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Topological Data Modelling for 3D GIS

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Abstract

3D data modelling is one of the key problems within research works of 3D GIS. This paper focused on the research of a topological model based on space partition for 3D GIS. Firstly the formal definition of 3D spatial features were given. Then the methods for calculating topological properties to support simplexes topologically equivalent to an n-manifold with (n-1)-pseudomanifold boundary cycles was introduced. After that the formal framework to describe the topological relationships between 3D spatial features was been given. Finally we gives geometric structure of 3D spatial features and presents the topology model based on geometric structure of 3D spatial features.

Key Words: 3D GIS, topological data model, space partition, topological properties, topological relationships

1. Introduction

Although our perception and actual universe is three-dimensional, spatial features in a geographic information system are for the time being mostly managed in a two-dimensional way. Therefore the present research activities pay special attention to the development of a 3D GIS. Within research works of them, the 3D data modelling is one of the key problems. 3D spatial data model is the tools to describe and represent 3D actual university in computer. In a computer-based GIS, the information for a geographic feature has four major component: its geographic position, its attributes, its spatial relationships and time. There are two fundamental approaches to represent them. One is the raster model, the other is vector model. It is difficult for the raster model to represent topological relationships whereas the vector model provided efficient encoding of topology. Vector model can be subdivided into three sub-categories: the spaghetti model, the triangulated irregular network (TIN) and the topological model. The topological model representing a subdivision with nodes, line, faces and bodies. It has been established that the topological model is richer than TIN network that it is richer than the spaghetti mode [Karine Zeitouni et al. , 1996], and as a result, more efficient implementation of operations that require topological information, such as the efficiency of spatial data organization, spatial analysis, spatial query, spatial reasoning and consistency test. So it is necessary to pay spatial attendee to 3D topological data model.

This paper focused on the research of a topological model based on space partition for 3D GIS. The rest of this paper is laid out as follows: Section 2 gives the formal definition of 3D spatial features. Section 3 introduces the methods for calculating topological properties to support simplexes topologically equivalent to an n-manifold with (n-1)-pseudomanifold boundary cycles. Section 4 gives the formal description of topological relationships between 3D spatial features. Section 5 presents the topology model based on geometric structure of 3D spatial features. Finally Section 6 gives the conclusion and future work plans.

2. The Formal Definition Of 3D Spatial Features

In terms of point-set topology, an n-dimensional manifold or n-manifold M^n is a topological space X in which any point $x \in X$ has a neighborhood homeomorphic to an open unit n-dimensional disk

$$D^n = \{(x_1, ..., x_n) \in R^n \mid (x_1^2 + ... + x_n^2)^{1/2} < 1\}.$$

An n-manifold M^n with boundary is defined in exactly the same way except points on the boundary have neighborhoods homeomorphic to an open unit n-dimensional hemi-disk

$$D^n_{1/2} = \{ (x_1, ..., x_n) \in R^n \mid (x_1^2 + ...+ x_n^2)^{1/2} < 1 \text{ and } x_1 \geq 0 \}.$$

From the number of important intrinsic and extrinsic topological properties of n-manifold M^n and the continuous map about homeomorphism and homotopy, the base space of the domain for 3D GIS are formed by n-manifolds and (n+1)- manifolds with one or more n-manifold boundary cycles ($0 \leq n \leq 2$). Each feature is comprised of one or more connected and/or disjoint subsets of an n-manifold ($0 \leq n \leq 3$), each of these subsets may have one or more boundary cycles that are immersed, compact, orientable, two-sided and connected (n-1)-manifolds term pseudomanifolds.

The simplex and the complex are both the representation tool and a tool for the expression of topological problems as combinatorial and algebraic problems. The definition of a closed n-dimensional simplex or n-simplex S^n is as the homeomorphs of the closed n-dimensional disk , i.e. smallest n-dimension convex point-set. Suppose the points V0, V1, V2, ..., Vn are vertex of n-simplex S^n, then S^n can be represented as:

$$S^n = (V0, V1, V2, ... , Vn)$$

The interior of S^n is composed by point x which satisfies the expression as below.

$$x = \lambda_0 V0 + \lambda_1 V1 + \lambda_2 V2 + ... + \lambda_n Vn$$

where $\lambda_i>0$ and $\lambda_0+\lambda_1+\lambda_2+......+\lambda_0=0$

The n-complex C^n is a pair (X, ξ) which satisfies the followed properties. Where X is the Euclidean manifold modelling space and ξ is a simplex decomposition of X.

(1) For each n-simplex $S^n \in \xi$ there is a homeomorphism *f* into X taking the interior of M^n homeomorphically onto the n-simplex S^n and each M^{n-1} boundary cycle into the union of (n-1)-simplex S^{n-1}.

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(2) The closure of each $S^n \in \xi$ intersects only a finite number of other simplexes.
(3) $A \subset X$ is closed if and only if every $A \leftrightarrow$closure of S^n is.

According to the definition of complex as above, We defined 3D spatial features as orientable k-pseudomanifold, where $0 \leq k \leq 3$. k-pseudomanifold is included in k-manifold, meanwhile, it is a k-complex be provided with favorable simplex structure. So any 3D spatial feature is a manifold and it can be subdivided into a serial of n-simplex through the method of space partition,. The relations between simplex and complex are the bases to construct geometric structure of 3D topological model. While, the topological properties of k-manifold and the continuous map about homemorphism and homotopy can conduct the topological properties of 3D spatial features.

Four feature types can be identified in 3D Euclidean manifold modelling space according to their degrees of freedom. The formal definition of 3D spatial features consists of:

Body Feature: Body Feature is a 3-complex which is homeomorphic to a 3-manifolds with 2- pseudomanifolds or orientable, two-sided 2-manifold boundary cycles and connected sets of both. It can be partition to the union of all 3-simplexes and can be represented as:

$$\text{Body}=C^3=\lambda_0 S^3_1+......+\lambda i S^3_i+......+\lambda_k S^3_k$$
$$=\lambda_0(V_{01},V_{11},V_{21},V_{31}) + ... +\lambda i(V_{0i},V_{1i},V_{2i},V_{3i})+......+\lambda_k(V_{0k},V_{1k},V_{2k},V_{3k})$$

Surface Feature: Surface Feature is a 2-complex which is homeomorphic to a connected sets of 2-manifolds with 1- pseudomanifolds or orientable, 1-manifold boundary cycles . It can be partition to the union of all 2-simplexes and can be represented as:

$$\text{Surface}= C^2 =\lambda_0 S^2_1+......+\lambda i S^2_i+......+\lambda_k S^2_k$$
$$= \lambda_0(V_{01},V_{11},V_{21}) +......+\lambda i (V_{0i},V_{1i},V_{2i})+......+ \lambda_n (V_{0n},V_{1n},V_{2n})$$

Line Feature: Line Feature is a 1-complex which is homeomorphic to a connected sets of 1-manifolds and 1-manifolds with boundary cycles. It can be partition to the union of all 1-simplexes and can be represented as:

$$\text{Line}= C^1 =\lambda_0 S^1_1+......+\lambda i S^1_i+......+\lambda_k S^1_k$$
$$=\lambda_0(V_{01},V_{11}) +......+\lambda i (V_{0i},V_{1i})+......+ \lambda_n (V_{0n},V_{1n})$$

Point Feature: Point Feature is a 0-simplex which is homeomorphic to a 0-manifolds. It has a spatial position but no spatial extension. Spatial position is defined by linking 0-simplex to spatial coordinates. Point Feature can be represented as:

$$\text{Point}=S^0=[x, y, z]$$

In order to decrease redundant data and discuss expediently, we define a geometric element: Face. Face is a set of union of 2-simplexes which lie in the same plane in Surface feature or the boundary of Body features. More complex spatial features could be formed by networks of these base features.

3. Topological Properties Of 3D Spatial Features

3D spatial features is a manifold with especial form. They have some very useful topological properties delivered by the definition and properties of manifold, simplexes and complexes as follows:

(1) Space partition: 3D spatial features is defined as orientable n-pseudomanifold ($0 \leq n \leq 3$). k-pseudomanifold is a k-complex be provided with fovorable simplex structure, so each of the spatial feature can be divided to a serial of k-simplex ($k \leq n$) and these simplexes are connective but not overlap. For example, Line Feature (1-complex) can be partited to some arcs (1-simplexes). Surface Feature (2-complex) can be partited to a series of triangles (2-simplexes) and Body Feature (3-complex) can be partited to some tetrahedrons (3-simplexes) . see Fig. 1.

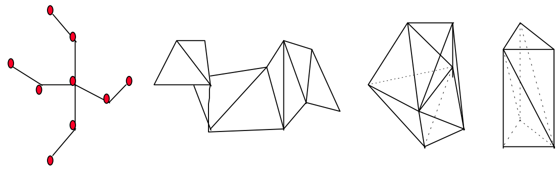

Fig.1 Space partition for 3D feature

(2)Orientability: Simplex can be oriented ....,i.e. Each k-simplex corresponding to k-complex should induce opposite orientation on any shared (k-1)-simplex, see Fig. 2. For the open 3-manifold modelling space, each 3-simplex corresponding to Body Feature should induce opposite orientations on any shared 2-simplex. Algebraically these induced opposite orientations of k-simplex cancel each other out, i.e. the sum of all oriented k-simplexes in a closed (k+1)-manifold is zero. This topological property forms an important consistency check [Corbett, 1979].

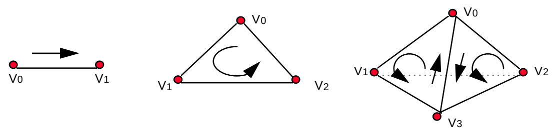

Fig. 2 The orientation of k-simplex

(3) Connectivity: For n-complexes C^n ($1 \leq n \leq 3$) , it is possible to construct homology group $Hq(C^n)$ (q=0, 1, 2,...., n) and $Hq(C^n)$ generates an abelian group. Homology group is concerned with a subset of the set of i-cycles in a n-complex which bound an i-dimensional "hole" in underlying topological space. $Hq(C^n)$ can be definition as :

$$Hq(C^n)= Zq(C^n) / Bq(C^n)$$

where $Zq(C^n)$ is a q-dimensional closen cycles group of C^n and $Bq(C^n)$ is a q-dimensional bounding closen cycles group of C^n . H_0 (C^n) can be also defined to be isomorphic to p-copies of the set of integers Z. i.e.

$$H_0(C^n) = \underbrace{Z \oplus Z \oplus Z \oplus ..... \oplus Z}_{p}$$

The number p is the rank of the qth homology group and is called the qth-betti number, it can be represented as β_q .

Homology group identify the different spaces underlying the complexes using the combinatorics of the complex itself and in some cases combine the combinatorial and

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connectivity information. If the spatial feature is connectivity, then

$$H_0(C^n) \cong Z$$

(4) . Euler characteristic: According the conception of topology, The Euler number for n-complex can be expressed as follow equation:

$$\chi\left(C^n\right) = \sum_{i=1}^{n} (-1)^n \alpha_i \quad ......(1)$$

where the α_i are the number of i-simplex. It describes the combinatorial relationship between the number of i-simplexes of an n-complex ($0 \leq i \leq n$). Meanwhile, the general form of the Euler characteristic for n-complex can be termed the Euler-Poincare equation:

$$\chi\left(C^n\right) = \sum_{q=1}^{n} (-1)^q \beta_q \quad ...... (2)$$

where the β_q is the qth-betti number for n-complex C^n. From formal (1) and (2), the Euler equation for 3D spatial feature can be represented as follow:

$$\chi\left(C^n\right) = \sum_{i=1}^{n} (-1)^n \alpha_i = \sum_{q=1}^{n} (-1)^q \beta_q$$

This equation describes the topological property of n-complex and is an important consistency check also.

(5) Composed relationships: Any n-simplex is bounded by n+1 geometrically independent (n-1)-simplexes and (n+1) 0-simplexes. see figure 3:

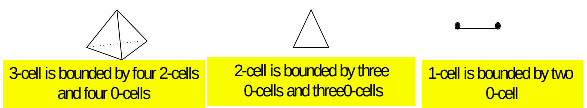

Figure 3.

(6) Shared relationships: Confining analysis to an n-dimensional space, an (n-1)-simplex can be shared by at most only 2 n-simplexes. see figure 4:

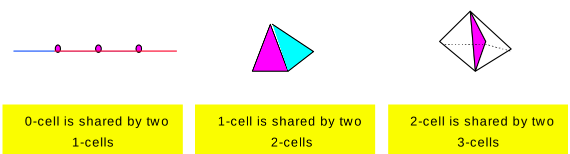

Figure 4.

(7) Boundary relationships: Boundary operator returns the set of (k-1)-simplexes incident to it when given a k-simplex.

(8) Coboundary relationships: coboundary operator is the dual of boundary operator, it returns the set of (k+1)-simplexes incident to it when given a k-simplex.

The topological properties of 3D spatial features is the bases to construct geometric structure and are used for consistency check while construct data structure.

4. A Formal Description of Topological Relationships

Topological relationships are spatial relations that are preserved under such transformations as rotation, scaling, and rubber sheeting. People have done many works in description of topological relationships(Gutting 1988, Wagner 1988, Egenhofer et al. , 1991-1996, Clementini et al., 1994), But these methods are in analogy with 2D situation. In 3D space, there are many problems to simply extend 2D formal frameworks such as the 9-intersections, which is a 3 by 3 matrix based on the intersections of feature A's boundary, interior and exterior with the boundary, interior and exterior of feature B (Egenhofer and Franzosa, 1993). For example, the boundary of closed surface feature M and the boundary of closed line feature L seeing in fig. 5 (a) are empty. The 9-intersections could not represent the relation between M and L effectively and no distinction can be made in fig. 5 (b) and fig. 5 (c) because both relations have same matrix.

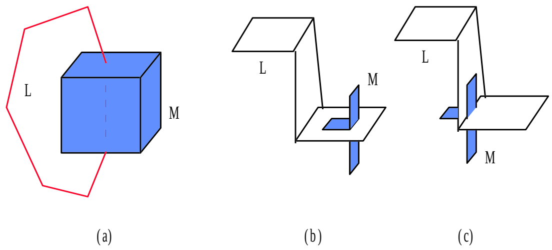

Figure. 5 Topological relationships between 3D spatial features

In this paper, we take into account the topological relationships between k-simplex ($0 \leq k \leq 3$) firstly. Suppose the boundary, interior and exterior of k-simplex A is denote by ∂A, A° and A^{-1}, the topological relationships between two k-simplexes A and B can concisely represent as criteria (1).

$$R(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^\circ & \partial A \cap B^{-1} \\ A^\circ \cap \partial B & A^\circ \cap B^\circ & A^\circ \cap B^{-1} \\ A^{-1} \cap \partial B & A^{-1} \cap B^\circ & A^{-1} \cap B^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad ...... \quad (1)$$

While S_{ij} (i,j=1,2,3) can be $\varnothing$, 0, 1, . . . , $dim(S_{ij})$•$dim(S_{ij})$ is a function which returns the dimension of a point-set S_{ij}, In case the point-set consists of multiple parts, then the highest dimension is returned. In 3D space, $MAX(dim(S_{ij}))=3$, so S_{ij} have 5 different cases $\varnothing$, 0, 1,2,3. Criteria (1) can mostly distinguish 5^9 topological relationships between two k-simplexes. Apart from some impossible case, all results can be partitioned into one of six relationship types as bellow.

Definition 1•The touch relationship•

<A, touch, B> $\Leftrightarrow (A^\circ \cap B^\circ = \varnothing) \wedge (A \cap B \neq \varnothing)$

Definition 2•The in relationship•

<A, in, B> $\Leftrightarrow (A^\circ \cap B^\circ \neq \varnothing) \wedge (A \cap B = A)$

Definition 3•The cross relationship•

<A, cross, B>$\Leftrightarrow dim(A^\circ \cap B^\circ) < (max(dim(A^\circ), dim(B^\circ)) \wedge (A \cap B \neq A) \wedge (A \cap B \neq B)$

Definition 4•The overlap relationship•

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<A, overlap, B> ⇔ (dim(A°∩B°)=dim(A°)=dim(B°)) ∧(A∩B≠A) ∧(A∩B≠B)

• •

Definition 5•The disjoint relationship•
<A, disjoint, B> ⇔ A∩B=∅

Definition 6•The equal relationship•
<A, equal, B> ⇔ <A, in, B>∧<B, in, A>

These six relationship types are multually exclusive, that is, it cannot be the case that two different relationships hold between two k-simplexes. Furthermore, they make a full covering of all possible topological situations, that is, given two k-simplexes, the topological relationships between them must be one of the six. Table 1 shows the possible topological relationships types between any two k-simplex ($0 \le k \le 3$) A and B.

According the topological properties of the 3D spatial features, each of the spatial features can be divided to a serial of k-simplex ($k \le n$) and these simplexes are connective but not overlap. The topological relationships between 3D spatial features can be described by the combination of relationships between k-simplexes. Suppose L and M are 3D spatial features which are composed by p and q k-simplexs. L•{ S_j^k| $0 \le k \le 3$, $1 \le j \le p$}•M= { $S_i^{k'}$| $0 \le k' \le 3$, $1 \le i \le q$}•the topological relationships between L and M can be represented by topological relationships matrix T as below.

$$T = \begin{bmatrix} T(S_1^k, S_1^{k'}) & T(S_2^k, S_1^{k'}) & \ldots\ldots & T(S_p^k, S_1^{k'}) \\ T(S_1^k, S_2^{k'}) & T(S_2^k, S_2^{k'}) & \ldots\ldots & T(S_p^k, S_2^{k'}) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ T(S_1^k, S_q^{k'}) & T(S_2^k, S_q^{k'}) & \ldots\ldots & T(S_p^k, S_q^{k'}) \end{bmatrix}$$

While $T(S_j^k, S_i^{k'})$ is the topological relationships between simplexes S_j^k and $S_i^{k'}$. Suppose $U = R_{disjoint} \cup R_{touch} \cup R_{in} \cup R_{cross} \cup R_{overlap} \cup R_{equal}$•∅ represent as empty•then topological relationships matrix T satisfied formula as follows.

$$\forall i,j \; T(S_j^k, S_i^{k'}) \subseteq U;$$

$$\forall i,j \; T(S_j^k, S_i^{k'}) \neq \emptyset;$$

Topological relationships matrix T are able to express all topological relationships between any two 3D spatial features.

5. Topological Data Modelling for 3D GIS

The simplexes and the complex are used to represent the geometric structure of the spatial features as well as provide an abstraction mechanism for data structure design in 3D GIS. The geometric structures delivered by simplexes and complexes are as follows:

The 0-simplex is the most fundamental geometric elements which has the attributes of 0-Simplex_ID and 3D coordinates of x, y, and z. It has 1:1 relation with the entity of Point Feature and has a 2:N relation with 1-simplex. 1-simplex is a part of the entity of Line Feature and has N:1 relationship related to the Line Feature . It is also the boundary of 2-simplex or Face and has N:3 relation with 2-simplex and N:1 relation with Face. 2-simplex is a part of Face and has N:1 relation with Face. 2-simplex and Face may be part of Surface Feature. Therefore, there is N:1 relationship between 2-simplex and Surface feature. So is the Face and Surface Feature. 3-simplex is the partition of Body Feature, it has N:1 relationship with Body Feature. The boundary of each 3-simplex is composed by four 2-simplex. thus 2-simplex has 4:N relationship with 3-simplex. Face is the part of the boundary of Body feature and has N:1 relationship related to the Body feature.

The topological data modelling for 3D GIS is obtained from the geometric structure of 3D spatial features description. The E-R diagram is shown as figure 6:

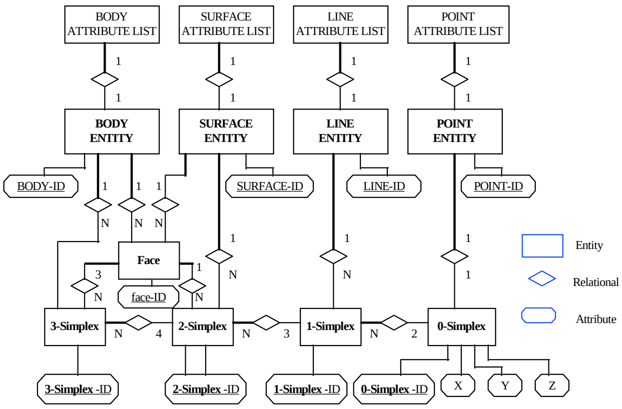

Figure 6: 3D topological data model based space partition

The topological data modelling based space partition for 3D GIS is a hybrid data model. It is well suited for the representation of regular shapes features and irregular shapes features . It is not only account for topology and thematic properties but also facilitate interpolation. so it is the primes for data organization, spatial database design, spatial query and spatial reasoning effectively.

4. Conclusion

This paper focused on the research of a topological model based on space partition for 3D GIS. First the formal definition of 3D spatial features are described. Then the methods for calculating topological properties to support simplexes topologically equivalent to an n-manifold with (n-1)-pseudomanifold boundary cycles and geometric structure of 3D spatial features are given. Finally the topology model based on it is presented.

Proving of the topological results and implementation of 3D topological data model, then the methods of querying and analyzing topological relationships can be seen in other paper.

References

1. Lee, Y.C. and Isdale, M., 1991, The Need for a Spatial Data Model. In: Proceedings of the Canada Conference on GIS. March 18-22, 1991, Ottawa, pp 530-540.
2. Oosterom,P.V., 1994, Integrated 3D Modelling Within GIS. In: Advanced Geographic Data Modelling .
3. Pigot, S.,1994, A topological Model for a 3D SIS.

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

4. Martien Molenaar, 1990. A formal data structure for three dimensional vector maps. Proc 4th int symp on spatial data handling, Zurich.pp 830-843
5. .Martien Molenaar, 1994. A query oriented implementation of a topologic data structure for 3D vector maps. INT.J.GIS. vol 8,no 3. pp 243-260
6. Rongxing Li, 1994. Three dimensional GIS : A simple extension in the third dimension ?
7. Ferenc Sarkozy, 1994. The GIS concept and 3D modeling. Comput.,Environ.,and Urban Systems. vol.18. pp 111-121
8. Christopher B. Jones, 1989. Data structure for 3D spatial information systems in geology. INT.J.GIS. vol 3. pp 15-31
9. Thomars R. Fisher, 1993. Use of 3D GIS in Hazardous Waste Site Investigations.Environ. Modeling with GIS OXFORD University Press.
10. Wei Guo. 1995. The Formal Description of Topological Spatial Relationships in 3D**.** Proceedings of the Forth International Symposium of LIESMARS, Wuhan, P.R. China.
11. Wei Guo. 1996. Three-Dimensional Representation of Spatial Object and Topological Relationships. International Archives of Photogrammetry and Remote Sensing, Vol. XXXI, Part B3. Vienna.
12. Jianya Gong, Zong-guo Xia. 1996. An Interated Data Model in Three-dimensional GIS. Proceedings of ‘Geoinformatics’96 Wuhan’-international Symposium. pp 107-120.
13. Qingquan Li, Deren Li, 1996. Study on Data Structure in 3D GIS. Proceedings of ‘Geoinformatics’96 Wuhan’-international Symposium. pp 229-237.
14. Wenzhong Shi. 1996. An Hybrid Data Model for Three-dimensional GIS. Proceedings of ‘Geoinformatics’96 Wuhan’-international Symposium. pp 400-409.
15. Zhijun Wang. 1996, Digital Photogrammetric Data Acquisition for 3D GIS. Proceedings of ‘Geoinformatics’96 Wuhan’-international Symposium. pp 490-49

Table 1. topological relationships types between any two k-simplex (0 ≤ k ≤3)A and B

B / A	0-Simplex	1-Simplex	2-Simplex	3-Simplex
0-Simplex	<A, equal, B> <A,disjoint, B>	<A, touch, B> <A,disjoint, B>	<A, touch, B> <A, in, B> <A,disjoint, B>	<A, touch, B> <A, in, B> <A,disjoint, B>
1-Simplex	<B, touch, A> <A,disjoint, B>	<A, touch, B> <A, equal, B> <A,disjoint, B>	<A, in, B> <A, touch, B> <A, cross, B> <A,disjoint, B>	<A, in, B> <A, touch, B> <A, cross, B> <A,disjoint, B>
2-Simplex	<B, touch, A> <B, in, A> <A,disjoint, B>	<B, in, A> <B, touch, A> <B, cross, A> <A,disjoint, B>	<A, equal, B> <A, in, B> <A, touch, B> <A, cross, B> <A,disjoint, B>	<A, in, B> <A, touch, B> <A, cross, B> <A,disjoint, B>
3-Simplex	<B, touch, A> <B, in, A> <A,disjoint, B>	<B, in, A> <B, touch, A> <B, cross, A> <A,disjoint, B>	<B, in, A> <B, touch, A> <B, cross, A> <A,disjoint, B>	<A, equal, B> <A, overlap, B> <A, in, B> <A, touch, B> <A,disjoint, B>