Comparison of Free Network Solutions by the Artificial Basis-Based Method with One by the Generalized Inverse in Video Metrology

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KEY WORDS: Free Network, Generalized Inverse, Basis Vectors, Null Space

ABSTRACT

In precise industrial measurement by close-range photogrammetry, it is common not to place control points but place only scale controls in an object space. This causes the rank defects of seven or six (when scale controls exist) in observation equations. For solving the least squares problem to determine object points coordinates, appropriate constraints on parameters are necessary to compensate for rank defects. A free network to constrain the parameter space with inner constraints is a very common method. Basis vectors in the null space of observation equations can be obtained in a numerical or analytical way. But the analytical vectors are valid only for infinitesimal rotation angles. For finite angles, the rotation matrices lose orthogonality in the iterative least squares process. This paper discusses a method to obtain the exact solutions by applying the sequential corrections to solutions to keep their orthogonality. And the paper further compares the solutions by this semi-analytical way with ones by numerical methods in terms of computation time and precision. The latter includes minimal constraints and the generalized inverse.

1. Introduction

In 3-D precise industrial measurement by close-range photogrammetry it is common not to place control points in the object space but set only scale controls. This leads to six rank defects (or seven in the case of no scale controls) in observation equations and normal equations. In other words the null space of the design matrix has six or seven dimensions, in which we can choose basis vectors freely. The most probable values of parameters are determined uniquely by the least squares unless we pose any overconstraints on the real space. But their variance and covariance matrix varies with the basis vectors we designate.

There are two ways to cope with rank defects. One is to add artificial minimal observation equations in the real space, while the other is to set artificial bases in the null space. The most common way for the former is to constrain six (or seven) coordinates of object points with large weights (minimal constraints). Of course it is possible to constrain the same number of exterior orientation parameters. The minimal constraints can be regarded as suppressing six (or seven) column vectors from observation equations or equivalently as setting the same number of independent unit basis vectors in the null space.

The latter way is called a free network. Most typical one is to constrain the null space by adding six (or seven) orthogonal independent basis vectors to observation equations as inner constraints. This is equivalent to using the Moore-Penrose generalized inverse, which produces the minimum norm least squares solutions.

We can alternatively constrain only object points coordinates, rather than the entire parameters. Geometrically it constrains the position of a centroid and the rotation of points around the centroid and their distance from the centroid if distance controls exist. Since this solution minimizes the mean variance of co-ordinates of object points, it is often used as an index of precision of object points coordinates(Fraser,1982). But it should be noted that this is obtained at the cost of swelling covariance between other parameters (exterior orientation parameters) and between object point coordinates themselves.
The intrinsic precision of parameters is given by their variance covariance matrix. It varies with posed inner constraints. Thus the difference of the precision is only appearance. If the network geometry is strong enough, a model similar or solid to the real space is constructed. The precision is determined by the network design (first order design). In this respect the mean variance of object points coordinates of the free network solution constraining equally the entire parameter space should be used as one number index of measurement precision.

There are two ways to calculate the Moore-Penrose generalized inverse or a pseudo-inverse; one is numerical and the other is analytical. Numerically basis vectors can be calculated by singular value decomposition(SVD) of a design matrix. On the other hand Granshaw showed analytical basis vectors Granshaw, 1980). For this purpose he defined a rotation matrix as a sequential product of small rotations, which is different from conventional Eulerian angle based matrix representation. But his expressions are valid only for small rotation angles, and the linearized collinear equation does not keep orthogonality of rotation matrices. Thus in the process of iterative least squares, rotation angles go apart from true values.

We extend this to be more applicable to general rotation angles by introducing successive correction of rotation matrices to keep orthogonality using the SVD technique. From practical point of view, we proceed with discussion using Eulerian angles around fixed axes, although other angle parameters, like Eulerian angles around rotating axes or quaternion based representation are available with minimal modification.

In Paragraph 2 general theory of free network is outlined to make the discussion self-contained. In Paragraph 3 the construction of semi-analytical basis vectors is discussed. And Paragraph 4 gives an experimental result on the properties of solutions by free networks by different computation methods as well as minimal constraints.

2. Free Network Solutions in Close-Range Photogrammetry

Assume that an object space is multiple photographed by a camera, interior orientation parameters of which are stable but unknown.

A system of observation equations (linearized collinearity equations together with, if any, distance observations) is expressed by

$$v + A_1x_1 + A_2x_2 + A_3x_3 = e \quad : \quad W \qquad (2.1)$$

or

$$v + Ax = e \quad : \quad W \qquad (2.2)$$

where x_1 $(n_1 \times 1)$, x_2 $(n_2 \times 1)$, x_3 $(n_3 \times 1)$ are vectors of exterior and interior orientation parameters and object points coordinates. v $(m \times 1)$ is an error vector, with m being the number of observation equations, A_1 $(m \times n_1)$, A_2 $(m \times n_2)$, A_3 $(m \times n_3)$ are respectively design matrices, $e_1 (m \times 1)$ is a discrepancy vector and W $(m \times m)$ is a weight matrix. The total number of parameters is $n = n_1 + n_2 + n_3$.

eq.2.2 has rank defects of six or seven (more generally r hereafter). We choose an arbitrary matrix, B $(n \times r)$, removing the rank defects;

$$B = \begin{bmatrix} B_1 \\ 0 \\ B_3 \end{bmatrix} \qquad (2.3).$$

Since A_2 has no rank defects, B_2 is set to 0. And let a matrix of basis vectors of the null space of A be denoted by $G\,(n \times r)$, which satisfies

$$G = \begin{bmatrix} G_1 \\ 0 \\ G_3 \end{bmatrix}, \quad AG = A_1G_1 + A_3G_3 = 0 \qquad (2.4).$$

And when we constrain the parameters x with B as

$$B^T x = 0 \quad \text{or} \quad B_1^T x_1 + B_3^T x_3 = 0 \qquad (2.5)$$

the least squares solutions of eq.2.2 is given with a Lagrangean multiplier λ by (Grafarend and Sanso,1985)

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} \left(A^T WA\right)_{rs}^{-} & G\left(B^T G\right)^{-1} \\ \left(G^T B\right)^{-1} G^T & 0 \end{bmatrix} \begin{bmatrix} A^T We \\ 0 \end{bmatrix} \qquad (2.6)$$

where $\left(A^T WA\right)_{rs}^{-}$ is a reflective symmetric generalized inverse, which is explicitly calculated by

$$\left(A^T WA\right)_{rs}^{-} = \left(A^T WA + BB^T\right)^{-1} - G\left(G^T BB^T G\right)^{-1} G^T \qquad (2.7).$$

The a posterior variance of observations with weight unity and the variance covariance matrix of the most probable values of parameters, $\Sigma_{\hat{x}}$, is

$$\hat{\sigma}_0^{\,2} = \frac{v^T Wv}{\operatorname{rank}(W) - \operatorname{rank}(WB)} \qquad (2.8)$$

$$\Sigma_{\hat{x}} = E\left\{xx^T\right\} = \hat{\sigma}_0^{\,2} \left(A^T WA\right)_{rs}^{-} \qquad (2.9).$$

We assume the constraint matrix B is given by the form

$$B = PG \qquad (2.10)$$

where P, a diagonal matrix with elements 0 or 1, gives constraints on specific parameters. The solutions $\hat{x}$ of eq.2.6 minimizes a weighted norm $\hat{x}^T P \hat{x}$, or minimizes the mean variance of parameters specified by P as

$$\sigma_m^{\,2} = \operatorname{trace}\left\{\left(A^T WA\right)_{rs}^{-} P\right\} / \operatorname{rank}(P) \qquad (2.11).$$

When P is taken to be a unit matrix, $\left(A^T WA\right)_{rs}^{-}$ becomes the Moore-Penrose generalized inverse or the pseudo inverse $\left(A^T WA\right)^{+}$. Geometrically it produces solutions with all the parameters constrained with loose and equal weights.

It is very common in industrial measurement to set the matrix P so as to constrain only object points coordinates, because only object points are of major concern in measurement. The solutions minimize the mean variance of object points coordinates. And it is often used as the precision of object points coordinates. But its minimum variance property is only apparent, and we have no reason to consider it better than the value for $P=I$ (all the parameters are equally constrained).

3. Expression of Analytical Basis Vectors

To extend Granshaw(1980) to be applicable to finite angles, we propose a semi-analytical method, which corrects the rotation matrices to strictly orthogonal ones every after the least square solutions are obtained.

The collinearity equation for an object point $P(X, Y, Z)$ and its image point $p(x, y)$ is expressed by

$$\left.\begin{aligned} F_x &= x + \delta x + c\frac{N_x}{D} = 0 \\ F_y &= y + \delta y + c\frac{N_y}{D} = 0 \end{aligned}\right\} \quad (3.1)$$

where δx, δy are corrections to lens distortions and c is a camera distance. Nx, Ny and D are defined by

$$\left.\begin{aligned} N_x &= (X - X_0)m_{11} + (Y - Y_0)m_{12} + (Z - Z_0)m_{13} \\ N_y &= (X - X_0)m_{21} + (Y - Y_0)m_{22} + (Z - Z_0)m_{23} \\ D &= (X - X_0)m_{31} + (Y - Y_0)m_{32} + (Z - Z_0)m_{33} \end{aligned}\right\} \quad (3.2)$$

where (X_0, Y_0, Z_0) is a perspective center and (m_{ij}) is a rotation matrix.

There are many ways to represent rotations. We adopt here azimus θ, elevation ϕ and rolling κ, because it is easy to give approximations in practical situations (See Fig.1). With them, a rotation matrix M is expressed by

$$M = M_\kappa M_\varphi M_\theta \quad (3.3),$$

where

$$\left.\begin{aligned} M_\kappa &= \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ M_\varphi &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \\ M_\theta &= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \end{aligned}\right\} \quad (3.4).$$

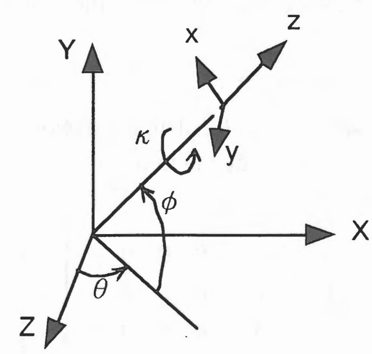

Fig.1 Coordinate System

Usually rotation angle θ, for instance, is updated in a addition form;

$$\theta_{i+1} = \theta_i + \Delta\theta \quad (3.5).$$

To find orthogonal vectors to a design matrix, we define the update of a rotation matrix in a product form;

$$M_{i+1} = M_i \cdot \Delta M \quad (3.6).$$

where

$$\Delta M = \begin{bmatrix} 1 & \Delta\kappa & -\Delta\theta \\ -\Delta\kappa & 1 & \Delta\varphi \\ \Delta\theta & -\Delta\varphi & 1 \end{bmatrix} \quad (3.7).$$

$\Delta\theta$, $\Delta\phi$ and $\Delta\kappa$ is correction angles. Not that eq.3.7 and the multiplication order in eq.3.6 are different from ones shown by Granshaw(1980), because he postulated conventional Eulerian angles around rotating axes.

From eq.3.6, we obtain directly

$$\left.\begin{aligned}\frac{\partial M}{\partial \theta} &= \begin{bmatrix} m_{13} & 0 & -m_{11} \\ m_{23} & 0 & -m_{21} \\ m_{33} & 0 & -m_{31} \end{bmatrix} \\ \frac{\partial M}{\partial \varphi} &= \begin{bmatrix} 0 & -m_{13} & m_{12} \\ 0 & -m_{23} & m_{22} \\ 0 & -m_{33} & m_{32} \end{bmatrix} \\ \frac{\partial M}{\partial \kappa} &= \begin{bmatrix} -m_{12} & m_{11} & 0 \\ -m_{22} & m_{21} & 0 \\ -m_{32} & m_{31} & 0 \end{bmatrix}\end{aligned}\right\} \qquad (3.8).$$

Using eq.3.8 as derivatives in linearizing eq.3.1, we get a system of observation equations for the entire image points;

$$v + A_1 x_1 + A_2 x_2 + A_3 x_3 = e \qquad (3.9).$$

An orthogonal matrix $G = [G_1\ 0\ G_3]$ to the design matrix $A = [A_1\ 0\ A_3]$ of eq.3.9 is given by the form;

$$\left.\begin{aligned} G_1^T &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & -Z_0 & Y_0 & 0 & 1 & 0 & \cdots \\ Z_0 & 0 & -X_0 & 1 & 0 & 0 & \cdots \\ -Y_0 & X_0 & 0 & 0 & 0 & 1 & \cdots \\ X_0 & Y_0 & Z_0 & 0 & 0 & 0 & \cdots \end{bmatrix} \\ G_3^T &= \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & -Z & Y & \cdots \\ Z & 0 & -X & \cdots \\ -Y & X & 0 & \cdots \\ X & Y & Z & \cdots \end{bmatrix} \end{aligned}\right\} \qquad (3.10).$$

Geometrically G_1 fixes the centroid of camera positions and rotations around the centroid, while G_3 fixes those of the object space.

The solutions must be updated by iteration. But in the course of updates, the rotation matrix M loses orthogonality, since eq.3.7 is not precisely orthogonal. To prevent this, we correct M to the nearest orthogonal one in terms of matrix norm. In general the orthogonal matrix R nearest to any matrix S minimizes the norm

$$\begin{aligned}&\mathrm{trace}\left\{(R-S)^T(R-S)\right\} \\ &= \mathrm{trace}\left\{R^T R - 2S^T R + S^T S\right\} \\ &= 3 - 2\mathrm{trace}\left\{S^T R\right\} + \mathrm{trace}\left\{S^T S\right\}\end{aligned} \qquad (3.11).$$

R is determined from the SVD of S as

$$\mathrm{trace}\left\{S^T R\right\} = \mathrm{trace}\left\{V \Lambda U^T R\right\} = \mathrm{trace}\left\{\Lambda U^T R V\right\} \quad (3.12).$$

Then

$$R = UV^T \qquad (3.13).$$

Note that if distance controls are placed in the object space, its linearized equation is not orthogonal to the matrix G in eq.3.10. Therefore the least squares solutions do not minimize the mean variance.

4. Numerical Basis Vectors and Moore-Penrose Generalized Inverse

Basis vectors of the null space of a design matrix A are numerically constructed by singular-value-decomposing A;

$$A = U \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} V^T \qquad (4.1),$$

where U and V are $n \times n$ orthogonal matrices and Λ $((n\text{-}r) \times (n\text{-}r))$ is a diagonal matrix with singular values as diagonal elements. Let the transformation matrix from right side, V, be expressed with n column vectors as $V = [v_1 \cdots v_{n-r}\ v_{n-r+1} \cdots v_n]$, and their last r vectors $v_{n-r+1} \cdots v_n$ make basis vectors in the null space. This calculation is very expensive because of large size of a design matrix, whereas the semi-analytical method solves basis vectors easily and the extra cpu time is negligible. But in the case of existence of scale controls, only the numerical method is available as the strict solution.

Further the Moore-Penrose generalized inverse is calculated from eq.4.1 as

$$A^+ = V \begin{bmatrix} \Lambda^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^T \qquad (4.2).$$

5. Experimental Comparison

The methods mentioned above were applied to a camera calibration experiment. In the following the properties of solutions and CPU times are compared. The object space is a square steel plate 750mm wide, on which 52 steel rods of different height from 0 through 300mm are erected. The camera used is Kodak DCS420 (color, 1,012 $\times$ 1,524pixels) with a Nikon 28mm lens. Nine photographs were taken at various positions from 2-3m off the plate with κ rotated by 90deg. at every exposure. The number of image points is 461 and the number of observation equations is 922. No scale controls were placed. Interior orientation parameters we employed are camera constant, c, principal point coordinates, x_p, y_p, radial and tangential lens distortion parameters, K_1, K_2 and P_1, P_2. The dimensions of unknown vectors are $n1$=54 (exterior orientation parameters), n_2= 7(interior orientation parameters) and n_3=156 (object points coordinates). In total $n=n_1+n_2+n_3$= 217.

The solutions are calculated for the following cases of rank defect compensations:

(A) Update of rotation matrices in the product form (eq.3.6)

(A1) Inner constraint with analytical basis vectors $G=[G_1 \;\; 0 \;\; G_3]$, which produces the minimal norm solutions for all the parameters.

(A2) Inner constraint with analytical basis vectors $G=[G_1 \;\; 0 \;\; 0]$, which produces the minimum norm solutions for exterior orientation parameters. This has no practical meanings, but was executed for comparison.

(A3) Inner constraint by analytical basis vectors $G=[0 \;\; 0 \;\; G_3]$, which produces the minimum norm solutions for object points coordinates.

(A4) Inner constraint by numerical basis vectors, which is expected to produce the same result as A1.

(A5) The same as A1 except that no orthogonality corrections were made to rotation matrices.

(B) Update of rotation matrices in the conventional form (update of angles in addition) (eq.3.5)

(B1) Use of the Moore-Penrose generalized inverse

(B2) Minimal constraints on seven object points coordinates with large weights

All observations were normalized for calculation stability. Coordinates of object points were divided by a typical distance from the camera to object space. Image coordinates and camera parameters were divided by a nominal focal length. Initial values of object points coordinates were quoted from a design sheet of the calibration space. The adjustments in every case converged in four times iterations.

The table 1 shows the list of mean variances

In all results the inner orientation parameters were restored stably as invariants. It means that any constraints are equivalent for camera calibration, and the restored space is similar to the actual space.

Table 1 Results of the experiments

	$\overline{\sigma}$ image	$\overline{\sigma}^2$x	$\overline{\sigma}^2$x1	$\overline{\sigma}^2$x2	$\overline{\sigma}^2$x3	$\overline{\sigma}$x3	CT
unit	[μm]	[mm2]	[mm2]	[mm2]	[mm2]	[mm]	[-]
(A1)	0.402124	0.015315	0.013165	0.000022	0.002126	0.0461	1.000
(A2)	0.402124	0.049208	0.011333	0.000022	0.037853	0.1946	0.984
(A3)	0.402124	0.074936	0.074419	0.000022	0.000495	0.0222	1.080
(A4)	0.402124	0.015315	0.013165	0.000022	0.002126	0.0461	6.184
(A5)	0.402204	0.015321	0.013170	0.000022	0.002127	0.0461	0.982
(B1)	0.402124	0.017631	0.015354	0.000022	0.002254	0.0475	1.716
(B2)	0.402124	0.077691	0.076623	0.000022	0.001047	0.0331	0.991

$\overline{\sigma}_{\text{image}}$: standard deviation of errors in image plane

$$\Sigma_{\hat{x}} = S_x = \begin{bmatrix} S_{x1} & * & * \\ * & S_{x2} & * \\ * & * & S_{x3} \end{bmatrix} \text{ : variance covariance matrix}$$

$\overline{\sigma}^2$x=trace(S_x)/n

$\overline{\sigma}^2$x1=trace(S_{x1})/$n1$

$\overline{\sigma}^2$x2=trace(S_{x2})/$n2$

$\overline{\sigma}^2$x3=trace(S_{x3})/$n3$

$\overline{\sigma}$x3=sqrt($\overline{\sigma}^2$x3)

CT : cpu time shown in ratio to (A1)

In (A) what is restored are not the rotation angles but rotation matrices. For evaluating the rotation angles we need decompose the matrices. (A1) and (B1) are completely equivalent from mathematical point of view. But the results were a little bit different, probably due to the difference of definition of rotation angles. (A1) takes less CPU time than (B1). This may be because in (A1) a non-singular normal equation is solved, while in (B1) a design matrix is singular-value-decomposed.

Comparing (A1) with (A4) in which basis vectors are numerically constructed, we see the latter takes six times more CPU time than the former, since it singular -value-decomposes a large matrix.

Comparing (A1) with (A5) in which no orthogonality corrections were made, we see the rms error of residuals of image points coordinates in the latter is a little bit larger than in the former. This means the solutions in (A5) do not converge to the least squares solutions. Though its increase is very small in this experiment (from 0.402124 to 0.402204), it is only because the initial values of object points coordinates are highly precise. For rough initial values, the deviation from least squares solutions is expected to be larger.

The mean variances of unknown parameters varies with the methods used for rank defect compensation. The minimal mean variance of object points coordinates was attained at (A3), which is 48% of the pseudo inverse (A1) and 67% of minimal constraints (B2). (A3) can be easily solved by forming basis vectors analytically.

The most practical method seems (B2). The CPU time is small and that the resultant mean variance of object points coordinates is similar to that of (A1). Further it can be used even if scale controls exist.

6. Conclusions

This paper discussed the construction of analytical basis vectors in the null space of a design matrix which has rank defects in close-range photogrammetry. The solutions are quite effectively calculated using the basis vectors as inner constraints. To keep the rotation matrices orthogonal, sequential corrections are performed to the resultant matrices every after least squares adjustment. This semi-analytical method is proved to take less CPU time than the conventional numerical method which calculates pseudo inverse. Its shortcomings are that rotation angles can not be restored directly and it does not produce strictly exact minimal norm solutions when scale controls exist.

Acknowledgment
The authors express the deep thanks to IHI corporation for the experiments on camera calibration.

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