CORRECTION OF NON-LINEAR DISTORTION IN DIGITAL IMAGES USING A PROPOSED NORMALIZATION ALGORITHM

Othman Alhusain
Budapest University of Technology and Economics, Dept. of Photogrammetry and Geomatics,
Muegyetem rkp. 3, I. 24., H-1521 Budapest, Hungary
Email: alhusain@eik.bme.hu

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ABSTRACT

In this paper, image formation will be investigated in detail. Data captured is a representation of data collected by two kinds of sensors widely in use in remotely sensed image processing applications. These are space borne optical sensors and one example of electro-optical sensors used in medical imaging applications. The investigation shows that non-linear distortion is materialized in two aspects related to data measured by the sensor system. These are the intensity of the signal measured at a given pixel and the location (coordinates) of that measured signal. The existence of such distortion is strange enough, taking into consideration that the algorithms implemented in image formation are all well designed and based on sound mathematical foundations.

The algorithm proposed to correct these non-linear discrepancies is a phase that follows data capture and image formation. It functions as a correction stage for non-linear distortion before the image is rendered into its final form. The principal idea of the algorithm is to perform a normalization process on either the coordinates of the measured data, the value (intensity) of the measured data, or on both. In the process, at each pixel, the values (coordinates, intensity) associated with the measured data are compared to those of a reference and/or neighbouring pixels, and are modified accordingly. Performing the normalization algorithm has resulted in significant reduction of non-linear distortion in the generated images. In many cases it has resulted in total elimination of the distortion.

1 INTRODUCTION

Compensation for non-linear distortion is best done through the optimum design of the circuitry (hardware) of the different elements involved in the data capture and image formation of the imaging system. Unfortunately, this goal is impossible to attain solely through the hardware alone due to factors related to the characteristics of the hardware implemented and to those of the captured data itself. A more amenable solution to the non-linearity problem can be achieved through the implementation of algorithms that correct image non-linearity caused by different stages of the imaging system. Algorithmic corrections are especially more attractive than hardware oriented solutions due to the fact that to implement them there is no need for any change on the hardware already installed. Algorithms are also easily modified when the need arises in later stages, and in many cases, are even less expensive to utilize.

Non-linear distortion happens in imaging systems mainly due to two reasons. First, non-linear and variable response of the "sensing" elements in the sensor system to different intensities of signals. This kind of distortion depends on physical properties of the "sensing" elements and can be modeled and compensated for with reasonable accuracy. Second, and most significant, is that sensors utilized in imaging systems, whether these are far-range (air- space borne) or close-range (terrestrial, medical), are generally designed in a way that each sensor is made up of rows or arrays of individual sensing elements ranging from few to several tens to provide for complete coverage of the scanned area.

Theoretically, individual sensing elements assembled in a single sensor are designed to be identical when they are required to perform the same function, as to giving equal output when scanning the same or similar areas. However, practically this is a goal hard to achieve and maintain, especially with aging of the sensor. This non-conformity among sensing elements results in variations in the physical and electronic characteristics of the individual sensing elements which in turn result in changing the conditions at which signals are measured, and consequently can lead to significant fluctuations in the response of each sensing element when measuring the same signal. Meanwhile in the ideal case the response of each sensing element and measurement of the signal should be identical. Thus non-linear spatial distortions cause noticeable non-uniformities in captured images. Various methods to correct for spatial non-uniformities utilize the concept of increasing or decreasing the density and/or intensity of the captured signals (events) in specific areas in the image where the distortion is most apparent (Graham et al., 1978, Morrison et al., 1971). However, non-uniformity resulting from non-linear spatial distortions is best corrected if the procedure applied to perform the correction can guarantee the removal of the distortion and relating the captured event to its appropriate location (Spector et al., 1972).

2 IMAGE GENERATION

Images in this study were originally formed by processing data generated by simulation of two different sensors. These are one example of photomultiplier tube based medical imaging cameras and one example of multispectral scanner used in remotely sensed satellite imagery. Since the characteristics of non-linear distortion present in the images of both sensors are similar in regard to the applicability of the proposed algorithm, the detailed discussion and images shown are mainly related to the photomultiplier tube based sensors and the correction of their data. On the other hand, it is easy to generalize the study to be applied to multispectral and other sensors taking into consideration the specifics of the sensors under investigation.

2.1 Photo Multiplier Based Sensors

One of the most common implementations of photomultiplier detectors is in scintillation cameras which are widely used in medical imaging. The general construction of these cameras is quit simple and did not go through much change over the years (NEMA,1979, Stover, 1990).

It consists of three main components, these are scintillation crystal with mirror, light guide glass, and a number of photomultiplier tubes connected through electronic circuitry and mounted over the light guide glass. Figure 1, shows a simple construction of photomultiplier based sensors. Each photomultiplier tube gives an output signal proportional to the quantity of visible photon stream present at its input. The relationship between the input and output of a given photomultiplier is govern by the Lambertian curve figure 2. In this curve, if the abscissa is the distance from the photomultiplier center and the coordinates is the relative intensity of the output and input signals, the curve is described in the following equation:

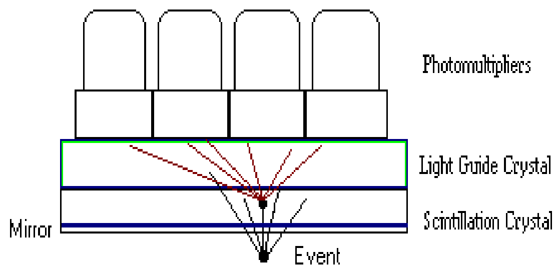

Figure 1. Photomultiplier Based Scintillation Sensor

$$f(x) = 1/e^{(\sigma x)x} \qquad (1)$$

where σ is a constant, and *x* takes the continuous values from: 0 -----> ∞. From this equation it is clear that when the event happens at the center of a photomultiplier (0 cm distance), then the relative intensity is equal to 1, or the output signal is equal to the input signal without any attenuation. As the place of the event move away from the photomultiplier center the relative intensity decreases until approaching near zero value at an approximate distance of 12 cm, thus there is no response at the output of the photomultiplier regardless of the input signal intensity. The photomultipliers are manufactured with an approximate diameter of 7.62 cm. The diameter of the useful field of view (UFOV) is slightly less than this number. Also, when the photomultipliers are mounted over the light guide glass of the detector, there is a gap between two neighbouring tubes that is uncovered.

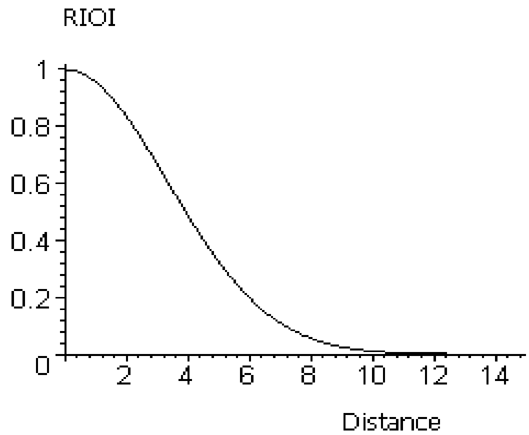

Figure 2. Photomultiplier Relative Intensity Response

In words, between the UFOV of any two neighbouring photomultipliers, there is a gap of an uncovered (inactive) area of the detector. This shortage in the coverage process is a major source of non-uniformity and artifact addition in images generated by such detectors and must be compensated for by the algorithm used to generate the image. Algorithms responsible for detection and positioning a given signal (event) are ideally supposed to measure the intensity and location of that specific event and accordingly recording these two values into the output image. As mentioned above, due to the presence of distortion in the output images, there is a need for continuous correction element in the image formation process. This can be achieved with reasonable satisfaction by implementing the non-linear correction algorithm whose task is to achieve this goal of distortion removal.

3 NON-LINEAR CORRECTION ALGORITHM

The role of the non-linearity correction algorithm is to correct the distorted coordinates of events (pixels) in the output image. These distorted coordinates make the input data that will be fed to the correction algorithm, while the output of the algorithm will constitute the corrected coordinates. The correction algorithm deploys a correction array, in this array the indices in x, y directions are the uncorrected coordinates, and the arrays elements will be the corrected locations (coordinates).

The input position is used as a key to find the four closest elements in the array. Those four elements include the positions within a square whose summits are the four array elements. An interpolation procedure is then used to calculate the corrected positions (coordinates). The choice of the interpolation procedure has to satisfy both accuracy and speed requirements. This condition is well satisfied by the *bi-linear* interpolation which is one of the most straight forward and best interpolations suitable for this purpose. After completing the interpolation process, the algorithm would execute an averaging and/or comparison process, especially if the intensity of the individual pixels is required to be corrected too, the averaging and comparison processes are best done when enough ancillary information about each pixel and its neighbourhood were available. Such a detailed information about the whole image area is best constructed in the form of a small data base (alhusain and Minda, 1995)

The interpolating array is built from the correct uniformed predicted locations of the pixels in the input image. After identification of the predicted locations, a first array is produced, linking the real position of the pixels with the distorted position computed by the image generation algorithm. The real positions follow a uniformed pattern either rectangular or hexagonal, while their corresponding computed positions show small deviations from the uniformed patterns, so that, if the spacing between the real positions is uniformed pattern (like a mesh), it is irregular for the computed positions to have a nonuniformed pattern.

3.1 Bilinear Interpolation

The basic principle of the bilinear interpolation is that the 2-D interpolation is broken down into three 1-D interpolations:

- Two *1-D* interpolations are done along the edges of two sides of the interpolating square.
- The two interpolated points on the two sides of the interpolating square lay on a straight line with the final interpolation point; so a *1-D* interpolation is done along this line giving the final interpolating point.

The bilinear interpolation algorithm is a straightforward procedure that is easy to implement through programming code and give predictable results. Let us explain the proposed bilinear interpolation algorithm in more details. First, the interpolation equations applied to conventional Cartesian coordinate system and then generalize them to an array system arrangement.

If we consider a cell of four points $A_1(0,0)$, $A_2\ (0,1)$, $A_3(1,0)$, and $A_4(1,1)$ located at the corners of the cell and drawn on x,y coordinate system, figure 3. At each one of these points, let us consider the function $f(x,y)$ which has a value *of* $f(0,0)$, $f(0,1)$, $f(1,0)$ and $f(1,1)$ at each point respectively. Suppose we want to determine the value of the function f(x, y) at an

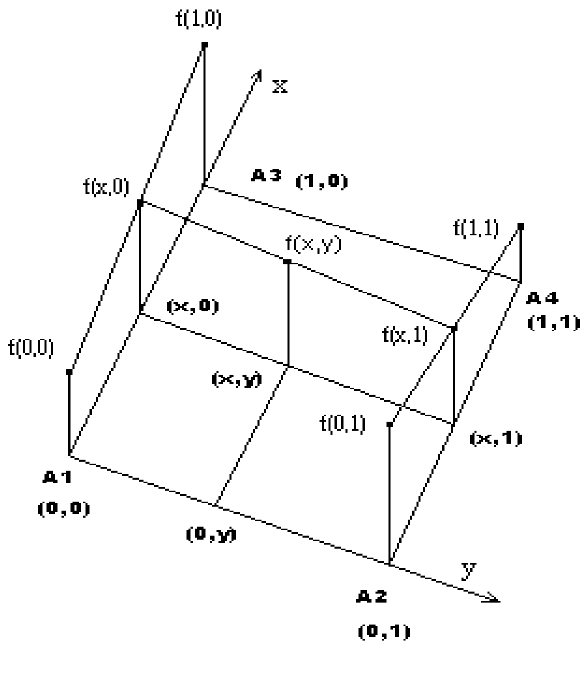

Figure 3, *2-D* Bilinear Interpolation

arbitrary point within the cell, this can be done through interpolation in two-dimensions process. The mathematical foundations of the two-dimension interpolation process can be found in many texts and references, one of the best is presented by Castleman (1979) as follows:

First, we perform *1-D* interpolation along x axis at $y = 0$, that is:

$$f(x, 0) = f(0, 0) + x[f(1,0) - f(0,0)] \qquad (1)$$

Second, we perform another *1-D* interpolation along x axis but at $y = 1$, that is:

$$(x, 1) = f(0, 1) + x[f(1,1) - f(0,1)] \qquad (3)$$

In the third step we perform another *1-D* interpolation along y at x that is:

$$f(x, y) = f(x, 0) + y[f(x,1) - f(x,0)] \qquad (4)$$

By substituting and simplifying we get the general *2-D* interpolation equation:

$$f(x, y) = [f(1, 0) - f(0,0)]x + [f(0, 1) - f(0, 0)]y + [f(1, 1) + f(0, 0) - f(0,1) - f(1,0)]xy + f(0,0) \qquad (5)$$

Which can be modeled as:

$$f(x, y) = ax + by + cxy + d \qquad (6)$$

Where:

$$a = f(1, 0) - f(0, 0) \qquad (7)$$

$$b = f(0, 1) - f(0, 0) \qquad (8)$$

$$c = f(1, 1) + f(0, 0) - f(0, 1) - f(1, 0) \qquad (9)$$

$$d = f(0, 0) \qquad (10)$$

To be able to implement the interpolation by software means, we need to substitute the mathematical equations by matrix ones. If we define a *2-D* array xx[i][k] where i[1, ...,m] and k[1, ...,n] and its first element xx[i][k] located at the point A_1 or coordinates *(0, 0)* then we can generalize equation *(5)* to a matrix set form as:

$$xx[i][k] = (xx[i+1][k] - xx[i][k])x + (xx[i][k+1] - xx[i][k])y + (xx[i+1][k+1] + xx[i][k] - xx[i][k+1] - xx[i+1][k])xy + xx[i][k] \qquad (11)$$

Where the coefficients in relationship to the matrix system become:

$$a = xx[i+1][k] - xx[i][k] \qquad (12)$$

b = xx[i][k+1] - xx[i][k] (13)

c = xx[i+1][k+1] + xx[i][k] - xx[i][k+1] - xx[i+1][k] (14)

d = xx[i][k] (15)

It is obvious that the bilinear general formula *(11)* gives interpolated values that are continuous through the edges of any interpolating cell. The derivatives of the interpolated position are not continuous, but show a jump at the border between two adjacent cells. Figure 4, shows a distorted image where the distortion is most evident in the form of darker clusters spread all over the area of the image. Application of the algorithm to remove this distortion resulted in significant improvement on the image, figure 5. This improvement is clearly apparent in the normalized distribution of pixels to uniformally cover the whole image area with equal density distribution.

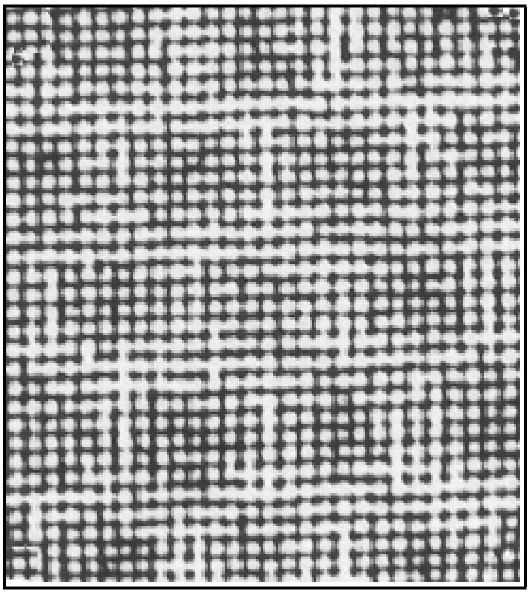

Figure 4. Distorted Image

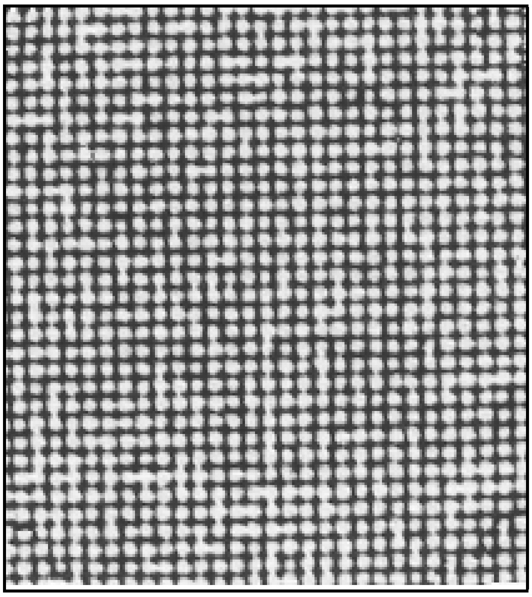

Figure 5. Corrected Image

3.2 Software Segment Implementation

```
/////////////////////////////////////////////
// 2-D Interpolation Function Written in C //
/////////////////////////////////////////////
#include <stdio.h>
#include <math.h>
#include "mathn1.h"
#include "mathn2.h"

#define N 512
#define M 512

int main(void)

{
int i,j,k;
float a,b,c,d,fixy,foxy;
unsigned long x,y;
float **xx, *xa,*xb, *ya;
```

```
float x11,x22;
char Strng1[20],Strng2[20];
xa=vector(1,M);
xb=vector(1,M);
xx=matrix(1,N,1,N);

//Read the file that contain the values for xa,ya
FILE* stream;
      stream = (FILE*) fopen( "events.input", "r" );
      if( stream == NULL )
      printf( "The file ...... was not opened\n" );
      else
      fseek( stream, 0L, SEEK_SET );
      // Read data back from file
      fscanf( stream, "%s %s", Strng1, Strng2 );

      for (j=1; j<=M;j++) {
            fscanf( stream, "%f %f", &x11, &x22);
            xa[j]=x11;
            xb[j]=x22;
      }

//Create array to be searched
      for (j=1;j<=N;j++)      {
            for (k=1; k<=N; k++) {
            xx[j][k]=xa[j]+xb[k];

            }
      }

//Calculate interpolation coeffecients
printf("\n%3s %3s %11s %11s %11s %11s %11s
%11s\n","j","k","a","b","c","d","fixy","foxy");
for (j=1;j<N;j++) {
            for (k=1;k<N; k++) {
                  x=0.5*j;
                  y=0.5*k;
a=xx[j+1][k]-xx[j][k];
b=xx[j][k+1]-xx[j][k];
c=xx[j+1][k+1]+xx[j][k]-xx[j][k+1]-xx[j][k];
d=xx[j][k];
fixy=a*(x/0.2)+b*(y/0.2)+c*(x/0.2)*(y/0.2)+d;
foxy=a*j+b*k+c*j*k+d;
printf("\n%3d %3d %11.6f %11.6f %11.6f %11.6f %11.6f %11.6f\n",
j,k,a,b,c,d,fixy,foxy);
            }
getchar();
      }

fclose( stream );
free_vector(xa,1,M);
free_vector(xb,1,M);
free_matrix(xx,1,N,1,N);

}
```

4 CONCLUSIONS

Non-linearity in digital images is a problem inherited in the design of imaging systems. Although this kind of distortion can happen at any stage of the imaging process, it is mainly encountered in the electro-optical sensors during the capture of signals present at their input and/or in the image formation process where the captured signals are transformed into an image. The algorithm presented in this study, relying on well established mathematical model, is easily implemented in software terms, and most significantly provides an efficient and flexible method for image generation and correction procedures. However, in some few cases, its application can be associated with some kind of suppression or loss of the fidelity of data, this loss of the fidelity is specially encountered if excessive execution of the algorithm is applied, this can happen if the algorithm is executed more than couple of times in correcting the same data. Thus keeping a balance between distortion reduction and maintaining fidelity of data is highly recommended. In this regard, knowledge of the data characteristics and caution in applying the algorithm to suite the application at hand are of utmost importance.

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