CONTROL SURFACE IN AERIAL TRIANGULATION

Jen-Jer JAW
Department of Civil Engineering, National Taiwan University,
Taipei, Taiwan, Republic of China
Jejaw@ce.ntu.edu.tw

Working Group III/2

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ABSTRACT

With the increased availability of surface-related sensors, the collection of surface information becomes easier and more straightforward than ever before. Thus, the integration of surface information into the photogrammetric workflow, the task which has been long time interesting as well as challenging, is gaining focus again within the photogrammetric community. In this paper, the author proposes a model in which the surface information is integrated into the aerial triangulation workflow by hypothesizing plane observations in the object space, the estimated object points via photo measurements (or matching) together with the adjusted surface points would provide a better point group describing the surface. Apart from releasing aerial triangulation from the necessity of identifying control points in the object space, the proposed algorithms require no special structure of surface points and involve no interpolation process. The proposed system is proven workable having data collection in the photogrammetric laboratory.

1 INTRODUCTION

More and more surface related sensors, such as airborne laser range finder, INSAR (INterferometric Synthetic Aperture Radar) with tightly integrated on-board GPS/INS system became commercially available during the last decade. By that, the booming research and applications mainly in generating elevation information for the area of interest and scene analyses, especially for the buildings in residential areas have been, among others, promising an era of sensors in which the collection of surface information becomes easier and more straightforward than ever before. Besides, due to the state-of-the-art of the sensor integration technique [Schwarz, 1995], the accuracy of the analyzed surfaces via airborne laser scanning system proves competitive with the scenario where a well-controlled data set and careful measurements by the operator are the necessities for the accuracy typical in traditional photogrammetric production line. Thus, how would photogrammetrists consider this newly available technique and its seemingly favorable data set apart from the aforementioned interests? One of the attractive thoughts follows: Can aerial triangulation by taking photo measurements benefit from this sensor dominant era and do a better job for the task of the surface reconstruction and how? These questions led the author into this study.

This very same idea and attempt had been carried out a decade ago at the Technical University of Munich, Germany, conducted by Ebner [Ebner/Strunz,1988][Ebner et al., 1991][Ebner/Ohlhof, 1994] even when the laser range data did not yet come to applications. Their algorithms focused mainly on the satisfaction of accuracy for the middle and small scale photogrammetry by minimizing the differences between the heights of the object-to-be-solved and the interpolated heights (bilinear interpolation) via the surrounding surface points, DEMs (Digital Elevation Models) in their case, as constraints. In this study, without interpolation on the surface points and requiring no special structure of the surface points, the author exploits different algorithms by hypothesizing planes (also called "control surface" in this paper) and assessing the uncertainties via checking the fitting planes with local surface points; the minimization takes distances along the surface normal as the target function when formulating the surface constraint.

The rest of this paper consists of the following: Section two introduces the surface constraint with formulating its mathematical as well as stochastic model. The integration of surface constraint into aerial triangulation, the least squares solution and the extended model by employing "tie surface" are explained in section three. Section four demonstrates the experimental test in the photogrammetric laboratory together with the accuracy (root mean square error) report and the analyses. Section five concludes this study by giving some observations of this research from this author's perspective.

2 MODEL FORMATION OF THE SURFACE CONSTRAINT

2.1 The First–Order Surface (Plane) Equation:

Mathematically, three points, let's say $P_1(X_1, Y_1, Z_1)$, $P_2(X_2, Y_2, Z_2)$ and $P_3(X_3, Y_3, Z_3)$, define a plane. The plane equation with a, b and c being the function of coordinate components of three points can be seen as equation (1).

$$aX + bY + cZ + d = 0 \tag{1}$$

To calculate the distance along the surface normal between a point $P_i(X_i, Y_i, Z_i)$ and the plane $aX + bY + cZ + d = 0$, one could use equation (2) shown as below,

$$l_i = \frac{aX_i + bY_i + cZ_i + d}{\sqrt{a^2 + b^2 + c^2}} \tag{2}$$

2.2 The Surface Constraint:

Analytically, if a point is known to lie on the known plane, equation (1) would be automatically realized. A direct result of that is that the distance calculated by using equation (2) would end up with zero.

Realistically, due to the measuring errors of the employed instrument, the physical truth of the plane could hardly be found by the collected data. Thus if the points are judged to lie on the known plane, one would compromise the observations by minimizing the distances along the normal, shown as equation (3), such that the imperfection of the measurements and the overall registration of the data set could be taken into account in an optimal way towards the solution. By that, the author proposes an algorithm in which the plane is hypothesized in the object space and confirmed by the surface points, thus forming the surface constraint into the adjustment for simultaneously determining the object points and the exterior orientation parameters via aerial triangulation procedures.

$$E\{l_i\} = E\left\{\frac{aX_i + bY_i + cZ_i + d}{\sqrt{a^2 + b^2 + c^2}}\right\} \tag{3}$$

2.3 The Functional as well as the Stochastic Model of the Surface Constraint

The symbols in equation (3) are classified into two groups, namely the observations and the unknowns. For this study, what is known are the surface points while those points $P_i(X_i, Y_i, Z_i)$ considered to lie on the planes are the unknowns through the photogrammetric measurements. One, therefore, is able to formulate functional model as well as stochastic model of the control surface constraint shown as equation (4)

$$w_{mx1} = B_{mx3q}\, y_{3qx1} = A_{mx3n} x_{3nx1} + Be_{3qx1}, \quad Be \sim (0, \overline{\Sigma_o} = B\Sigma_0 B^T = \sigma_{o_o}{}^2 B P_o^{-1} B^T) \tag{4}$$

where
n : the number of object points from the photo measurements;
m : the number of surface constraints;
q : the number of registered surface points;
w : the discrepancy ((l_i^o) derived with the approximations of the unknowns and the observations;
y : the observations of the known surface points;
B : the coefficients of partial derivatives with respect to the surface points;
A : the coefficients of partial derivatives with respect to the unknowns of the object points (may include $0_{mx3(n-m)}$ for those object points unable to find registered surface planes);
x : the unknowns of the object points ; e : the random errors of the surface points;
Σ_o : the dispersion matrix of the surface points; P_o : the weight matrix of the surface points;
$\sigma_{o_o}{}^2$: the variance component of the surface points;

Yet from the prediction point of view, the closer the unknown object point is to the registered surface point, the more reliable the constraint information would become . The error-propagated uncertainty assessment of the surface constraint based on the aforementioned model does not find itself the same conclusion. Therefore a modification of the stochastic model compromising the likely conflict due to the imperfect analytical model is needed [Jaw, 1999]. The modification performs in a way such that the Σ_{oo}**,** as shown below, reflects the deviation of the plane formation by three surface points from the fitting plane by all the neighboring surface points. We therefore come to the modified model as follows,

$$w = By = A\text{x} + Be, \qquad Be \sim (0, \overline{\Sigma_{oo}} = B\Sigma_0 B^T + \Sigma_{oo} = \sigma_{o_o}{}^2 (BP_o^{-1}B^T + P_{oo}{}^{-1}) = \sigma_{o_o}{}^2 \overline{P_{oo}}^{-1}) \tag{5}$$

where
P_{oo}: the weight matrix of the added uncertainty;
$\overline{P_{oo}}$ stands for the weight matrix of surface constraint after the modification;

Due to the restricted space in this paper, the author encourages the interested readers refer [Jaw, 1999] for more detailed explanation of the model modification.

3 CONTROL SURFACE IN AERIAL TRIANGULATION

3.1 Aerial Triangulation System Formulation

With the formulation of surface constraint model, the photogrammetric measurements can be joined into the system performing the aerial triangulation task. The combined model is expressed as below,

$$y_{(2k+m)x1} = \begin{bmatrix} y_1 \\ w \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} \text{x}_{1(6p x1)} \\ \text{x}_{2(3n x1)} \end{bmatrix} + \begin{bmatrix} e_1 \\ Be_2 \end{bmatrix}, \begin{bmatrix} e_1 \\ Be_2 \end{bmatrix} \sim (0, \Sigma), \tag{6}$$

$$\Sigma_{(2k+m)x(2k+m)} = \begin{bmatrix} \Sigma_{1(2kx2k)} = \sigma_{o_1}{}^2 P_1^{-1} & 0_{(2kxm)} \\ 0_{(mx2k)} & \overline{\Sigma}_{oo\,(mxm)} = \sigma_{o_o}{}^2 \overline{P_{oo}}^{-1} \end{bmatrix}$$

where
y_1 : increments of the photo observations, namely the difference between the original observations and the derived observation via the approximations;
k : number of the photo point measurements;
p : number of the photos ; n : number of the object points to be determined;
A_{11} : the coefficient matrix derived from taking partial derivatives with respect to the exterior orientation parameters;
A_{12} : the coefficient matrix derived from taking partial derivatives with respect to the object point coordinates;
$\text{x}_1 = [\Delta X_{o_1}, \Delta Y_{o_1}, \Delta Z_{o_1}, \Delta\omega_{o_1}, \Delta\varphi_{o_1}, \Delta\kappa_{o_1}, \ldots, \Delta X_{o_p}, \Delta Y_{o_p}, \Delta Z_{o_p}, \Delta\omega_{o_p}, \Delta\varphi_{o_p}, \Delta\kappa_{o_p}]^T$: unknowns of exterior orientation. Δ indicates the increment of the parameters; $\text{x}_2 = [\Delta X_1, \Delta Y_1, \Delta Z_1, \Delta X_2, \Delta Y_2, \Delta Z_2, \ldots \Delta X_n, \Delta Y_n, \Delta Z_n]^T$: unknowns of object points;
e_1: the random errors of the photo point measurements;
$\sigma_{o_1}{}^2$: the variance component of the photo point measurements;
P_1: the weight matrix of the photo point measurements; w : the discrepancy of the surface constraint;
A_{22} is A in equation (4) plus $0_{mx3(n-m)}$ for those object points unable to find registered surface planes;
m : number of the surface constraints; e_2 : the random errors of the registered surface points;
$\sigma_{o_o}{}^2$: the variance component of registered surface points;
$\overline{P_{oo}}$: the weight matrix of the modified surface constraints;

With the above system formulation, the least squares method would lead to the solutions as follows,

The estimation of the parameters

$$\hat{\xi}_1 = [N_{11}^{-1} + N_{11}^{-1}N_{12}(N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1}N_{21}N_{11}^{-1}]\cdot C_1 - [N_{11}^{-1}N_{12}(N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1}]\cdot C_2 \tag{7}$$

$$\hat{\xi}_2 = [-(N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1}N_{21}N_{11}^{-1}]\cdot C_1 + [(N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1}]\cdot C_2 \tag{8}$$

where

$$N_{11} = A_{11}^T P_1 A_{11}, \qquad C_1 = A_{11}^T P_1 y_1$$

$$N_{12} = A_{11}^T P_1 A_{12} = N_{21}^T,$$

$$N_{22} = A_{12}^T P_1 A_{12} + A_{22}^T \overline{P_{oo}} A_{22}, \qquad C_2 = A_{12}^T P_1 y_1 + A_{22}^T \overline{P_{oo}} w$$

The dispersion of the parameters,

$$\sigma_o^{-2} \cdot D\left\{\hat{\xi}_1\right\} = N_{11}^{-1} + N_{11}^{-1}N_{12}(N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1}N_{21}N_{11}^{-1} \tag{9}$$

$$\sigma_o^{-2} \cdot D\left\{\hat{\xi}_2\right\} = (N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1} \tag{10}$$

The predicted residuals:

$$\tilde{e}_1 = y_1 - A_{11}\hat{\xi}_1 - A_{12}\hat{\xi}_2 \tag{11}$$

$$\tilde{e}_2 = -P_2 B^T \overline{P_{oo}}(A_{22}\hat{\xi}_2 - w) = -(P_o^{-1} + Q_{oo})B^T \overline{P_{oo}}(A_{22}\hat{\xi}_2 - w) \tag{12}$$

The estimated variance component:

$$\hat{\sigma}_o^2 = \frac{\tilde{e}_1^T P_1 \tilde{e}_1 + \tilde{e}_2^T P_2 \tilde{e}_2}{2k + m - 6p - 3n} \tag{13}$$

The estimated dispersion of the exterior orientation parameters ($\hat{\xi}_1$):

$$\hat{D}\left\{\hat{\xi}_1\right\} = \hat{\sigma}_o^2(\sigma_o^{-2} \cdot D\left\{\hat{\xi}_1\right\}) \tag{14}$$

The estimated dispersion of the object point coordinates ($\hat{\xi}_2$):

$$\hat{D}\left\{\hat{\xi}_2\right\} = \hat{\sigma}_o^2(\sigma_o^{-2} \cdot D\left\{\hat{\xi}_2\right\}) \tag{15}$$

3.2 Best Fitting of Object Points and Surface Points

The system solved by the above least squares method not only estimates the unknowns of object points measured via photos but also adjusts the registered surface points combining collinearity property and surface plane constraints. Thus the collection of the estimated object points and adjusted surface points, see Figure 1, would theoretically provide a better surface points group from the data fusion point of view. However, the correlation between these two data sets due to adjustment procedure should not be forgotten if the overall surface points are to be further processed, such as interpolation or surface analyses. Based on the least squares solution, the fusion of the data sets and their variance-covariance matrix can be further derived [Jaw, 1999].

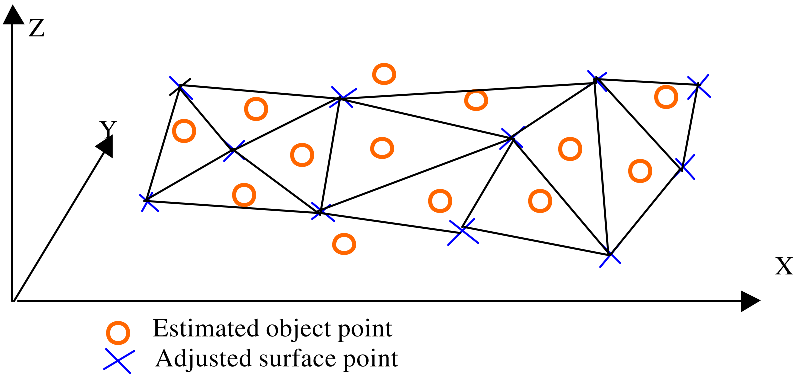

Figure 1. Combined data set (estimated object points + adjusted surface points)

3.3 The extension of the Model

Although the above system is derived based on the assumption that the stochastic constraints in which the unknown object points are registered with surface planes are employed. Yet for some applications where plane hypotheses are confirmed by other information, such as the interpretation of the human operator or given scene knowledge, instead of surface points, the associated stochastic constraints involved only parameters could be formulated taking the plane measurements into account. In case the plane measurements are performed by overlapping models, an analogue to tie points for traditional aerial triangulation, the "tie surface", in this scenario, would tie the overlapping models together, thus contributing to the estimations of both related exterior orientation parameters and object points (see Figure 2). Without going into the detail, the stochastic constraints of this kind could be explained as following,

Given a set of plane measurements, $\bar{p} = \{P_1, P_2, \ldots P_n\}$

$P_1, P_2, \ldots P_n$ are the points measured on $\bar{p}$ plane, thus one plane constraint for every four points could be formulated in a similar way as the author introduced in section three, the only difference is the three surface points are replaced by three object points, therefore formulating an "observation equation". The total number of independent constraints is (n-3). Since the similarity of the constraints the functional as well as stochastic model of "tie surface" can be joined into the original stochastic constraints without losing the generalization of the system towards the solution, the author would not repeat the derivation of the system solution in this manner.

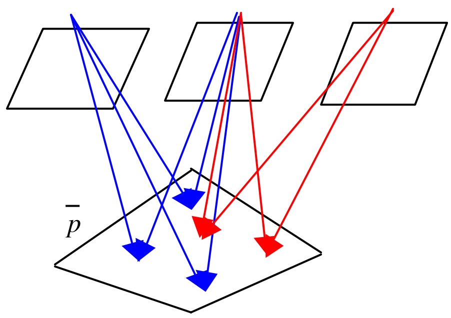

Figure 2. Tie Surface Plane

4 EXPERIMENTAL RESULTS AND ANALYSES

The author conducted an experiment using an analytical plotter for collecting both the ground truth and the measuring data. The measurements involving 4 consecutive stereomodels were performed by the Zeiss C120 Analytical Plotter in the photogrammetric laboratory at the Department of Civil and Environmental Engineering and Geodetic Science, the Ohio State University. The photographs with about 1/4000 scale were taken at Ocean City, Maryland in April, 1997. The area of interest has been aerially triangulated by using highly accurate GPS site measurements as the control points. Accordingly, the author treats the exterior orientation parameters as the true values for the collection of the object points, then comparing them with the solved parameters in order to assess the empirical accuracy (root mean square error). The coverage of the test area and the measured plane features, as marked, can be seen in Figure 3.

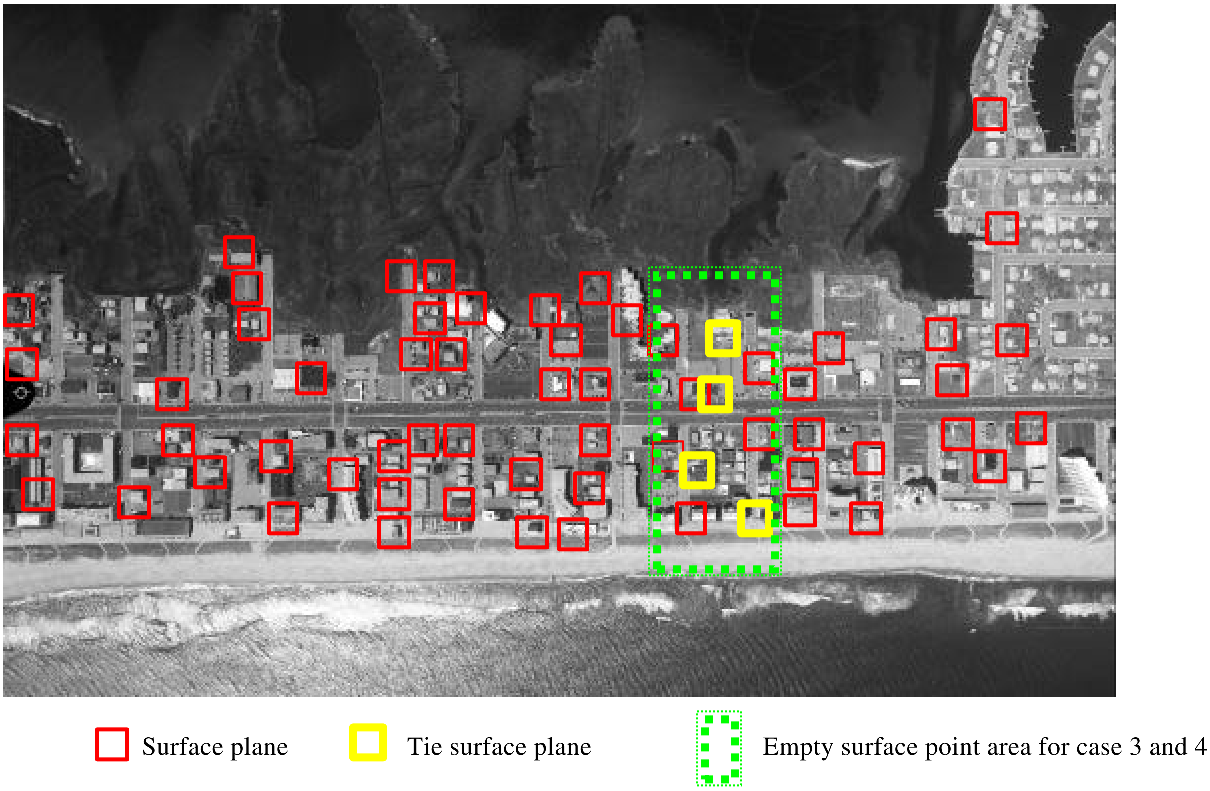

Figure 3. The test area and the distribution of the surface measurements

4.1 Description of Four Investigated Cases

With the data set arranged as previously mentioned, the following four cases, are used to test how well the proposed model would obtain the solution.

Case 1: Only known surface planes serve as constraints in adjustment, meaning four tie surface planes would not participate in the data set.

Case 2: Both known surface planes and unknown tie surface planes are observable.

Case 3: Only the tie surface planes are observable in the area with empty surface points as illustrated in Figure 3. Therefore, in this case, the unknown tie surface planes really try to bridge the models by only geometrically available plane constraints (namely without knowing the orientation of the planes).

Case 4: Exclude the measurements of the tie surface planes from case 3 so that the area indicated by the dotted line in Figure 3 is with empty information.

4.2 Results

	Xo	*Yo*	*Zo*	w	j	k	*X*	*Y*	*Z*	*System Redundancy*
Case 1	0.22	0.20	0.14	0.000274	0.000334	0.000167	0.087	0.107	0.079	84
Case 2	0.19	0.16	0.12	0.000214	0.000287	0.000146	0.074	0.093	0.074	104
Case 3	0.21	0.19	0.13	0.000245	0.000323	0.000172	0.090	0.110	0.086	90
Case 4	0.29	0.34	0.17	0.000544	0.000468	0.000210	0.108	0.131	0.092	70

{ X_o, Y_o, Z_o, w, j, k }: exterior orientation parameters

{X, Y, Z} object point coordinates

Positional unit: meter; Angular unit: radian

Table 1. Accuracy assessment (R.M.S.E.) based on the adjustment

	Xo	*Yo*	*Zo*	w	j	k	*X*	*Y*	*Z*	*System Redundancy*
Case 1	0.11	0.12	0.09	0.000136	0.000416	0.000212	0.133	0.057	0.072	84
Case 2	0.11	0.10	0.08	0.000121	0.000428	0.000196	0.121	0.053	0.065	104
Case 3	0.10	0.10	0.07	0.000161	0.000407	0.000208	0.120	0.051	0.072	90
Case 4	0.08	0.12	0.08	0.000140	0.000365	0.000190	0.138	0.074	0.076	70

{ X_o, Y_o, Z_o, w, j, k }: exterior orientation parameters

{X, Y, Z} object point coordinates

Positional unit: meter; Angular unit: radian

Table 2. Accuracy assessment (R.M.S.E.) based on the ground truth

Note that the following common parameters were used in this test:

Focal length of the camera = 0.152764m;
S.D. of photo point measurement = 7 micrometers;
S.D. of surface points in X component = 0.07m; Y=0.07m; Z=0.12m.
(S.D. Stands for Standard Deviation)
Variance component of photo measurement = 1; surface points = 1.

4.3 Analyses

- The system works without the necessity of employing control points, the ingredient of indispensable for traditional aerial triangulation.
- In overall, the higher the system redundancy is, the higher the accuracy obtained.
- By comparing case 1 and case 2, the tie surface planes, adding the constraints into the adjustment increases the system performance by higher redundancy, thus leading to the better parameter estimation both for the exterior orientation and the object points.
- The tie surface planes, without knowing the orientation of the planes in the object space, show their importance in tying models where known surface points are not available. The achieved accuracy does not deteriorate significantly due to the lack of registering those points to the object space.
- Case 4 involves none of the control surface or the tie surface observations, for that information-empty area shows the worst accuracy overall for the object points due to the least favorable geometry among all cases. Therefore, the contribution of the tie surface for the overall solution can not be neglected if observable.
- Even with the narrow strip, the unfavorable geometry by traditional aerial triangulation, the solution is not weakened at all.

5 CONCLUSIONS

Aerial triangulation can be performed using the control surface rather than the control points, thus no identification of measured surface points with respect to their object truth is necessary.

The control surface hypothesizing planes (first-order surface) in the object space may seem to oversimplify the surface at the first glance. Yet with increasingly available surface collectors, such as laser range finder and INSAR, the denser surface points data set would come into application field inevitably in the nearest future. Thus the available surface information would be analyzed in a more local scale in which the first-order surface, namely the plane, seems to be the most likely shape among all the others. Furthermore, for the residential area where artificial features dominate, the surfaces of the buildings including the roofs and the side walls, the surfaces of the road and many other man-made constructions provide abundant plane-like features not only in number but also in orientation, the proposed model shows its potential for the application in this area.

The proposed model solves not only the newly added object points via photogrammetric methods but also has the surface points adjusted through the registration of points from the photo space onto the object space. Thus the combined data set, the solved object points and the adjusted surface points together with their variance and covariance matrix [Jaw, 1999], would provide a better surface description for the task of surface analyses or surface reconstruction especially when image matching is to be exploited for the thorough surface generation. It is this author's strongest emphasis for the contribution of this study on getting better estimation of the object space as a whole instead of watching out for every single point and/or the exterior orientation parameters.

The extension of the proposed model integrating the tie surface constraints empowers bridging the overlapping photo models where the surface points are not available. For the gap area without surface information the plane observation via scene interpretation or possibly from matching, if applicable, would find themselves useful towards the solution by the extended model.

Last but not least, the plane hypotheses and confirmation in the proposed model need no gridded DEM or special structure of surface points as a prerequisite, thus the original data set without being filtered or interpolated can be directly input to the system for the preservation of the firsthand accuracy.

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