_1} - R_{Y_1} L_{9_1} & L_{6_1} - R_{Y_1} L_{10_1} & L_{7_1} - R_{Y_1} L_{11_1} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ L_{1n} - R_{X_n} L_{9n} & L_{2_n} - R_{X_n} L_{10_n} & L_{1_n} - R_{X_1} L_{11n} \\ L_{1n} - R_{X_n} L_{9n} & L_{6_n} - R_{y_n} L_{10_{n1}} & L_{7_n} - R_{y_n} L_{11_n} \end{bmatrix}$$

$$l = \begin{bmatrix} R_{x\,1} - L_{4\,1} \\ R_{y\,1} - L_{8\,1} \\ \dots \\ \dots \\ R_{x\,n} - L_{4\,n} \\ R_{y\,n} - L_{8\,n} \end{bmatrix} \qquad x = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

This system always has a lesser number of unknowns than that of the equations and therefore is resolved using the method of least squares.

At the end of the calculations the program provides:

- the estimated X, Y, Z coordinates which AutoCAD reads and utilizes for the restitution;
- the residual errors of the equations, s_0^2, and the standard deviation of X, Y e Z.

Limits of the D.L.T

Having already listed the advantages to using the D.L.T. now some of its main disadvantages must be indicate.

First and foremost, the D.L.T. needs at least 6 three dimensional control points in order to estimate the parameters: this involves an increase in the topographic work which, however, in the case of a strictly didactic exercise could be considered negligible.

Another negative factor (which is common to many photogrammetric applications), is the fact that the D.L.T. is very sensitive to the spatial distribution of the control points. A situation to be avoided is where all of the control points lie on a plane or are next to one another. In this case the solution becomes unstable due to the poor conditioning of the normal system. The points therefore must have a distribution that is the most varied possible compatibly with the depth the field of the photogram. Moreover, the control points should be positioned around the object to plot. This aspect is very important from a didactic point of view, in so far as it involves a particular attention to the geometric configuration of the survey on the part of the students. The indication of the value of the determinant of the normal matrix, carried over on the screen of the results of the orientation calculation, provides an index of the geometric configuration linked to the conditioning of the system.

What it will do

NM3-digit was realized as a “development instrument” where it is possible to experiment with algorithms and impose new applications.

The availability of the source code in Pascal that can be had upon request allows for the writing of new forms of the program.

In particular, the software could evolve itself according to the probable lines of development that are listed below:

- calculate the epipolar plane of every point ploted and show the outline of the image;
- allow for auto correlation

Even the part of the calculation could be the object of experimentation, directed at improving the characteristics: other algorithms of the calculation could be used, as an alternative to the D.L.T., classic collinear equations, in order to make use of, for example, photocameras.

Compared with the D.L.T. condition equations could also be inserted, in order to reduce the instability of the system; the possibilities are: define the condition equations on the object, give the opportunity to ponder an indicative value of the main distance and of the set point, introduce pass points.

But the most substantial additions to the program, in order to bring it closer to the more expensive software, is the integration of adjunctive software directly within the principle code to allow it to execute: the aerial triangulation, the automatic generation of the D.T.M., the normalization of the photographs, the rectificatin of the photographs, the orthophoto, and the projection of the images onto non-planar surfaces.

The authors

The authors of this paper have occupied themselves in the development of the analytical part and the designing of the software, based on the didactic experiences carried out in the courses of Professor Clemente di Thiene and Carlo Monti. The NM3-Digit has been written by Francesco Guerra and Eugenio Mario for the graduating thesis of the latter. In particular, Francesco Guerra oversaw the design of the numeric calculation and Eugenio Mario took care of the graphic interface.

Laboratory test
The first test was done using 7 images oriented with 23 control ponts surveied with angles and distances from 4 stations. The images are scanned camera Rollei 6006 photographs.
The restitution accuracy are shown in the next tables where can be read the topographi coordinates, the photogrammetic coordinates and their differences.

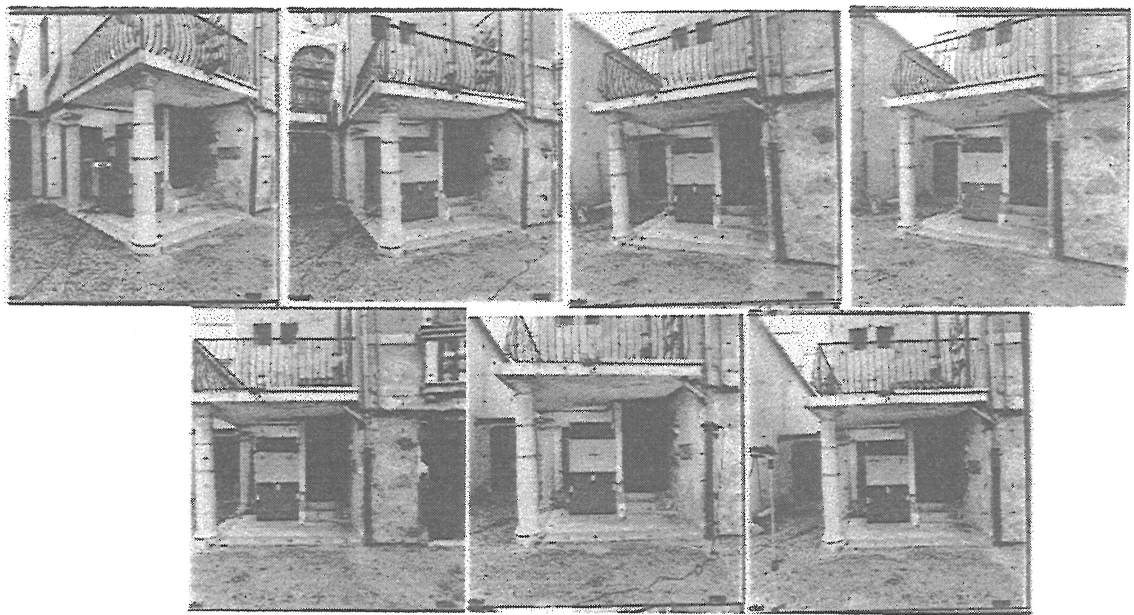


Fig. 1. Images

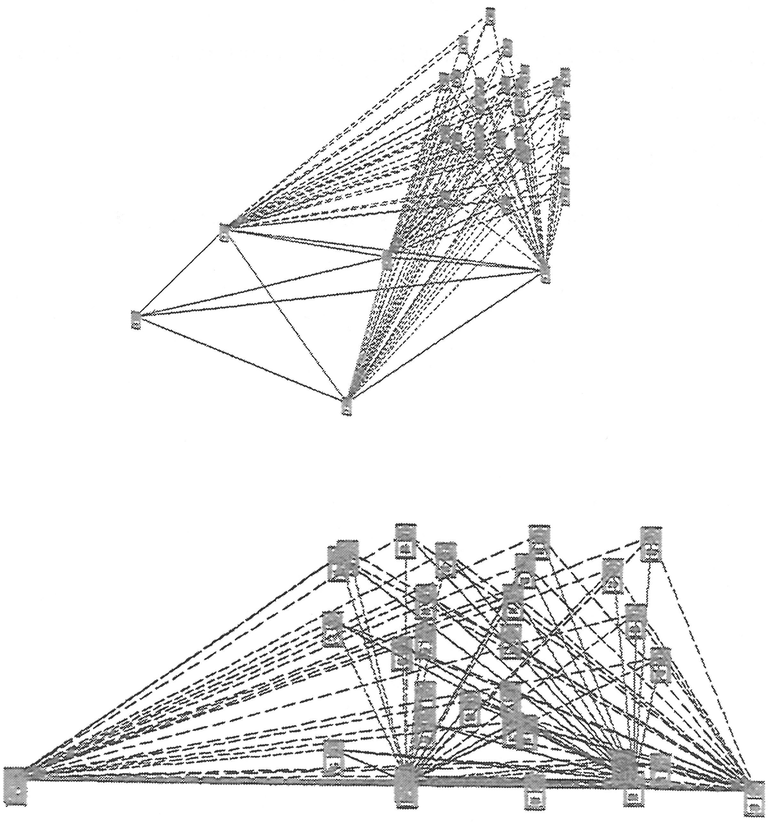


Fig. 2. The topographic net

Point	Topographic coordinates			Photogrammetric coordinates			Differences		
	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)	Δx (m)	Δy (m)	Δz (m)
1	3,978790	3,538196	2,524979	3,980104	3,535950	2,522782	-0,001314	0,002246	0,002197
2	5,340784	3,898351	2,522043	5,339027	3,905748	2,518710	0,001757	-0,007397	0,003333
3	6,497713	4,203215	2,518584	6,498951	4,209677	2,521002	-0,001238	-0,006462	-0,002418
4	3,973667	3,655477	0,026720	3,974153	3,655190	0,026537	-0,000486	0,000287	0,000183
5	6,605795	4,447138	0,172297	6,606476	4,450539	0,170592	-0,000681	-0,003401	0,001705
6	3,947624	3,713668	1,368121	3,947215	3,714857	1,370303	0,000409	-0,001189	-0,002182
7	6,597227	4,409111	1,266480	6,596841	4,417929	1,267077	0,000386	-0,008818	-0,000597
8	6,333075	5,122012	1,715597	6,334065	5,122827	1,716937	-0,000990	-0,000815	-0,001340
9	5,213523	5,390990	0,559818	5,214445	5,394856	0,558153	-0,000922	-0,003866	0,001665
11	5,081204	5,449670	1,899971	5,081162	5,454173	1,902543	0,000042	-0,004503	-0,002572
12	4,190128	5,199926	1,894403	4,187728	5,202873	1,895607	0,002400	-0,002947	-0,001204
13	4,186075	5,203496	0,574225	4,187086	5,205615	0,574249	-0,001011	-0,002119	-0,000024
14	5,079735	5,454494	0,589865	5,078818	5,450538	0,587850	0,000917	0,003956	0,002015
21	6,196828	5,448353	0,216831	6,195087	5,453011	0,216030	0,001741	-0,004658	0,000801
22	4,645674	5,328687	0,802078	4,649520	5,327522	0,800508	-0,003846	0,001165	0,001570

Fig. 3. Thopografical and photogrammetrical comparison

The table underlines the software is able to execute right orientation and restitution. But it's important to understand that the test condition are particular:

- high accuracy of control points;
- optimal distribution of control points related to D.L.T.;
- marked points are collimated.

These are not the pratical condition of an architectural survey: in fact the accurancy is worse.

The survey of the facade of Villa Della Torre Chapel

This is a pratical exemple. The surveied object is the principle facade of Villa Della Torre in Fumane (VR) designed by Michele Sammicheli.

Images

The photogrammetrical model is obteined scanning 3 images (Rollei 6006m camera)
The 12 control points have a uniform distribution on the facade plane.

Fig. 4. Image1, Image2, Image3

Restitution

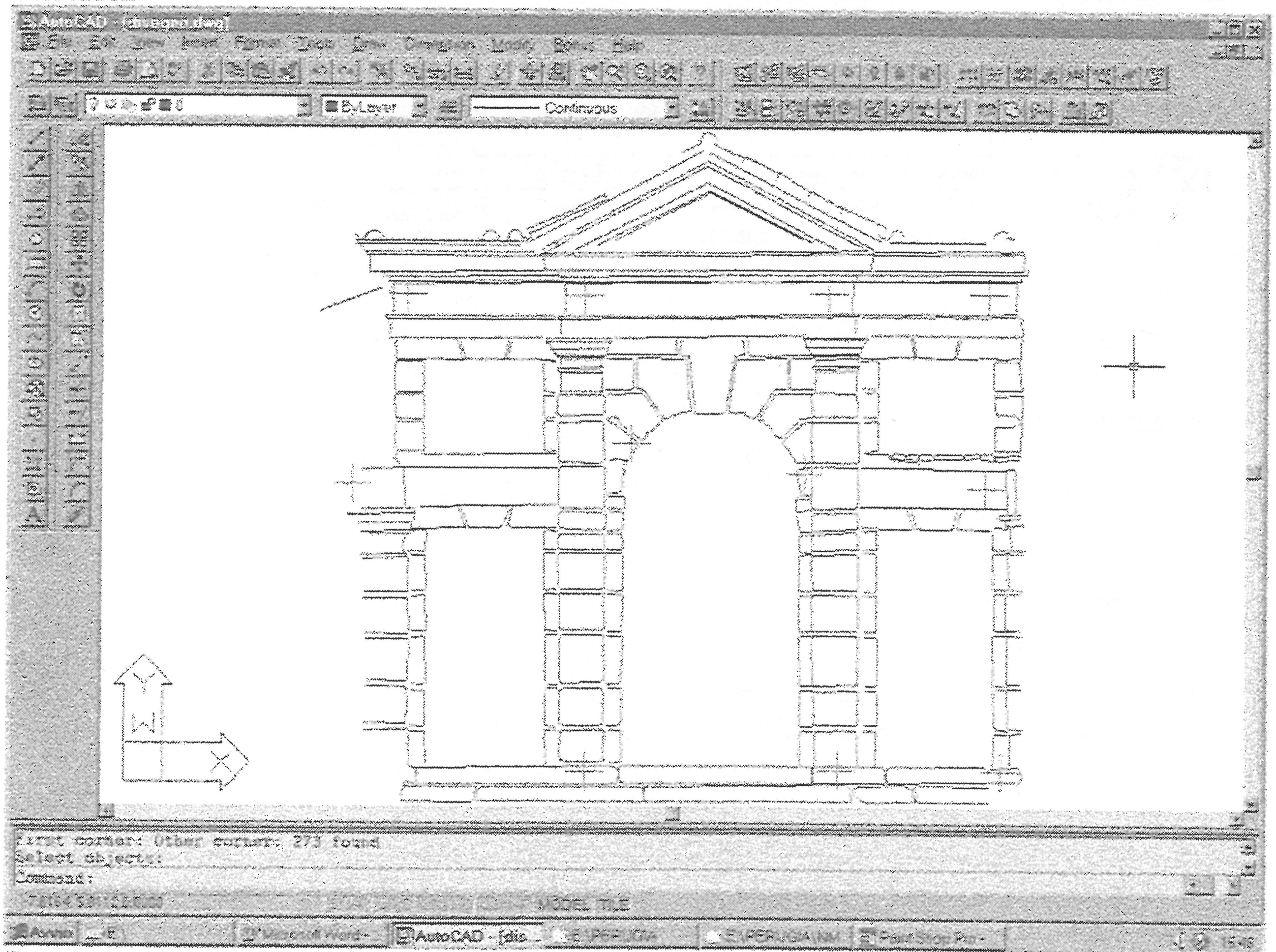


Fig.5.Vectorial drawing from the restituited individual points

Orientation

The orientation parameters are shown in a window where even the D.L.T. parameters are contained

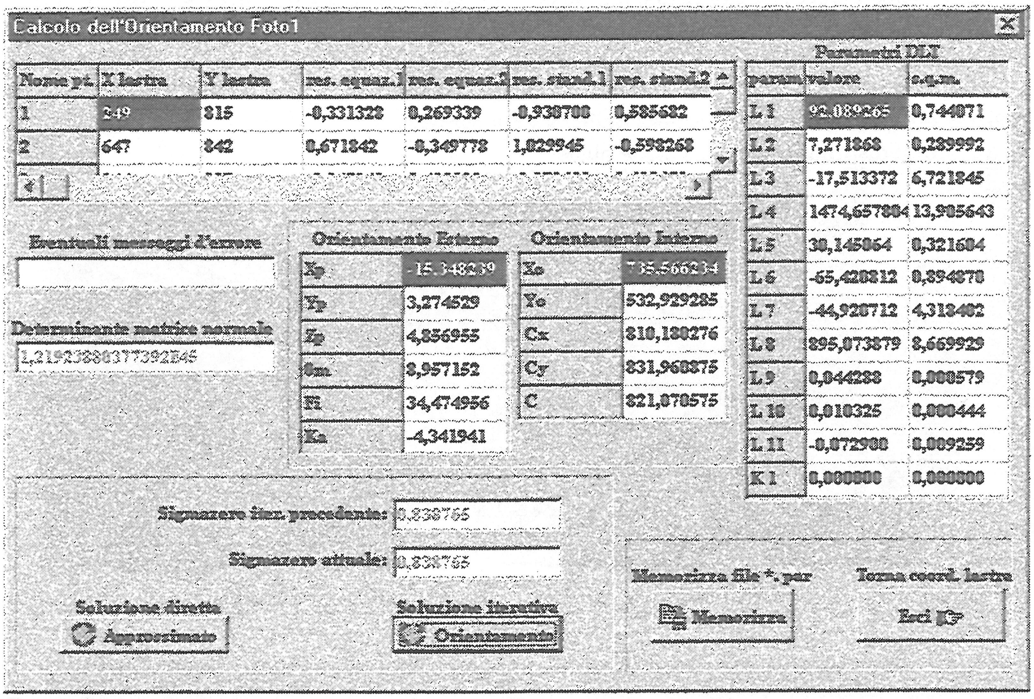


Fig. 6. Image 1 orientation.

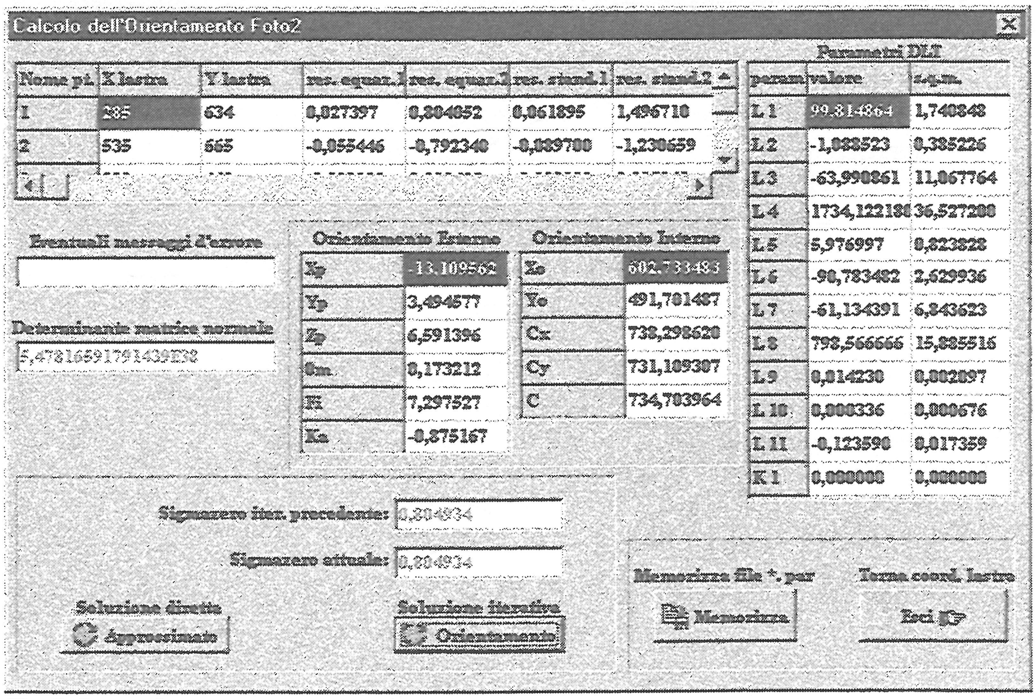


Fig.7. Image 2 orientation.

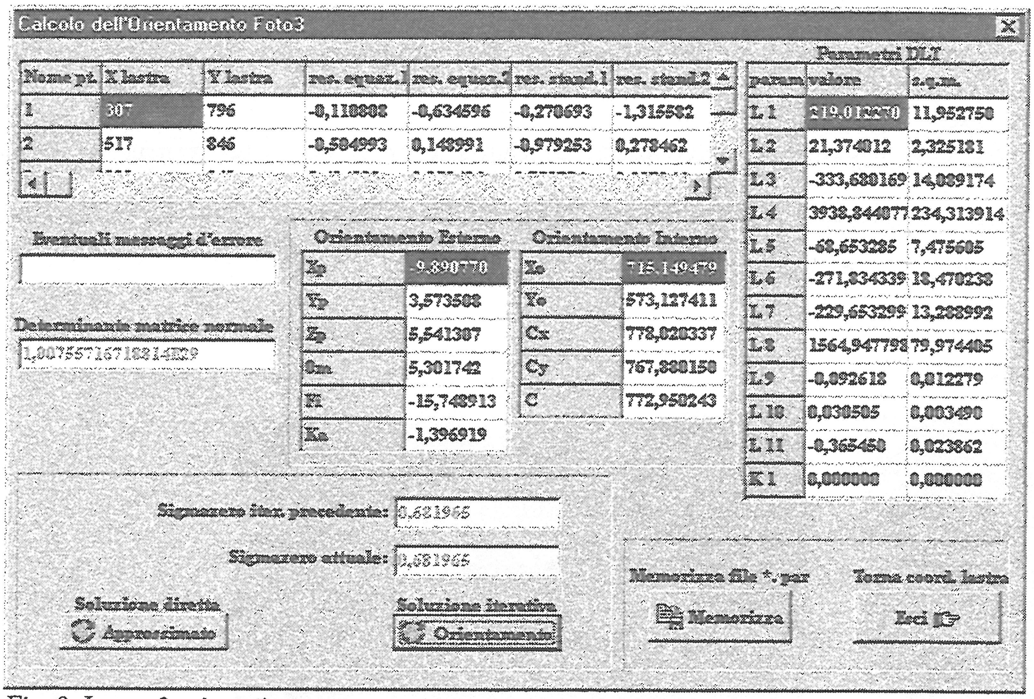


Fig. 8. Image 3 orientation.

Numerical restitution

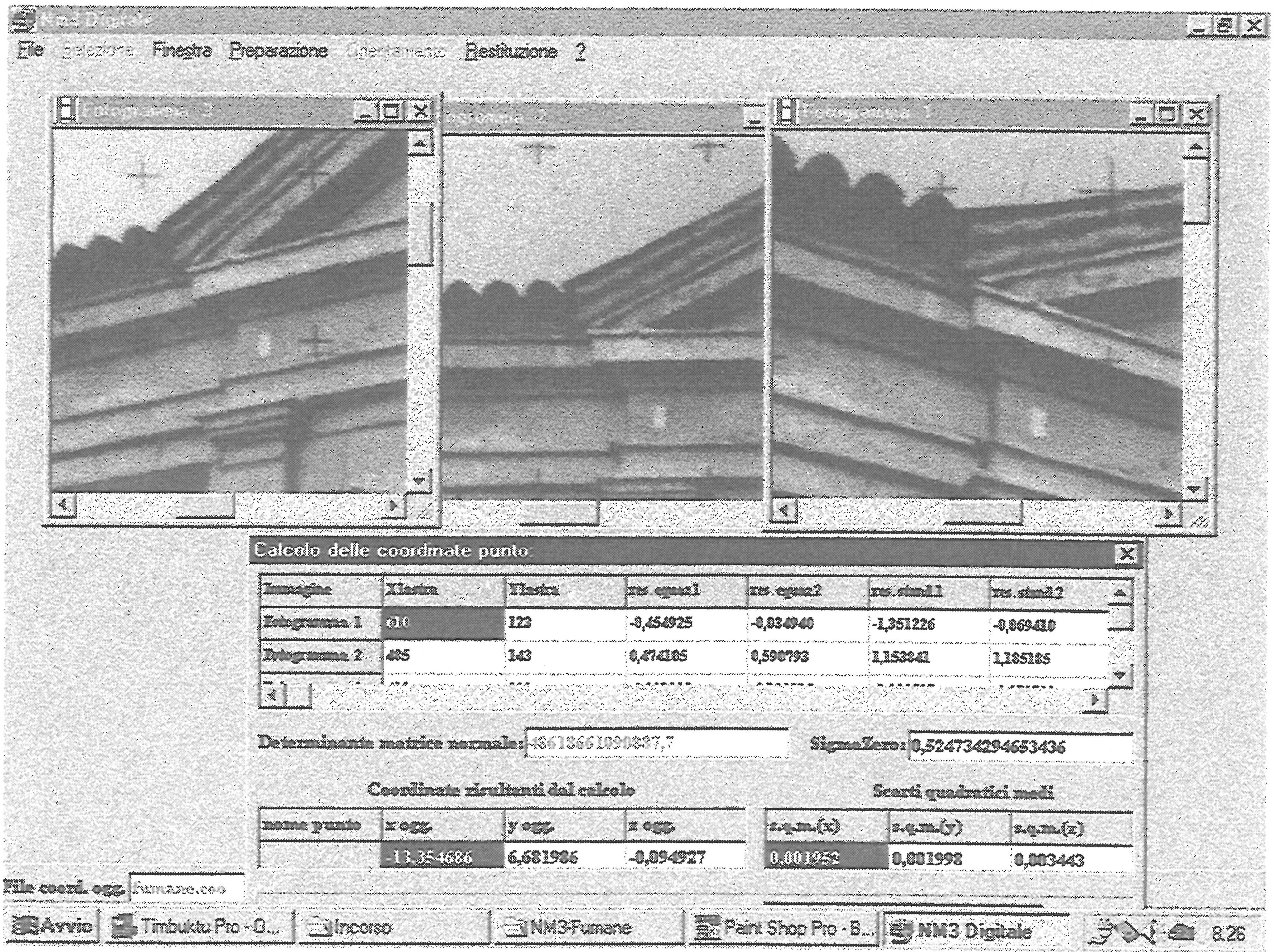


Fig. 9. Numerical restitution window.

Deformed images test
The scanned image are modified to introduce geometrical deformation. These deformation are:

- a different scale in x and y axes,
- a shear between x and y axes.

The aim is to test the algorithm, modelling the deformation, that allows to orient the images guaranteing a right restitution.
Original and deformed images had been oriented with the same control points. Then the same points had been restituted from the 2 set of images. The coordinates comparison underlines it's possible to have a right restitution starting from deformed images, and the collimation differences are caused by casual errors.

Deformation
Image 2: deformation exemple.

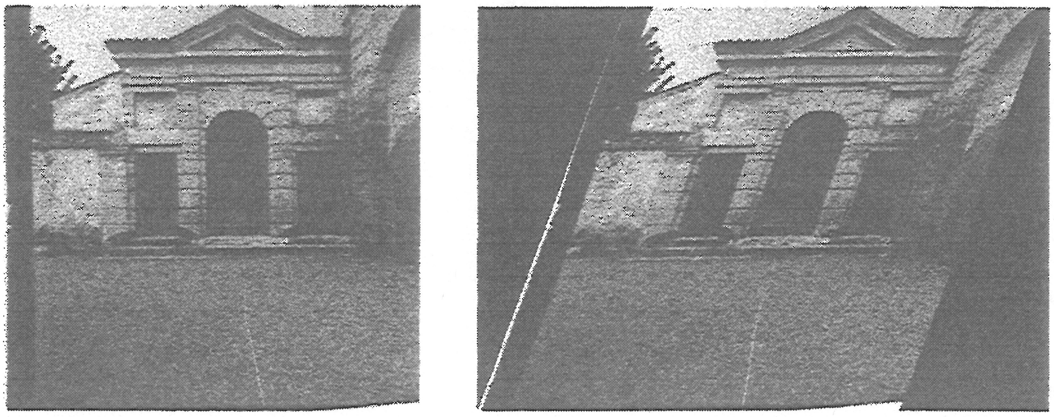


Fig. 10. Shear between axes: case A.

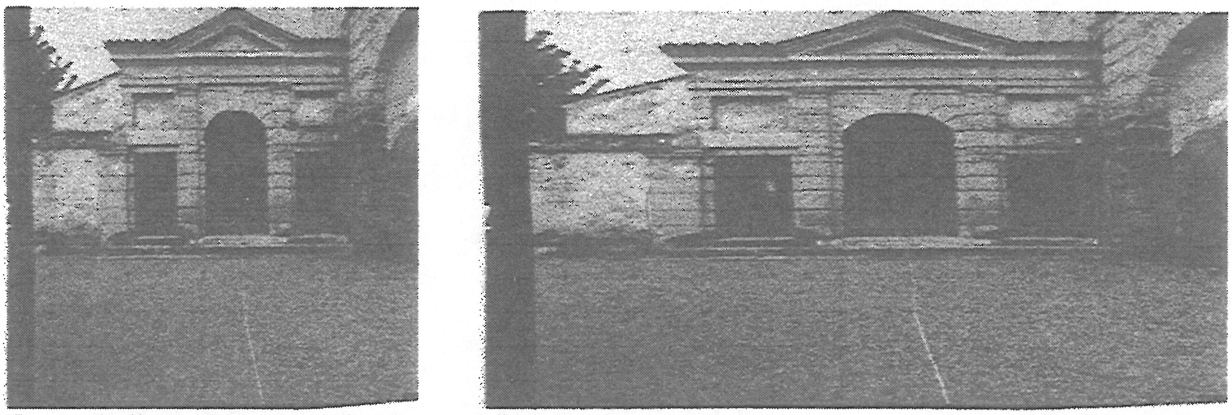


Fig. 11. Different scale in x and y axes: case B.

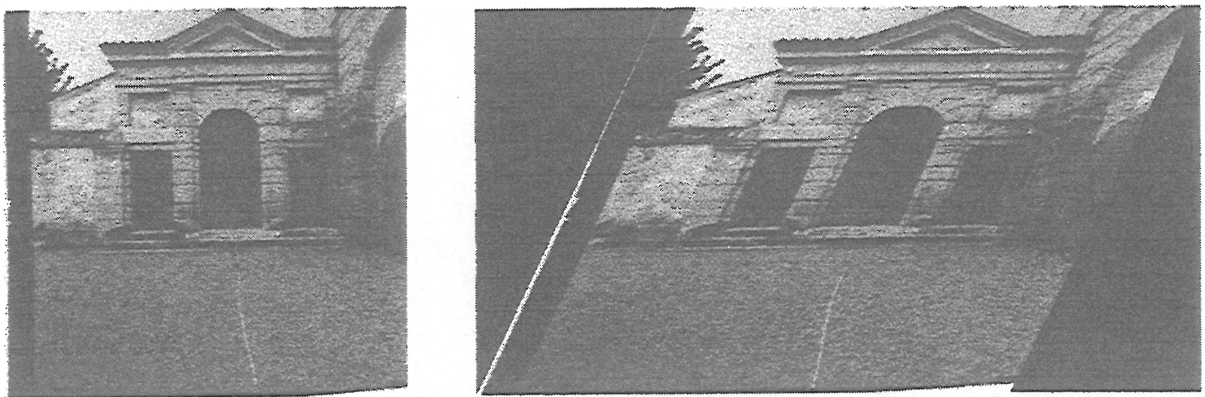


Fig. 12. Both above-mentioned deformations: case C.

Deformated images orientation.

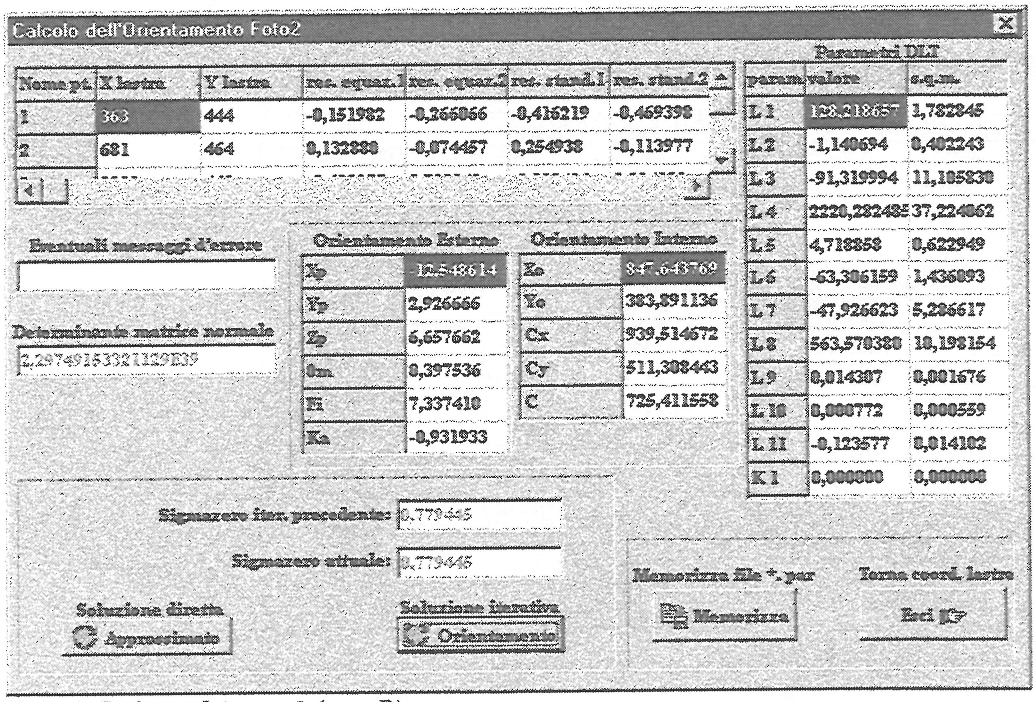
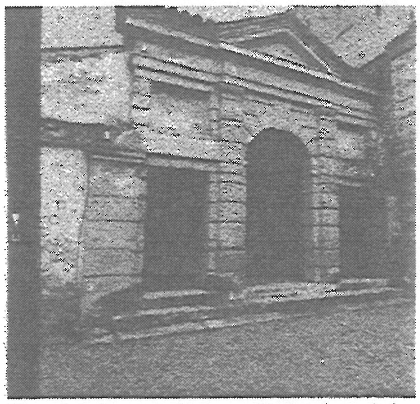


Fig. 13. Deformed image 2 (caso B).



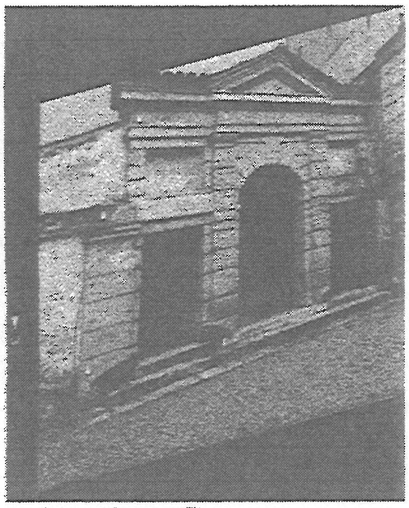


Fig. 14. Image 1:the original and the deformed one (caso C).

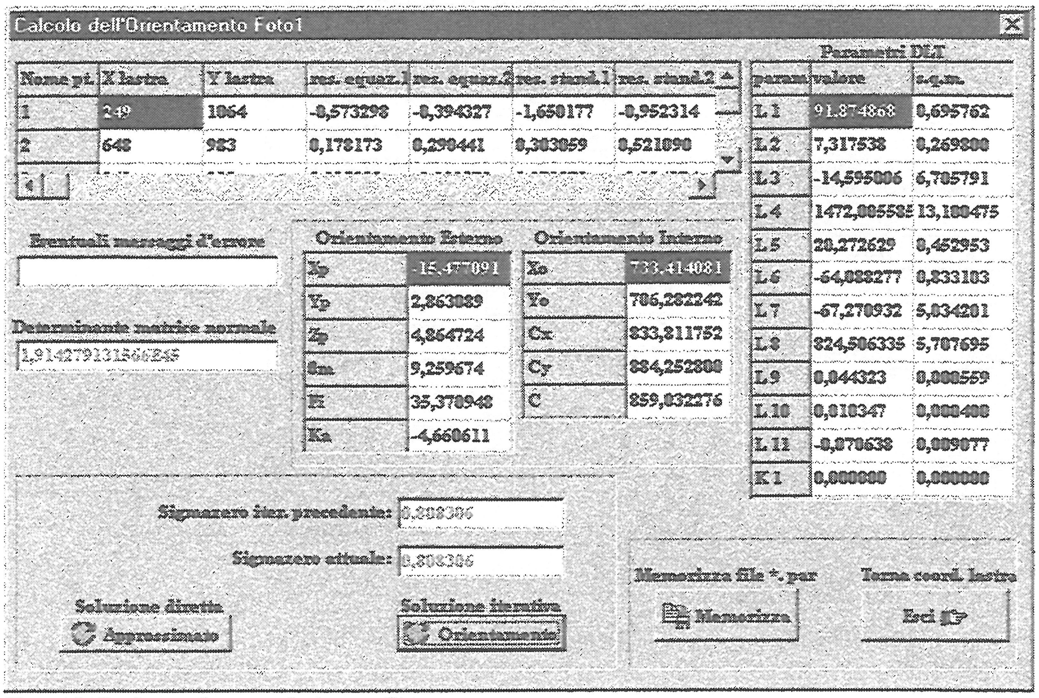


Fig. 15. Deformed image I (caso C).

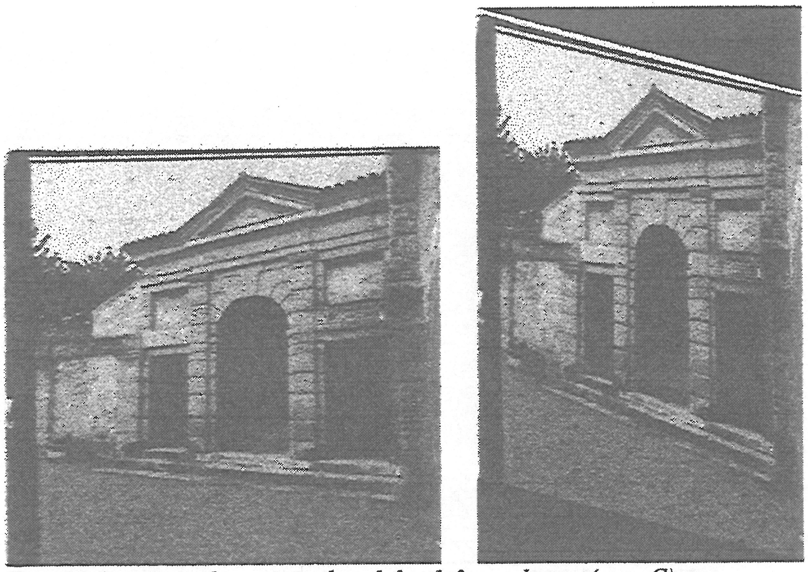


Fig. 16. Image 3: the original and the deformed one (case C).

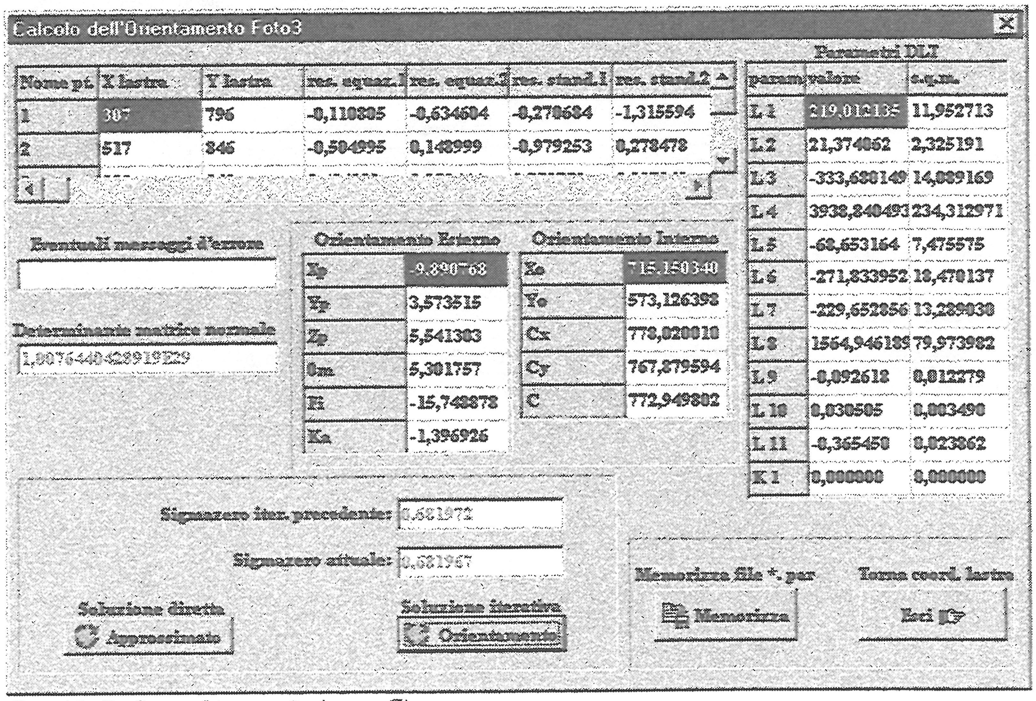


Fig. 17. Deformed image 3 (caso C)..

Deformed image restitution:

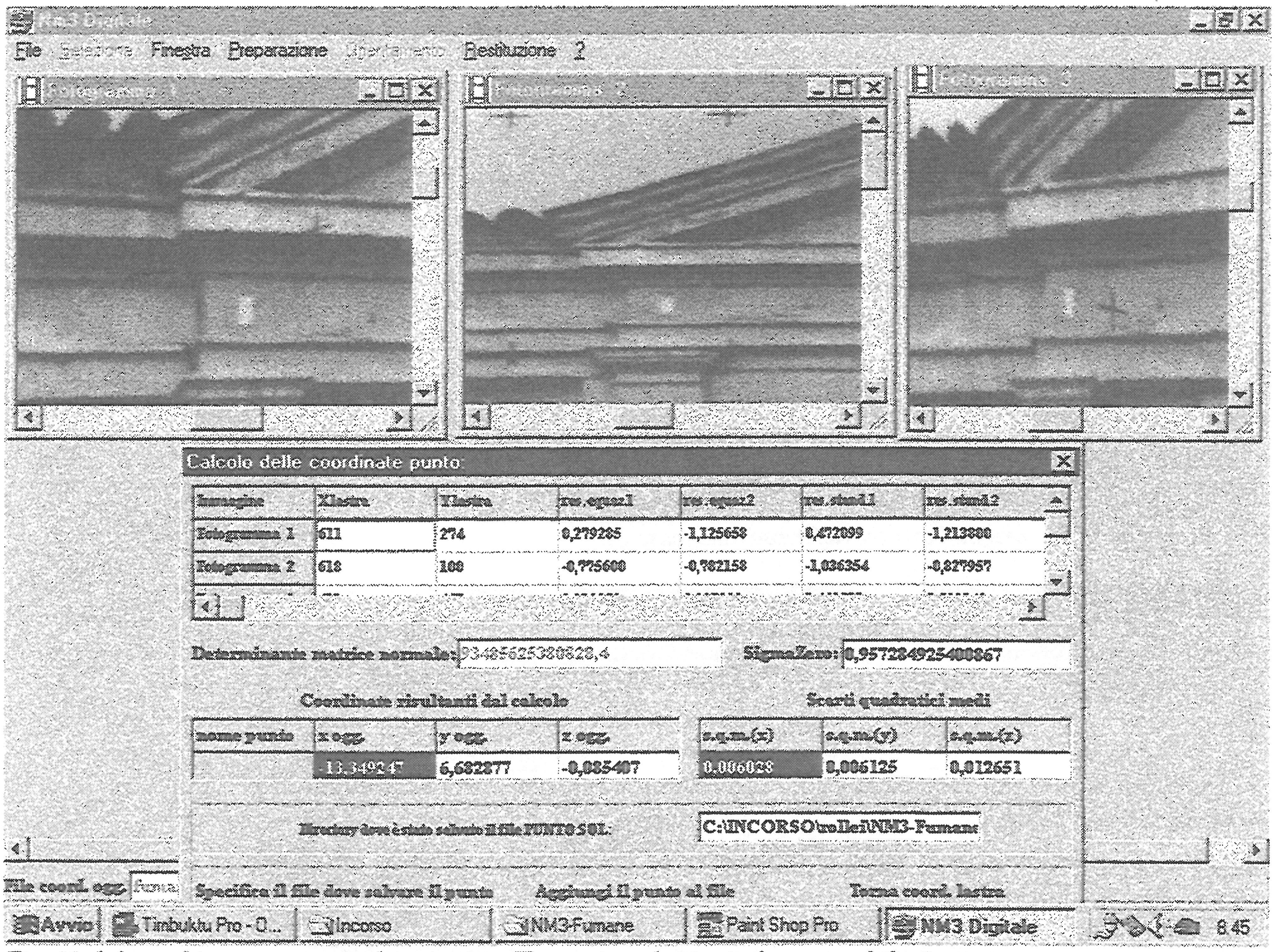


Fig. 18.deformed images numerical restitution. The restituted point is the same of fig.9.

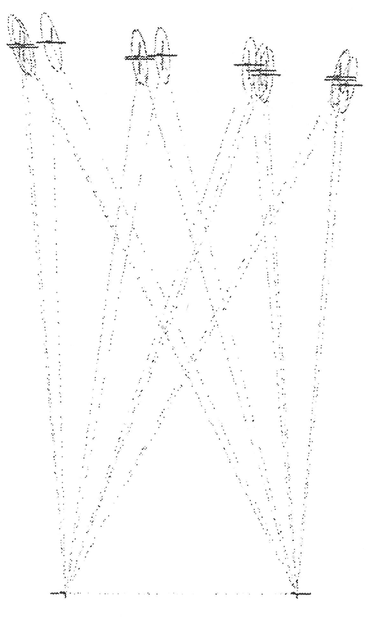


Fig. 19. Image control points scheme.