ADAPTIVE SUBPIXEL CORRELATION BASED ON PRELIMINARY SEGMENTATION

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ABSTRACT

This work deals with the subpixel point correspondence problem. Subpixel matching methods such as Least-Squares Correlation [2] or Adaptive Subpixel Cross-Correlation [1] use six-parameter geometric transformation and two-parameter radiometric transformation of the whole image patches to achieve subpixel matching accuracy. However different regions inside image patch may have different distortion parameters. The method developed in this paper uses the images preliminary segmented into regions. Each region possesses its own unknown distortion parameter set that can be found by solving the correlation coefficient maximization problem. Two different kinds of correlation: widely used normalized cross-correlation and morphological correlation are to be considered. In both cases the consecutive correlation application results in problem of a finding a vector of the amendments of the parameters as a generalized eigenvector problem. The theoretical decision of this problem in view of specific structure of matrices obtained by linearization is offered.

1. INTRODUCTION

Precise points matching on the images of a stereopair is one of central problems in the area of machine vision and digital photogrammetry. A lot of publications is devoted to investigations of this problem. Among well-known classical approaches the conventional normalized cross-correlation method occupies first place due to its fundamental importance and vast utilising in practice during several decades. However, revealing drawbacks of the method connected with non-adaptive geometric properties have brought the creating of new more powerful methods, for example, adaptive least squares correlation [2]. One of this article goals is to provide consequential extension of classical normal cross-correlation that it could gain subpixel accuracy and adaptive geometric properties.

A subject of the given work is a situation, when the rough decision of a correspondence problem is already received and it is required to reach extreme possible accuracy of matching. It can be achieved by using information about preliminary image segmentation.

2. ADAPTIVE SUBPIXEL CROSS-CORRELATION

The adaptive subpixel cross-correlation method in a point correspondence problem was first described in [1]. The method uses normalized cross-correlation function as a similarity measure of two image patches.

Let us denote $f(x, y)$ - intensity distribution on left image patch (also called template). Futher for the simplicity assume that the average intensity of the template is equal to zero: $\bar{f}=0$. For this purpose we subtract template average intensity from each intensity value

$$f(x,y) \rightarrow f(x,y) - \bar{f} \tag{1}$$

Let place an origin of a rectangular coordinate system (x,y) in the middle of central pixel of the template. Denote $g(x_1,y_1)$ - intensity distribution on the right image patch which corresponds to the template. The shape of this patch differs from the shape of the template for the reason of perspective distortions. An origin of coordinate system (x_1,y_1) will be placed in the center of the right image patch. Coordinate systems (x,y) and (x_1,y_1) are connected by an unknown transformation (2), where **p** - transformation parameters vector

$$\begin{aligned} x_1 &= x_1(x,y,\mathbf{p}) \\ y_1 &= y_1(x,y,\mathbf{p}) \end{aligned} \tag{2}$$

It is necessary to find a vector of parameters **p** by maximizing of a normalized cross-correlation of the patches

$$k(\mathbf{p}) = \frac{\sum\limits_{(x,y)} f(x,y)g(x_1,y_1)}{(\sum\limits_{(x,y)} f^2(x,y))^{1/2}(\sum\limits_{(x,y)} g^2(x_1,y_1) - N\bar{g}^2)^{1/2}} \tag{3}$$

In this formula $\sum_{(x,y)}$ designates summation on all pixels of the template, N - total number of pixels belonging to the template, $\overline{g}$ - average intensity of the right patch.

The correspondence problem can be formulated as follows: to find

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} k(\mathbf{p}) \qquad (4)$$

To take into account the patch shape distortion it is offered in [2] to use affine transformation of a kind (5).

$$\begin{aligned} x_1 &= a_1 + a_2 x + a_3 y \\ y_1 &= b_1 + b_2 x + b_3 y \end{aligned} \qquad (5)$$

For the solving of a problem (4) it is necessary to find a vector of parameters $(a_1, a_2, a_3, b_1, b_2, b_3)^T$.

Suppose that an initial approximation of the parameter vector - $(a^*, 1, 0, b^*, 0, 1)^T$ is known after first step of conventional cross-correlation. Let us denote $g^*(x,y)$ intensity distribution on the right image patch which position is set by an initial vector of parameters. Let us denote g^*_x, g^*_y - partial derivatives of $g^*(x,y)$.

Consider linearization of unknown function $g(x_1,y_1)$ with respect to $g^*(x,y)$ taking into account parameters of transformation (5).

$$g(x_1, y_1) \approx \mathbf{g}^T \Delta\mathbf{p} \qquad (6)$$

Where

$$\mathbf{g}^T = \begin{bmatrix} g^* & g^*_x & xg^*_x & yg^*_x & g^*_y & xg^*_y & yg^*_y \end{bmatrix};$$

$\Delta\mathbf{p}^T = \begin{bmatrix} 1 & \Delta a_1 & \Delta a_2 & \Delta a_3 & \Delta b_1 & \Delta b_2 & \Delta b_3 \end{bmatrix}$ - vector of the transformation (2) parameter amendments.

After substitution (6) in (3)

$$k(\Delta\mathbf{p}) = \frac{\sum_{(x,y)} f\mathbf{g}^T \Delta\mathbf{p}}{(\sum_{(x,y)} f^2)^{1/2} (\sum_{(x,y)} \Delta\mathbf{p}^T \mathbf{g}\mathbf{g}^T \Delta\mathbf{p} - N\Delta\mathbf{p}^T \overline{\mathbf{g}}\overline{\mathbf{g}}\Delta\mathbf{p})^{1/2}} \qquad (7)$$

After equivalent transformations

$$k'(\Delta\mathbf{p}) = \left(k(\Delta\mathbf{p})\right)^2 \sum_{(x,y)} f^2 \qquad (8)$$

(4) looks

$$k'(\Delta\mathbf{p}) = \frac{\Delta\mathbf{p}^T (\sum_{(x,y)} f\mathbf{g})(\sum_{(x,y)} f\mathbf{g}^T)\Delta\mathbf{p}}{\Delta\mathbf{p}^T (\sum_{(x,y)} \mathbf{g}\mathbf{g}^T - N\overline{\mathbf{g}}\overline{\mathbf{g}}^T)\Delta\mathbf{p}} = \frac{\Delta\mathbf{p}^T \mathbf{A}\Delta\mathbf{p}}{\Delta\mathbf{p}^T \mathbf{B}\Delta\mathbf{p}} \qquad (9)$$

where

$\mathbf{A} = \mathbf{r}\,\mathbf{r}^T$ - singular matrix of dimensions 7x7. Note that rank of **A** is equal to one,

$\mathbf{r} = \sum_{(x,y)} f\mathbf{g}^T$ - vector of dimension 7,

$\mathbf{B} = \sum_{(x,y)} \mathbf{g}\mathbf{g}^T - N\overline{\mathbf{g}}\overline{\mathbf{g}}^T$ - matrix of dimensions 7ō7.

The matrix **B** - symmetric and positively determined. The latter follows due to the denominator of the formula (9) is a value proportional to intensity dispersion of the right image patch. For real images a matrix **B** is supposed to be non-singular (determinant of **B** is not equal to zero).

Thus (4) is reduced to a problem (10) which is equivalent to the generalized eigenvalues problem (11).

$$\lambda = \frac{\mathbf{x}^T \mathbf{A}\mathbf{x}}{\mathbf{x}^T \mathbf{B}\mathbf{x}} \to \max \qquad (10)$$

$$\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{B}\,\mathbf{x} \qquad (11)$$

The following statement was proved in [1].

Statement

Consider any vector **a** of dimension n and symmetric, positively defined and non-singular matrix **B** of dimensions n x n. Then for solutions of a generalized eigenvalues problem (11) where

$$\mathbf{A} = \mathbf{a}\mathbf{a}^T \qquad (12)$$

the following statements are valid:

1) There are two generalized eigenvalues: λ_1=0 of n-1 fold and λ_2>0 of 1 fold;

2) Generalized eigenvector corresponding to λ_2 is given by the formula

$$\mathbf{x} = \mathbf{B}^{-1}\mathbf{a} \qquad (13)$$

3) $\lambda_2=\mathbf{a}^T\mathbf{x}$, where $\mathbf{x}$ - eigenvector corresponding to λ_2.

From the statement follows, that the decision of the problem (6) is under the formula

$$\Delta\mathbf{p} = \mathbf{B}^{-1}\mathbf{r} \tag{14}$$

The effective algorithm of the numerical solution of the problem (14), based on triangular Cholecky decomposition of **B** matrix was also offered in [1].

3. USING OF AN IMAGE SEGMENTATION IN THE CORRESPONDENCE PROBLEM

Consider the more complicated a priori data: besides the original images a result of segmentation of one of the images (for example - left) is obtained. The segmentation is a result of low-level or semantic analysis of the image. Segmentation splits the image into not crossed regions. The development of algorithms of segmentation is beyond the scope of this work. Therefore it is considered that the result of segmentation is given a priori.

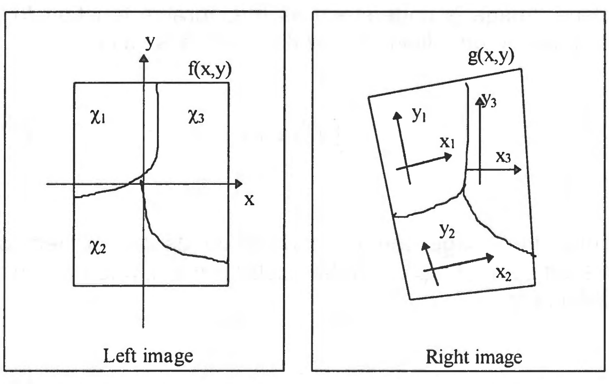

Fig.1 a Designation of coordinates systems in the model with preliminary segmentation

Let us designate *s* - segmentation of the template, which represented by union of n not crossed regions χ_i, $i=1,..,n$:

$$\chi_i \bigcap \chi_j = \varnothing,\quad i \neq j$$
$$s = \bigcup_{i=1}^{n} \chi_i \tag{15}$$

The model used in case of three regions is schematically shown on Fig.1. Taking into account the geometrical distortions of images caused by a perspective projection and the three-dimensional form of scene objects for each region χ_i we consider its own affine distortion (16).

$$x_i = a_{1i} + a_{2i}x + a_{3i}y$$
$$y_i = b_{1i} + b_{2i}x + b_{3i}y \tag{16}$$

The transformation (16) enters in each region χ_i a system of coordinates (x_i,y_i), determined by a vector of parameters $\mathbf{p}_i = (a_{1i},a_{2i},a_{3i},b_{1i},b_{2i},b_{3i})^T$. As in a case with only one region (see Section 2), the initial approximation of a vector of parameters $(a^*,1,0,b^*,0,1)^T$ and initial distribution of intensity $g^*(x,y)$ are known.

4. LINEARIZATION STEP

Consider linearization of unknown function $g(x_i,y_i)$ in region χ_i with respect to $g^*(x,y)$ taking into account parameters of transformation (16). In the differential form expression (16) is

$$\Delta x_i = \Delta a_{1i} + x\Delta a_{2i} + y\Delta a_{3i}$$
$$\Delta y_i = \Delta b_{1i} + x\Delta b_{2i} + y\Delta b_{3i} \tag{17}$$

Linearization of $g(x_i,y_i)$ in region χ_i yields

$$g(x_i,y_i) \approx g^* + g_x^*\Delta a_{1i} + g_x^* x\Delta a_{2i} + g_x^* y\Delta a_{3i} + g_y^*\Delta b_{1i} + g_y^* x\Delta b_{2i} + g_y^* y\Delta b_{3i} = \mathbf{g}_i^{\,T}\Delta\mathbf{p} \tag{18}$$

where

$$\mathbf{g}_i^T = \left[0 \;\ldots\; 0 \;\; g^* \;\; g_x^* \;\; xg_x^* \;\; yg_x^* \;\; g_y^* \;\; xg_y^* \;\; yg_y^* 0\ldots0\right] \tag{19}$$

$$\begin{aligned}\Delta\mathbf{p}^{\mathrm{T}} = [&1\ \Delta a_{11}\ \Delta a_{21}\ \Delta a_{31}\ \Delta b_{11}\ \Delta b_{21}\ \Delta b_{31}\ \ldots\\ &1\ \Delta a_{1i}\ \Delta a_{2i}\ \Delta a_{3i}\ \Delta b_{1i}\ \Delta b_{2i}\ \Delta b_{3i}\ \ldots\\ &1\ \Delta a_{1n}\ \Delta a_{2n}\ \Delta a_{3n}\ \Delta b_{1n}\ \Delta b_{2n}\ \Delta b_{3n}\ ]\end{aligned}$$

The vector $\mathbf{g}_i$ of dimension 7n has the following structure: first 7(i-1) of components are equal to zero, 7 coefficients of linearization further follow, last 7(n-i) of components are also equal to zero. The vector $\Delta\mathbf{p}$ of dimension 7n is assembled from transformation parameter amendments of all areas.

5. TRANSFORMATION OF A CROSS-CORRELATION COEFFICIENT

In view of the entered splitting on regions (3) can be transformed as follows

$$k(\Delta \mathbf{p}) = \frac{\sum_i \sum_{(x,y)_i} f(x,y) g(x_i, y_i)}{(\sum_{(x,y)} f^2(x,y))^{1/2} (\sum_i \sum_{(x,y)_i} g^2(x_i,y_i) - N\bar{g}^2)^{1/2}} \quad (20)$$

Here double summation $\sum_i \sum_{(x,y)_i}$ designates summation on all regions and on all pixels inside each region of the template.

Linearization of expression (20) taking into account (18) yields

$$\bar{g} \approx \frac{1}{N} \sum_i \sum_{(x,y)_i} \mathbf{g}_i^T \Delta \mathbf{p} = \bar{\mathbf{g}}^T \Delta \mathbf{p} \quad (21)$$

After transformation (8) expression (20) looks

$$k'(\Delta \mathbf{p}) = \frac{\Delta \mathbf{p}^T (\sum_i \sum_{(x,y)_i} f \mathbf{g}_i)(\sum_i \sum_{(x,y)_i} f \mathbf{g}_i^T) \Delta \mathbf{p}}{\Delta \mathbf{p}^T (\sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T - N \bar{\mathbf{g}} \bar{\mathbf{g}}^T) \Delta \mathbf{p}} = \frac{\Delta \mathbf{p}^T \sum_i (\sum_{(x,y)_i} f \mathbf{g}_i)(\sum_{(x,y)_i} f \mathbf{g}_i^T) \Delta \mathbf{p}}{\Delta \mathbf{p}^T (\sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T - N \bar{\mathbf{g}} \bar{\mathbf{g}}^T) \Delta \mathbf{p}} = \frac{\Delta \mathbf{p}^T \mathbf{A} \Delta \mathbf{p}}{\Delta \mathbf{p}^T \mathbf{B} \Delta \mathbf{p}} \quad (22)$$

Where

A = $\sum_i \mathbf{r}_i \mathbf{r}_i^T$ - matrix of the dimensions 7n õ 7n,

$\mathbf{r}_i = \sum_i \sum_{(x,y)_i} f \mathbf{g}_i$ - vector of dimension 7n,

$\mathbf{B} = \sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T - N \bar{\mathbf{g}} \bar{\mathbf{g}}^T$ - matrix of the dimensions 7n õ 7n.

The second equality in (22) follows from the fact that e matrices $\mathbf{g}_i \mathbf{g}_j^T$, i≠j are zero-matrices (see (19))

The matrix B has the following structure

$$\mathbf{B} = \sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T - \frac{1}{N} (\sum_i \sum_{(x,y)_i} \mathbf{g}_i)(\sum_i \sum_{(x,y)_i} \mathbf{g}_i^T) = \sum_i \left(\sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T - \frac{1}{N} \sum_{(x,y)_i} \mathbf{g}_i \sum_{(x,y)_i} \mathbf{g}_i^T \right) \quad (23)$$

Thus, the matrices **A** and **B** have block structure of a kind

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & \mathbf{A}_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & \mathbf{B}_n \end{bmatrix}. \quad (24)$$

where $\mathbf{A}_i$, $\mathbf{B}_i$ - matrices of the dimensions 7 õ 7. The properties of matrices **A** and **B** follow from the analysis which has been carried out in Section 2. Matrix **B** - symmetric, positively determined and non-singular. The rank of each matrix $\mathbf{A}_i$ is equal to one (see Section 2). From here follows, that the rank of a matrix **A** is equal to n.

6. TRANSFORMATION OF A MORPHOLOGICAL CORRELATION COEFFICIENT

The Pytiev morphological approach [3] to the problem of comparison of the images is actively developed nowadays. In the contrary to the considered cross-correlation method which is sensitive to non-linear change of images contrast, the algorithms based on the morphological approach allow to solve the problem of images comparison in conditions of strong radiometrical variability.

Consider the basic concepts of the morphological analysis. Let a field of view X be a compact set on a plane. Image is understood as measurable function *f(x))* determined on a field of view X for which is valid

$$\int_X f^2(x) dx < \infty \cdot \quad (25)$$

Thus, the image can be considered as an element of Hilbert space $L^2_\mu(x)$ with the scalar product and the norm defined as

$$(f_1, f_2) = \int_X f_1(x) f_2(x) dx \quad (26)$$
$$\|f\| = \sqrt{(f, f)}$$

Let us define the image *«segmentation based shape»* the splitting of X by not crossed regions χ_i with constant intensity value. Therefore we can present *f(x)* as

$$f(x) = \sum_{i=1}^n c_i C_i(x) \cdot \quad (27)$$

where ñ$_i$ - constant intensity of the region χ_i, C_i - characteristic function of the region χ_i:

$$C_i(x) = \begin{cases} 1, & x \in \chi_i \\ 0, & x \notin \chi_i \end{cases}; \quad (28)$$

n - number of regions with constant intensity on a field of view X.

Let F - class of one-dimensional functions. Let's consider the images $f(x)\in L^2_\mu$ and $F(f(x))\in L^2_\mu$, $x\in X$, $F\in F$. The «shape» of the image $f_1(x)$ is said to be *not more complex* than the «shape» of the image $f(x))$ (denoted $f_1<f$) if there exists $F\in F$ that $f_1(x)=F(f(x))$.

Consider a set of the images which shape is not more complex than the shape of $f(x))$. We denote this set by V_f. V_f is obtained from f with the help of various functions F from a class F. Let the class of functions F the following restrictions are imposed on:

1. $\{F(z)=z, z\in R_1\} \in F$
2. if $F_1 \in F$, $F_2 \in F$, then $F_1(F_2(z)) \in F$
3. if $F_1 \in F$, $F_2 \in F$, then $\alpha_1 F_1(z)+ \alpha_2 F_2(z) \in F$, $\alpha_1,\alpha_2 \geq 0$.

The set V_f formed with the help of the various functions from the class F satisfying 1-3 is the closed convex set in L^2_μ. Then for anyone $\varphi\in L^2_\mu$ there exists a unique element from V_f nearest to φ. This determines the projection operator P_f on the set V_f. It is defined from a condition:

$$\|P_f\varphi - \varphi\| = \inf_{g\in V_f} \|g - f\| \tag{29}$$

The ***shape*** of f is defined as the projection operator P_f on the set V_f.

The projection operator P_f satisfies to the following properties:

1. $V_f=\{\varphi:\ \varphi=P_f\,\varphi\}$
2. $\|P_f\,\varphi\| \leq \|\varphi\|$, $\varphi\in L^2_\mu$; $\|P_f\,\varphi\| = \|\varphi\| \Leftrightarrow \varphi<f$
3. $\|P_f\,\varphi - \varphi\| \geq 0$, $\varphi\in L^2_\mu$

In the case of preliminary image segmentation (27) the explicit form of operator $P_f\,\varphi$ is

$$P_f\varphi = \sum_i \frac{(\varphi,\chi_i)}{(\chi_i,\chi_i)}\chi_i \tag{30}$$

From properties of the projection operator follows, that the size $\|P_f\,\varphi - \varphi\|$ is a measure of distinction of images f and φ, and the following characteristic

$$0 \leq \frac{\|P_f\varphi\|}{\|\varphi\|} \leq 1 \tag{31}$$

can be considered as a similarity measure of two images.

The result of segmentation s (15) can be considered as a shape of the template $f(x,y)$. Therefore morphological correlation coefficient between $f(x,y)$ and $g(x,y)$ is calculated as follows

$$k_M(\mathbf{p}) = \frac{\|P_s g\|}{\|g\|} \tag{32}$$

We denote N_i - number of pixels in the region χ_i, $\overline{g}_i$ - average intensity of the image $g(x,y)$ in the region χ_i. By definition of the projection operator

$$\begin{aligned}\|P_s g\|^2 &= \sum_i N_i \overline{g}_i^2 = \sum_i \frac{1}{N_i}\left(\sum_{(x,y)_i} g(x_i,y_i)\right)^2 \approx \\ &\approx \sum_i \frac{1}{N_i}\left(\sum_{(x,y)_i} \mathbf{g}_i^T \Delta\mathbf{p}\right)^2 = \\ &= \Delta\mathbf{p}^T \sum_i \frac{1}{N_i}\left(\sum_{(x,y)_i} \mathbf{g}_i\right)\left(\sum_{(x,y)_i} \mathbf{g}_i^T\right)\Delta\mathbf{p}\end{aligned} \tag{33}$$

$$\|g\|^2 = \sum_i \sum_{(x,y)_i} g^2(x_i,y_i) \approx \sum_i \sum_{(x,y)_i} \left(\mathbf{g}_i^T \Delta\mathbf{p}\right)^2 = \Delta\mathbf{p}^T \sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T \Delta\mathbf{p}$$

Thus, the expression for a square of morphological correlation coefficient is

$$k_M^2(\Delta\mathbf{p}) = \frac{\Delta\mathbf{p}^T \mathbf{A} \Delta\mathbf{p}}{\Delta\mathbf{p}^T \mathbf{B} \Delta\mathbf{p}}, \tag{34}$$

where

$$\mathbf{A} = \sum_i \mathbf{r}_i \mathbf{r}_i^{\mathbf{T}}, \quad \mathbf{r}_i = \frac{1}{\sqrt{N_i}} \sum_{(x,y)_i} \mathbf{g}_i, \quad \mathbf{B} = \sum_i \sum_{(x,y)_i} \mathbf{g}_i \mathbf{g}_i^T,$$

The matrices **A** and **B** have the same properties as corresponding matrices from Section 5.

7. CORRELATION COEFFICIENT MAXIMIZATION

The correlation coefficient maximization problem (22), (34) has a kind of a problem (10) which is reduced to the generalized eigenvalues problem (11). The block structure of matrices (24) allows to write down the generalized eigenvalues problem for each submatrix

$$\mathbf{A}_i\mathbf{x} = \lambda\mathbf{B}\mathbf{x}, \quad i=1...n, \tag{35}$$

Where $\mathbf{A}_i = \mathbf{r}_i\mathbf{r}_i^{\mathbf{T}}$. For the problem (35) the statement from the Section 2 is valid. The non-zero eigenvalue λ_i, $i=1...n$ is under the formula

$$\lambda_i = \mathbf{r}_i^T \mathbf{B}^{-1} \mathbf{r}_i, \qquad (36)$$

And corresponding eigenvector

$$\mathbf{x}_i = \mathbf{B}^{-1} \mathbf{r}_i \qquad (37)$$

Thus, each region χ_i derivates an eigenvector and an eigenvalue of the problem (11), that is similar to application of the subpixel correlation method described in the Section 2 for one particular region χ_i. For calculation of correlation of the whole set of regions we shall construct a vector of parameters $\Delta\mathbf{p}$ of each region eigenvectors:

$$\Delta\mathbf{p} = \sum_i \mathbf{x}_i = \mathbf{B}^{-1} \sum_i \mathbf{r}_i \qquad (38)$$

Then the correlation between patches is

$$\frac{\Delta\mathbf{p}^T \mathbf{A} \Delta\mathbf{p}}{\Delta\mathbf{p}^T \mathbf{B} \Delta\mathbf{p}} = \frac{\left(\mathbf{B}^{-1}\sum_i \mathbf{r}_i\right)^T \mathbf{A} \left(\mathbf{B}^{-1}\sum_i \mathbf{r}_i\right)}{\left(\mathbf{B}^{-1}\sum_i \mathbf{r}_i\right)^T \mathbf{B} \left(\mathbf{B}^{-1}\sum_i \mathbf{r}_i\right)} =$$

$$= \frac{\sum_i \mathbf{r}_i^T \mathbf{B}^{-1} \sum_{\mathbf{i}} \mathbf{r}_\mathbf{i} \mathbf{r}_\mathbf{i}^T \mathbf{B}^{-1} \sum_i \mathbf{r}_i}{\sum_i \mathbf{r}_i^T \mathbf{B}^{-1} \sum_i \mathbf{r}_i} = \qquad (39)$$

$$= \frac{\sum_i (\mathbf{r}_i^T \mathbf{B}^{-1} \mathbf{r}_\mathbf{i})(\mathbf{r}_\mathbf{i}^T \mathbf{B}^{-1} \mathbf{r}_\mathbf{i})}{\sum_i \mathbf{r}_i^T \mathbf{B}^{-1} \mathbf{r}_\mathbf{i}} = \frac{\sum_i \lambda_i^2}{\sum_i \lambda_i}$$

8. CONCLUSION

Both considered matching methods:

- cross-correlation;
- morphological correlation,

can be extended to adaptive subpixel form even when the preliminary image segmentation is obtained. The coefficient correlation maximization problem in both method can be reduced to generalized eigenvalues problem with specific matrices structure.

The subpixel estimation of corresponding points results in more accurate measurement of interesting points like corners and edges of three - dimensional objects.

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