KNOWLEDGE-BASED 3D SURFACE RECONSTRUCTION

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ABSTRACT

The reconstruction of complex surfaces in $\Re^3$ is still a rather uncovered area in the field of Photogrammetry and Geodesy. Whereas other disciplines, such as CAD, Computer Sciences, Medicine, Geology and others, have developed methods, suitable for their special needs and applications, no satisfactory solutions exist for natural topographic surfaces. This work offers an approach for the reconstruction of 3D-surfaces, designed to fulfil the requirements of Photogrammetry and Geodesy.
The main idea is the use of as much knowledge as possible for the reconstruction of the surface from the digitised points. This knowledge includes constraints and assumptions about the original surface (e.g. smoothness of the surface), about data sampling (specific characteristics of different data sources) and about additional information (e.g. measured lines). The knowledge is splitted into elementary and autonomous statements, so-called rules. These rules assign evidences in favour or against the shape of the reconstructed surface.
The surface is modelled with a triangular mesh, which offers the necessary flexibility when modelling natural surfaces. To find the 3D-triangulation a tetrahedral tessellation of the data is computed in a first step. The main reason is the reduction of the amount of possible triangles. From the triangles of this tessellation the ones belonging to the surface are extracted. For this purpose the above rules are applied. The inference of a decision, whether a triangle belongs to the surface or not, uses standard techniques from the field of Artificial Intelligence and Probabilistic Reasoning.

1 INTRODUCTION

'Reverse Engineering" is a term, commonly used for the reconstruction of surfaces from 3D-point clouds. Engineering can be seen as the skill to construct a physical object from a digital representation, thus Reverse Engineering is the generation of a digital representation of a real world object. This task is necessary for geometric analysis of the object, for comparison with other objects or for visualisation. Reverse Engineering has to be performed in two steps:

1. Measurement of the object, e.g. determination of co-ordinates of points on the surface. The measurements are always discrete, hence they do not completely describe the surface.
2. Calculation of a digital representation from the measurements, which fits to the original surface as good as possible. Figure 1 demonstrates this step, where a 3D-triangulation is generated from point- and line-data.

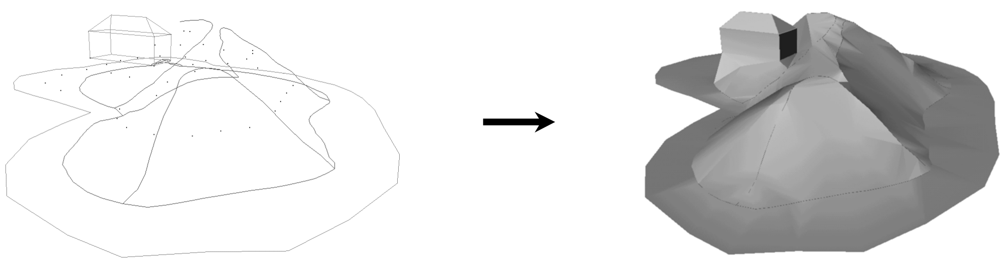


Figure 1, Reconstruction of a surface from point- and line-measurements, using a 3D-triangulation. The data consists mainly of lines: breaklines, representing a C^1-discontinuity, formlines, which indicate a high curvature of the surface across the line, and a borderline.

1.1 State of Research

Many different methods for Reverse Engineering have been developed in several disciplines. In CAD and CAGD algorithms have been developed for surface representation and reconstruction, an overview is given by Várady et al.(1997) and by Várady et al.(1998). Ekoule et al (1991) present an approach for surface reconstruction in medical applications,

which deals with the triangulation of multiple slices. Developments in Photogrammetry concentrated mainly on 2.5D-surfaces, which covered most of the possible applications. Only in recent years the reconstruction of man-made objects has become very important and several methods have been presented: Lang and Förstner (1996), Rottensteiner (1998) and Englert (1998). These methods are fully three-dimensional, but can not be used for arbitrary surfaces.
For the triangulation of points, sampled on an arbitrary surface in $\Re^3$, various algorithms are existing. An excellent overview of the most important methods is given by Mencl and Müller (1997b). Other methods are presented e.g. by Edelsbrunner and Mücke (1994), Choi et al. (1988) and Uray and Pinz (1995).

Some of these algorithms are designed for specific needs or certain data sources, others are of a more general purpose. But none of them is designed to fulfil especially the requirements of applications in the field of photogrammetry.

1.2 Problem Definition

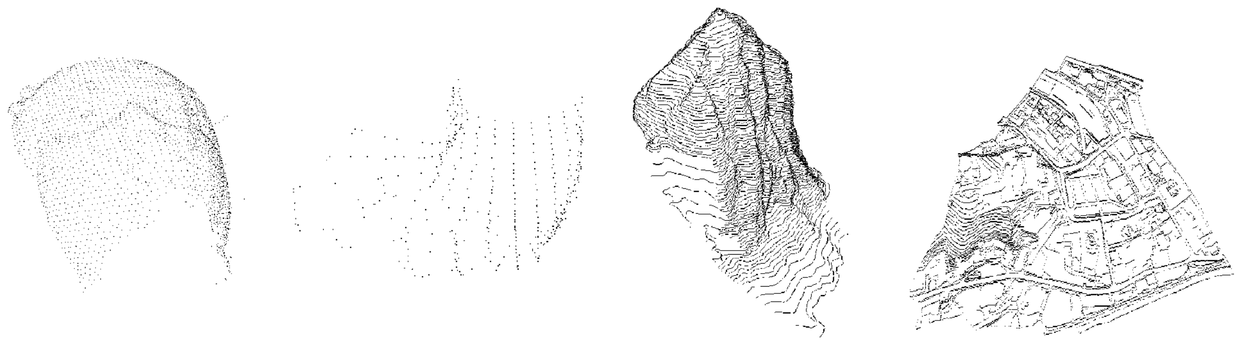
For modelling arbitrary surfaces in $\Re^3$ a flexible approach is necessary. One of the best approaches is a triangulation of the data points. A triangulation is an irregular topologic structuring of the data, but it can immediately be used for surface representation: the triangles can be used as planar facets or they can be replaced by curved triangular surface-patches (s. Pfeifer and Pottmann 1996). Hence the aim of this work is, to find a 3D-triangulation of a given point set.
The algorithm has to fulfil some additional requirements:

- Inclusion of lines: the measurement of topographic lines is one of the most important advantages of photogrammetric measurement. These lines have to be included in the triangulation as constraints.
- Use of additional information: e.g. from different sources of measurement, different hints can be deduced how the triangulation has to look like.
- Surface representation: the triangulation is the base of the actual surface representation, hence it has to be constructed considering the geometry of the surface.

2 KNOWLEDGE ABOUT THE DATA

Different kinds of measurement have different characteristics. These characteristics should be exploited as far as possible when generating the triangulation. If the algorithm is capable to include this information, this capability will be called knowledge: the program will *know* how to generate a triangulation, if a certain precondition is satisfied.
Every reconstruction algorithm uses some knowledge about the expected input. This knowledge is incorporated in the algorithm how to compute the triangulation. Seldomly, this knowledge is formulated explicitly. An example for the explicit formulation of rules is given by Mencl and Müller (1997a).
In this work it has been tried to formulate all possible knowledge. Seven categories have been introduced, from which the most interesting aspects will be discussed shortly:

- **Knowledge about the properties of the original surface**: assumptions about the shape and properties of the surface: C0, C1- continuity, open or closed surfaces, no self-intersections.
- **Knowledge about the measurement of the surface**: important are three aspects: discretization, point distribution and data source. Discretization has to meet the requirements of some Sampling Theorem (s. Tempfli, 1982, or Boissonnat, 1984). Data source, i.e. the type of measurement, gives the most important conditions for the surface reconstruction. For photogrammetric needs the following types have been identified as necessary: contourlines by digitisation or direct measurement, topographic measurement, automated measurement and profiles (s. Figure 2).



a b c d

Figure 2, examples for (a) automatic data sampling, (b) profile measurement, (c) contourlines and (d) photogrammetric measurement.

These different categories have been considered in this work. The main characteristics are:
Contourlines: the complete surface is sampled by intersection with multiple horizontal planes.

Topographic measurement: methods, which explicitly measure topographic details, such as local maxima or ridge lines, i.e. photogrammetric or terrestric measurement.
Automated measurement: determination of object points by automatic processes, e.g. image correlation.
Profiles: the object is scanned in various profiles, which can be arbitrary placed and shaped, located in parallel planes or symmetric to an axis of rotation.

- **Knowledge about the properties of lines on the original surface**: lines impose topological and geometrical constraints: breaklines indicate a C^1-discontinuity, formlines indicate a high curvature across the line and borderlines determine the border of the surface. A special case are contourlines, because they usually represent a whole surface (or a big part of it) without other kinds of data.
- **Knowledge about the measurement of lines**
- **Knowledge about the way of modelling**: the surface will be represented by a triangulation. The triangulation has to fulfil some conditions of validity (s. Hoschek and Lasser 1992), but it should also represent the surface in an optimal way.
- **Knowledge about the modelling of lines**: lines have to be included as series of constraint edges in the triangulation.
- **Additional knowledge**: this includes information which is commonly not available, such as: surface normals in the data points (can be delivered through certain image matching techniques), Line-Of-Sight conditions (i.e. viewpoint for each point on the surface), fixed type of the surface (torus, plane, ...), etc.

3 RULES

After having identified all information that can be used for reconstruction, the next question is how to formulate this knowledge. A well studied approach for this purpose is the use of rules.

3.1 Syntax of rules

A rule is a single piece of knowledge, which has to be (according to Buchanan and Shortliffe 1985):

- *elementary*: the content of a rule has to be as simple as possible.
- *primitive*: a rule should perform only operations, as simple as possible - i.e. without sub-routines or loops.
- *independent*: a single rule must not be dependent from any other rule.
- *not correlated*: different rules must contain different statements. If two rules state the same thing, the result will be biased.

The syntax of any rule can be defined as (Lucas and Gaag 1991, p. 102):

<*rule*>	:=	**if** <*antecedent*> **then** <*consequent*>
<*antecedent* >	:=	<*disjunction*> [**and** <*disjunction* >]
<*disjunction* >	:=	<*condition*> [**or** <*condition* >]
<*consequent* >	:=	<*conclusion*> [<*conclusion* >]
<*condition* >	:=	<*predicate*> (<*variable*>, <*constant*>)
<*conclusion* >	:=	<*action*> (<*variable*>, <*constant*>)
<*predicate*>	:=	*same* | *not same* | *greater than …*
<*action*>	:=	*add* | *remove …*

Example: if a triangle $T_i \in S$ has an area smaller than all triangles $T_j \in S$ ($i \neq j$), than a certain evidence e can be assigned to T_i. e has to be a measure for the evidence that T_i belongs to the surface. This yields the following:
antecedent: $\text{area}(T_i) < \text{area}(T_j),\ \forall\ T_j \in S,\ T_i \in S,\ i \neq j$.
consequent: $add(e_i, E)$, where e_i is the evidence of triangle T_i belonging to the surface and E some constant.

3.2 probabilistic reasoning

As can be seen in the example above, the statements, contained in the rules, are seldomly deterministic. This makes necessary the introduction of a system which allows the assessment, evaluation and propagation of uncertainties. In the field of Expert Systems, various approaches are in use. In this work three of them have been implemented and the so-called CF-method proved to be the most appropriate. The applied methods are:

- Subjective Bayesian method, an extension of Bayes' Theorem from classical Probability Theory (s. Lucas and Gaag 1991).
- Certainty-factor model (CF-model), which has been developed for the MYCIN-system, a system for medical diagnosis (s. Buchanan and Shortliffe 1985).
- A system, similar to the INTERNIST-model, which has also been developed for medical diagnosis (s. Puppe 1993).

All approaches define measures for evidence and functions for propagation of evidence, where four types of propagation are necessary:

1. Propagation by conjugation (**and**).
2. Propagation by disjunction (**or**).
3. Propagation by inference, i.e. determination of evidence, when applying a rule.
4. Combination of evidences: when several rules assign different evidences to a diagnosis, an overall evidence has to be computed.

3.3 Inference of decisions

For the application of the rules several different techniques are possible. These approaches differ in their strategy to select rules from the rule-base to evaluate. Mainly two approaches are used:

- Forward Reasoning: a data- or symptom-driven approach. All rules, whose pre-conditions are fulfilled by the data, are selected and executed.
- Backward Reasoning: a task- or diagnosis-driven approach, where all rules are selected and executed which contain the task in their *consequent*-part.

Both approaches can be refined, to minimise the amount of rules executed: by the use of given priorities, a certain order of the rules, content of the rules or an organisation of the rules, due to some semantic context. In this approach a Forward Reasoning method has been implemented following a rather fixed strategy of rule selection: a so-called rule-tree.

4 3D-TRIANGULATION

In the presented approach the triangulation is performed in two steps: at first a tetrahedral tessellation is calculated and secondly those triangles will be extracted from the tessellation, which belong to the surface.

4.1 Tetrahedral tessellation

The use of a tetrahedral tessellation (calculation see Edelsbrunner and Mücke 1994 or Hoschek and Lasser 1992) as the base of the triangulation has several reasons:

- The amount of possible triangles is reduced to the ones, contained in the tessellation.
- Lines are already contained as constraints in the tetrahedral tessellation.
- Intersecting triangles are impossible, hence it is easier to ensure a valid triangulation.
- A tetrahedral tessellation is a topological structure which allows efficient analysis of the data. Also many calculations and queries are supported by this structure.

Due to the expected large amounts of data, it had been necessary to implement the tessellation algorithm with a paging-mechanism. The tessellation is divided into tiles of different size by the use of an Octree-structure. Tiles, which are currently not used by the algorithm, will be written temporarily onto hard-disk. Thus, large amounts of data can be processed.

4.2 Extraction of triangles

From the triangles, included in the tetrahedral tessellation, the ones belonging to the surface are extracted by application of the rules. This is done step by step, always adding one triangle after the other to the already extracted surface. This incremental approach has the advantage that properties of the triangle, related to its neighbour on the surface, can be exploited, e.g. the angle between these two triangles. A similar, stepwise algorithm is presented by Boissonnat (1984) or by Mencl and Müller (1997a), only the set of triangles to be tested is limited otherwise.

For each step the inference mechanism is applied for all candidate triangles and the one with the highest evidence is added to the extracted surface. A set of candidate triangles is defined as all triangles belonging to one edge of the border of the already extracted surface. Figure 3 shows an intermediate step for a simple closed surface. About half of the triangles are already extracted from the tetrahedrons.

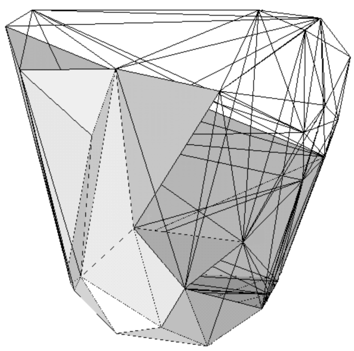


Figure 3, an intermediate step in surface extraction.

5 RULE-BASE

The rules, which are used to separate the triangles of the surface from the other ones, are the most important part of the program. These rules should include the knowledge we want to use for extraction. Corresponding to the categories of knowledge, which have been introduced in chapter 2, some important rules will be presented (current amount of rules: about 40).

5.1 Knowledge about the properties of the original surface

Self-Touching Rule	
Description	It will be tested, if the candidate triangle T_i touches the already extracted surface, or not. Self-touching or intersection is not allowed (all terms are explained in Figure 4).
Antecedent	(P_i belongs to surface) **and** (P_i lies not on the border) **or** (($k_{i,1}$ belongs to surface) **and** ($k_{i,1}$ lies not on the border)) **or** (($k_{i,2}$ belongs to surface) **and** ($k_{i,1}$ lies not on the border))
Consequent	The evidence e, that T_i belongs to the surface, is set to zero.

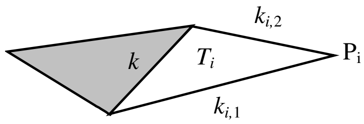


Figure 4, a candidate T_i, with its elements.

Intersection Rule	
Description	Due to the incremental strategy, it can occur, that extracted triangles are overlapping (s. Figure 5). To test the overlapping, a spatial criterion, according to Mencl and Müller (1997a), is used.
Antecedent	The currently extracted triangle T_i overlaps a triangle T_j of the surface
Consequent	The evidence e, that T_i belongs to the surface, is set to zero.

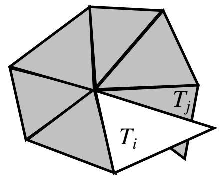


Figure 5, the triangle T_i overlaps the triangle T_j.

5.2 Knowledge about the measurement of the surface

Minimum Area Rule	
Description	According to the Sampling Theorem, it can be stated, that points lying close together (small Euclidean distance) are more likely to be neighbours on the surface than points lying far away from each other. Hence, triangles with small areas should be enforced.
Antecedent	The triangle T_i has the smallest area of all candidate triangles
Consequent	The evidence e, that T_i belongs to the surface, is increased by a certain value

Scanner Rule	
Description	Scanner data (e.g. from laser scanning) is generally arranged in an array: there are scan-lines l_1, ..., l_n and m points per scan line.
Antecedent	One edge of the triangle T_i connects two consequent points of a scan-line.
Consequent	The evidence e, that T_i belongs to the surface, is increased by a certain value.

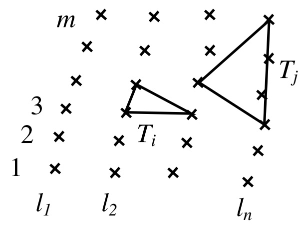


Figure 6, Scanner Data: the triangle T_i is much more likely than the triangle T_j.

5.3 Knowledge about the properties of lines on the original surface

A lot of knowledge can be used, when the object has been sampled with contourlines. The reconstruction has to solve two questions:

1. Determination of the neighbourhood relations between all contourlines.
2. Determination of the surface (in our case the triangulation) between two neighbouring lines.

ad 1
It can be shown, that three types of neighbourhood-relations are sufficient (s. Figure 7): vertical neighbours, horizontal neighbours and lines which are related over their endpoints (gaps in the data).

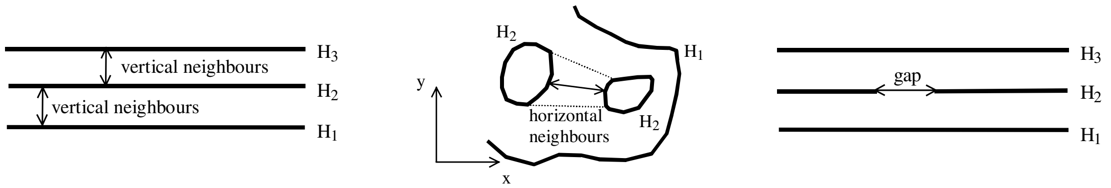


Figure 7, the three types of neighbourhood relation, which are sufficient for modelling a surface with contourlines.

ad 2
Due to the limited types of neighbourhood, there is also a limited amount of possible valid triangles between these lines (only seven types are allowed).

The facts about neighbourhood of contourlines and the triangles between can easily be formulated in a set of rules.

5.4 Knowledge about the way of modelling

The triangulation has to fulfil certain conditions of validity. Furthermore it should represent the surface in an optimal sense. For this optimisation different criterions are possible:

Delaunay Rule	
Description	The Delaunay criterion in 2D is equivalent to the maximisation of the minimum angle in each triangle. In 3D it is also desirable to gain equally shaped triangles.
Antecedent	T_i is the triangle of all candidates with the biggest minimum angle.
Consequent	The evidence e, that T_i belongs to the surface, is increased by a certain value.

Crease angle Rule	
Description	To gain a smooth surface, triangles with crease angles near to π/2 are enforced s. Figure 8).
Antecedent	T_i is the triangle of all candidates with the biggest crease angle β.
Consequent	The evidence e, that T_i belongs to the surface, is increased by a certain value.

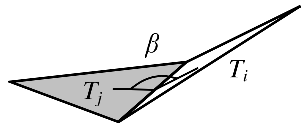


Figure 8, crease angle β between the triangle T_i and the already extracted triangle T_j.

The evidence, assigned to T_i, is composed from two aspects: at first the importance of the symptom β itself in regard of the diagnosis (part of the surface, or not). Secondly, the actual value of β: a short value produces a low evidence, a value near to π/2 a high evidence. The diagram of Figure 9 shows the evidence of β (the evidence ranges from 0.0 to 1.0) in a histogram. In this example β is used in favour of and against a membership to the surface, i.e. a value of β between 0° and 100° votes against a membership, a value between 100° and 180° votes pro.
The user can manipulate the evidences and according histograms, to trim the extraction in respect to the current data set. The program offers the user a training-mode, where a given triangulation is analysed and suggestions for the evidences are determined.

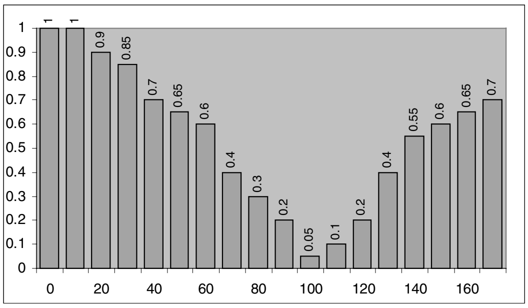


Figure 9, evidence of the angle β, depending on its value.

6 EXAMPLES

The presented method has been implemented and tested with various data sets. Some of these examples will be shown. The three presented methods for probabilistic reasoning have been implemented. In the following examples, always the certainty-factor model has been used.

6.1 Torus

This data set has been provided by Ernst P. Mücke (it is available - together with other data sets - under: http://www.geom.umn.edu/software/cglist/GeomDir/data_1.1.tar.gz). Due to the homogeneous point distribution, the surface could be reconstructed without any problems - Figure 10.

6.2 Bust

This data set is also one of E. P. Mücke. The point distribution is also homogeneous (s. Figure 11). Some points on the ears could not be inserted.

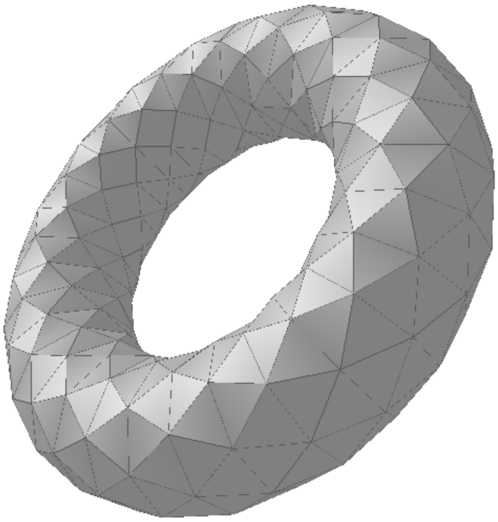


Figure 10, reconstructed torus.

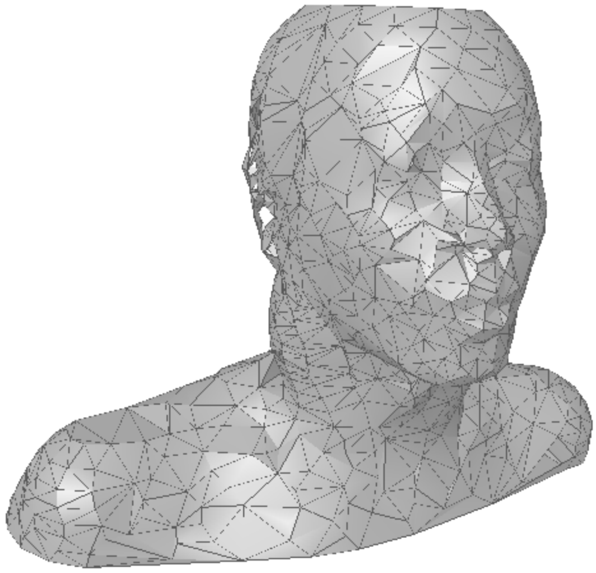


Figure 11, reconstructed bust.

6.3 Bischofsmütze

“Bischofsmütze” is the name of a prominent mountain in Salzburg, Austria. Because of movements of the rock and threatening rockfall it has to be monitored periodically. The data capturing has been done by the Engineering Company Linsinger, St. Johann / Pongau, by photogrammetric compilation. In multiple stereo-models contourlines of intervals of 2 and 5 meters have been measured.

This data set revealed some problems: at some locations the evidence of belonging to the surface is for all concerned triangles too small, so that no triangle could be inserted into the triangulation. Therefore small holes remained in the triangulation. These holes have to be filled automatically in a post-processing step.

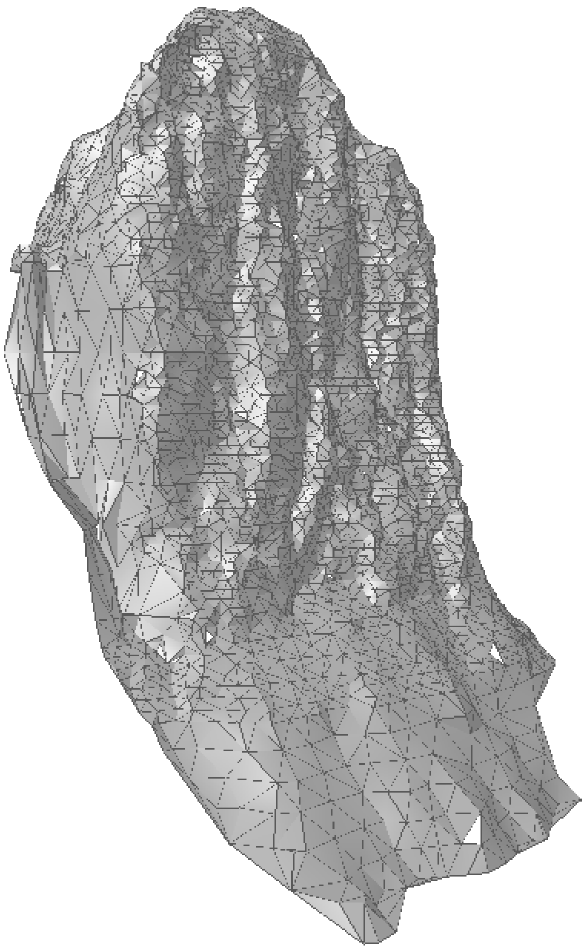


Figure 12, 3D-triangulation of the mountain “Bischofsmütze”.

7 CONCLUSION

The presented approach has some major advantages:

1. It is a flexible approach, useable for different applications.
2. New knowledge can be included easily.
3. The method can deal with most photogrammetric applications, which are generally difficult to handle.
4. The implemented method of probabilistic reasoning allows the usage of uncertain knowledge.

Necessary improvements are an increase of efficiency - for the tessellation as well as the extraction - and a more general strategy of rule-selection.

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