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DESCRIBING TOPOLOGICAL RELATIONS WITH VORONOI-BASED 9-INTERSECTION MODEL

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ABSTRACT

The Voronoi-based 9-intersection model (V9I) is a modified version of the original point-set-based 9-intersection model. The modification is made by replacing the complement (exterior) of an entity with its Voronoi region. It is shown in this paper that it is possible to solve the three major problems associated with the original 9-intersection model with the new V9I model.

1 INTRODUCTION

Over the past two decades, research has been conducted on how to apply fundamental mathematical theories for modeling and describing spatial relationships (Peuquet 1986, Jungert 1988, Chang et al. 1989, Lee and Hsu 1990, Egenhofer and Franzosa 1991, Egenhofer and Al-taha 1992, Smith and Park 1992, Cui et al. 1993, Kainz et al. 1993). A point-set topology based formal representation of topological relations has been developed (Guting 1988, Pullar 1988, Egenhofer 1989). The results of such formalization are the so-called 4-intersection and 9-intersection models (Egenhofer and Franzosa 1991). The latter is an improvement on the former (Egenhofer et al. 1993). In these models, the topological relations between two entities A and B are defined in terms of the intersections of A's boundary (∂A), interior (A^0) and exterior (A^-) with B's boundary (∂B), interior (B^0) and exterior (B^-). The exterior of an entity is then represented by its complement.

The 9-intersection model is the most comprehensive model for topological spatial relations so far. It has been used or extended for examining the possible topological relations between regions in discrete space (Egenhofer and Sharma,1993; Winter, 1995), modeling conceptual neighborhoods of topological line-region relations (Egenhofer and Mark, 1995), grouping the very large number of different topological relationships for point, line and area features into a small sets of meaningful relations (Clementini et.al. 1993), describing the directional relationships between arbitrary shapes and flow direction relationships (Abdelmoty and Williams, 1994; Papadias and Theodoridis, 1997), deriving the composition of two binary topological relations (Egenhofer,1991), describing changes to topological relationships by introducing a Closest-Topological-Relationship-Graph and the concept of a topological distance (Egenhofer and Al-Taha, 1992), analyzing the distribution of topological relations in geographic datasets (Florence III and Egenhofer, 1996), as well as formalizing the spatio-temporal relations between the father-son parcels during the process of land subdivision (Chang and Chen, 1997). The findings of these investigations have significantly contributed to the development of the state-of-art spatial data models and spatial query functionality (Egenhofer and Mark, 1995; Mark et. al., 1995; Papadis and Theodoridis, 1997).

However, even the 9-interscetion model has problems both in theory and in practice. Examples of the former are difficulties in distinguishing different disjoint relations and relations between complex entities with holes. An example of the latter is the difficulty or impossibility of computing the intersections with an entity's complement since the complements are infinite. To improve this situation, a modified model, called the Voronoi-based 9-intersection model (V9I model), was proposed by the authors by replacing the complements of spatial entities with their Voronoi regions (Chen et al., 1997).

The new V9I model is introduced in the following section based on the analysis of the three major problems associated with the original 9-intersection model.

2 V9I MODEL

It was found that there exist there fundamental shortcomings associated with the original point-set based 9-intersection model. One is that it fails to distinguish certain disjoint relations, such as in Fig.1. Three different disjoint relations between object A and object B have the same 9-intersection due to the infinitive complements of the objects. A's complement intersects with the boundaries, interiors and complements of object B. It's boundary and interior also intersect with the complements of objects B. The five complement-related sets $\partial A \bullet B^-$, $A^0 \bullet B^-$, $A^- \bullet \partial B$, $A^- \bullet B^0$ and $A^- \bullet B^-$ always take the non-empty value $-\varnothing$. The result is that the complements could not play roles in distinguishing these disjoint relations. However, about 80% of spatial relations are disjoint [Florence and Egenhofer, 1996].

The second problem associated with the complement-based 9-intersection is that it fails to identify the topological relations between two regions with holes as shown in Figure 2. For the cover and covered-by relations between A and B in Fig. 2a, two distinct 9-intesections can be found. In that case, A and B are simple homogeneously 2-dimensional, connected areas objects [Egenhofer and Franzosa 1991, Clementini et al. 1993]. However, the 9-intersection would be the same when A or B has a hole, as shown in Fig.2b. The interior and boundary of the object (i.e., A) falling in the hole do not intersect the interior of the other object (i.e., B), so $\partial A \bullet B^0 = \varnothing$, $A^0 \bullet B^0 = \varnothing$, and $A^0 \bullet \partial B = \varnothing$. Moreover, the five

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complement-related intersections all take non-empty values. Some other examples are illustrated in Fig.2c, 2d and 2e. Indeed, this model can only deal with simple entities such as homogeneous, 2-dimensioanl and connected areas, and lines with exactly two end points (Egenhofer 1993). If there exists a hole in a region, each of these regions are seperated into its generalized regions and holes, and the combinatorial intersections of the generalized regions and holes are then examined (Egenhofer et al., 1994).

It is also quite computational intensive or even impossible to compute the five intersections with an entity's complement $\partial A \bullet B^-$, $A^0 \bullet B^-$, $A^- \bullet \partial B$, $A^- \bullet B^0$ and $A^- \bullet B^-$ since the complement of an object is the set of all points of R^2 not contained in that object. On one hand, it is difficult to calculate the intersections of two object's components from their geometric data automatically. Qualitative analysis is required for deriving the 9-intersections. On the other hand, it is also difficult to derive automatically those spatial objects who satisfying given 9-intersection. In other words, it is difficult to manipulate the spatial relations with the complement-based 9-intersections.
The following two requirements for the exterior must be met if one would solve the above problems: the exterior of an entity should be as small as possible; and the exteriors of two disjoint entities must be exclusive. The Voronoi region as shown in Fig.3 seems to be the 'if and only if' candidate for the exterior of a spatial entity because the Voronoi region of an entity has a special meaning – the 'influence region' of itself and is defined as the area containing all locations closer to itself than to any other. Suppose we have a set of spatial objects, SO={O_1,…, O_i ,…, O_n} {$1 \le n \le \infty$ }, in IR^2, O_i may be a point object O_P, or line object O_L or area object O_A. An area object is not necessarily convex, and may have holes in which another area may exist. The object Voronoi region of O_i (called O^V) can be defined as

$$V(O_i)=\{p|ds(p, O_i) \le ds(p,O_j), j\neq i, i, j\in In \} \quad (1)$$

Indeed, a Voronoi-based tesselation (Voronoi Diagram) is closer to human perceptions, poses a variety of challenges to the 'usual way of doing things' in GISs (Gold, 1989, 1991, 1992; Wright and Goodchild, 1997) and has found wide applications [Yang and Gold,1994; Gold et al, 1996,; Edwrads et al., 1996; Hu and Chen, 1996; Chen and Cui, 1997]. By replacing the complement of an object with its Voronoi region, a new Voronoi-based 9-Intersection (called V9I briefly) framework can be formulated as the following:

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^o & \partial A \cap B^V \\ A^o \cap \partial B & A^o \cap B^o & A^o \cap B^V \\ A^V \cap \partial B & A^V \cap B^o & A^V \cap B^V \end{bmatrix} \quad (2)$$

where A^V is object A's Voronoi region, B^V is object B's Voronoi region. $\partial A \bullet B^V$, $A^0 \bullet B^V$, $A^V \bullet \partial B$, $A^V \bullet B^0$ and $A^V \bullet B^V$ are the five new intersections related to Voronoi regions and they can be easily computed.

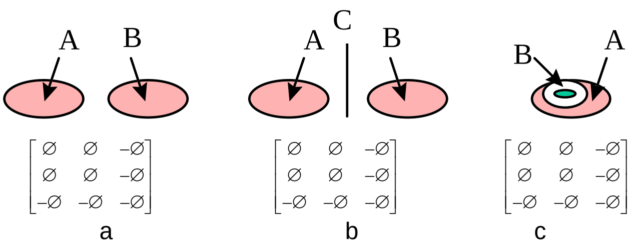

Fig. 1: ***Different disjoint relations with same 9-intersection***

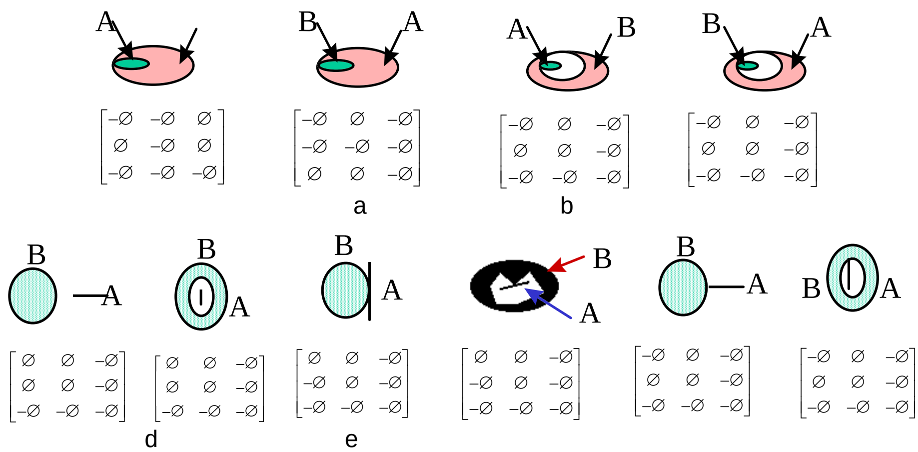

Fig. 2: ***Problems caused by the objects with holes***

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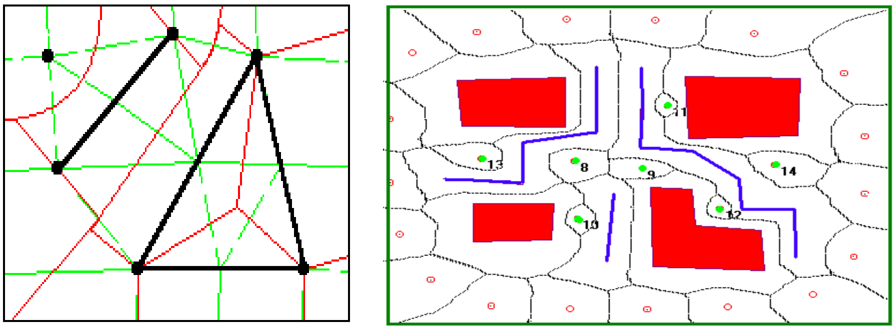


Fig.3: ***Voronoi diagram of point, line and area objects***

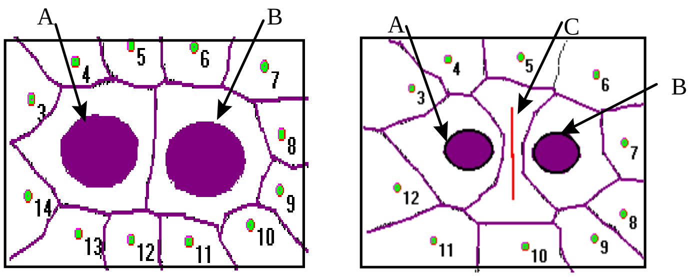
3 DISTINGUISHING TOPOLOGICAL RELATIONS WITH V9I

3.1 Disjoint relations

One of the advantages of the new V9I model is that it is possible to distinguish disjoint relations since each object has limited neighbors instead of having relations with all other objects. $A^v \bullet B^v$ would be non-empty when two objects are adjacent, such as A and B in Fig. 4a. It is because the Voronoi region of A share the same boundary with that of B. When there is an object C between A and B (Fig.4b), their Voronoi regions are separated by that of C and $A^v \bullet B^v$ is empty. It is therefore possible to distinguish adjacent relations from other disjoint relations with the V9I. More adjacent relations could be defined and derived with the Voronoi diagram, such as immediate neighbor, nearest neighbor, second-nearest, lateral neighbor, tracing neighbor, etc. [Chen et. al., 1997].

3.2 Relations between complex objects with holes

The other advantage of the V9I is being able to deal with objects with holes as shown in Fig.5. When the boundary of object A meets with that of object B, their Voronoi regions would also meet according to the definition of Voronoi diagram. $\partial A \bullet \partial B$ and $A^v \bullet B^v$ would both take non-empty values in this case as shown in Fig.5a. In addition, the boundary of object A meets with B^v and B's boundary meet with A^v. The Voronoi-based 9-intersection for the *meet* relation between area objects is therefore different from the original 9-intersection. If object A's boundary meets the inner boundary of B which has a hole as shown in Fig.5b, A's Voronoi region intersects B's inner boundary resulting in $\partial A \bullet \partial B = -\varnothing$, $A^v \bullet \partial B = -\varnothing$, $A^v \bullet B^v = -\varnothing$ and $\partial A \bullet B^v = -\varnothing$. Moreover, the whole body of A is contained in the hole of B, we say A's interior overlaps with the convex of B and $A^o \bullet B^v = -\varnothing$. The example shown in Fig.5b has the same original 9-intersection, but has a different Voronoi-based 9-intersection than Fig.5a. The example illustrated in Fig.5d is a *contained-by* relation which has the same 9-intersection with the *contains* relation shown in Fig.5e. The Voronoi regions touch and there is not intersection of boundaries and interiors between the two objects. However, the boundary and interior of the contained object intersect with the Voronoi convex of the other object. Another example is given by Fig. 5f where a line meets a homogeneously 2-dimensional and connected area B and the line falls into an area's hole in Fig.5g. It can be seen from these examples that it is possible to distinguish relations between complex objects with holes using the Voronoi-based 9-interscection model.



$$\begin{bmatrix} \varnothing & \varnothing & -\varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \quad \begin{bmatrix} \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & -\varnothing \end{bmatrix} \qquad \begin{bmatrix} \varnothing & \varnothing & -\varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \quad \begin{bmatrix} \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \end{bmatrix}$$

a b

Fig.4: ***Distinguishing disjoint relations with V9I***

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

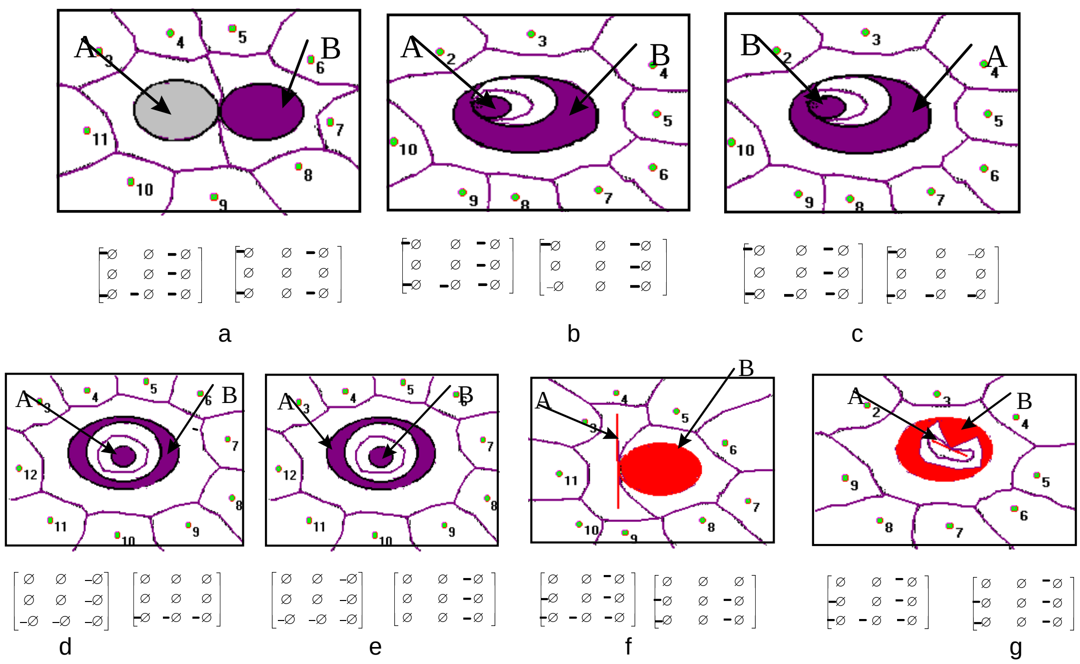


Fig.5: ***Distinguishing relations between complex objects with V9***

4 FORMAL REPRESENTATION OF TOPOLOGICAL RELATIONS WITH V9I

Topological relations between point, line and area objects have been formalized with the new V9I model, including relations between area-area, line-line, line-area, point-point, point-line and point-area objects. Some of the results are listed in Table 1.

Tab.1: ***Distinguished topological relationships using V9I***

	Case	Result
AA	Area/Area	13
LL	Line/Line	8
LA	Line/Area	13
PP	Point/Point	3
PL	Point/line	4
PA	Point/Area	5

Among the thirteen topologically distinct relationships between two areas characterized with the V9I, seven of them could not be distinguished using the original 9-interscetion model. Some of the distinguished relations between line-line objects are shown in Fig.6

The superiority of the V9I model can also be explained methodologically. That is, this model is an integrated model of the two possible approaches to formalize spatial relations - *intersection*-based and *interaction*-based models as classified by Abdelmoty et.al. (1994). In the former, an entity is represented in terms of its components and the relationships and the relationships are the result of the combinatorial intersection of those components, with the original 9-intersection model being the typical example. In the latter, the body of an entity is considered as a whole and is not decomposed into its components. This is proved by the fact that the four sets $\partial A \bullet \partial B$, $\partial A \bullet B^0$, $\partial B \bullet A^0$ and $A^0 \bullet B^0$ of the V9I take the intersection between objects into account and the interactions between adjacent objects can be distinguished with the five sets $\partial A \bullet B^V$, $A^0 \bullet B^V$, $A^V \bullet \partial B$, $A^V \bullet B^0$ and $A^V \bullet B^V$ by generating a Voronoi region for the whole body of each spatial entity.

5 FURTHER INVESTIGATION

It is possible or easier to compute the five exterior-based intersections (i.e., $\partial A \bullet B^V$, $A^0 \bullet B^V$, $A^V \bullet \partial B$, $A^V \bullet B^0$ and $A^V \bullet B^V$) since the Voronoi regions of each object can be generated and manipulated. It makes it easier to query and manipulate about the topological relations between two given objects with the new 9-interscetion model. However, lots of issues regarding the proposed V9I model remain to be examined and developed.

5.1 Development of a special toolkit for manipulating spatial relations with V9I

It is very helpful if the 9-intersection and its corresponding semantics of spatial relations could be derived when two objects are located. On the other hand, when a 9-intersection is given, it is also very essential to retrieve the spatial objects who satisfy such spatial relation. A special toolkit is under development by the authors now for derive the V9I from the geometry or spatial location of geographic entities. With this tollkit, users can then examine whether a specific spatial relation occurs with their expectations and GIS researchers can easily investigate what spatial relation exist for a given spatial dataset.

5.2 Inferencing spatial relations with the V9I

Since the Voronoi diagram forms a pattern of packed convex polygons covering the whole space, all spatial objects are linked together by their Voronoi regions. This makes it possible to deduce or reason about the

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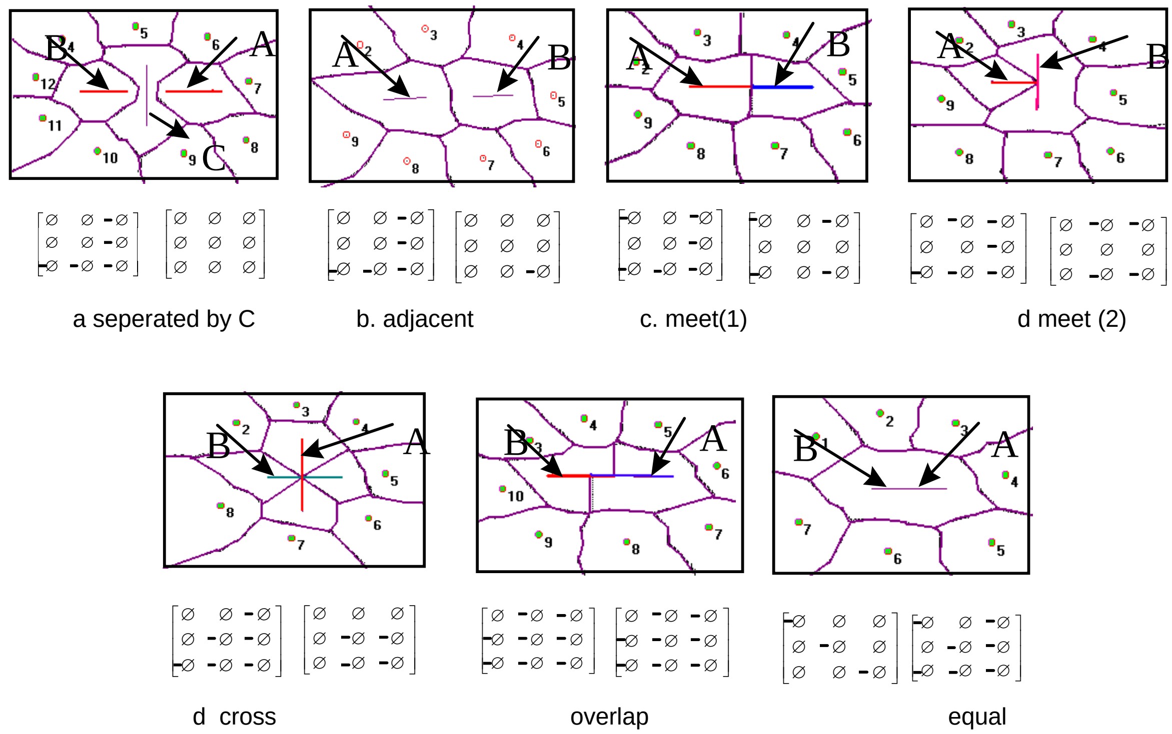

Fig.6: ***Some relations between line-line objects with V9***

relations between any two spatial objects with the Voronoi tessellation. The constraints among objects under a given spatial relation can be translated into VNI templates (or primitives), i.e, a pattern of empty, non-empty or arbitrary intersections representing the constraints among interiors, boundaries and Voronoi regions. Aggregates of two or more V9I templates would be used for defining the compound spatial relations. One of the applications would be deriving the contiguity, connectivity and inclusion directly from the spaggette data set.

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