

# Least-Squares Matching with Airborne Laserscanning Data in a TIN Structure

Hans-Gerd Maas  
Institute of Photogrammetry and Remote Sensing  
Dresden Technical University  
Helmholtzstr. 10  
D-01062 Dresden, Germany  
e-mail hmaas@rcs1.urz.tu-dresden.de

## Keywords:

Airborne laserscanning, least-squares matching, TIN, accuracy

## Abstract:

A number of tasks in airborne laserscanning require the establishment of correspondences between point data from neighbouring strips, or referencing between point clouds and object models. These tasks may be solved by interpolating laserscanner data, which are usually irregularly distributed  $2^{1/2}$ -D points, to a regular grid and applying standard photogrammetric matching techniques. Instead, the paper presents a formulation of least squares matching based on the original data points in a triangulated irregular network structure, thus avoiding degrading effects caused by the interpolation. The technique determines shifts in all three coordinate directions together with their covariance matrix. It can be shown that applying matching techniques to laserscanner data causes large systematic errors of the shift parameters in the case of partial occlusions. The presented formulation on the basis of a TIN structure allows for manifold extensions to solve this problem.

The technique and a number of extensions have been implemented and applied to the measurement of strip errors in an airborne laser scanner dataset with moderate point density, consisting of 20 strips including crossing strips. The paper shows the results from this test, discusses the advantages of the presented technique and the limitations of matching techniques applied to laserscanner data. Special attention has to be paid to problems caused by height discontinuities in the data and by the fact that the design matrix in least squares matching is derived from observations with stochastic properties. The latter leads to precision figures that are usually much too optimistic. A detailed analysis of the design matrix and extensive testing lead to better funded precision figures for the standard deviation of the obtained shift parameters. These are in the order of one centimeter in height direction and one decimeter in horizontal direction, corresponding to about  $1/20$ th of the average point spacing.

## 1. Introduction

Least-squares matching (LSM) is a technique that is being applied regularly by photogrammetrists for the establishment of correspondences between images taken from different viewing points, or between subsequent images of an image sequence. Formulated for two-dimensional greyscale images (F`rstner 1984, Grhn 1985), it determines the parameters of an affine transformation between corresponding patches of two or more images. Typical application fields of LSM are the determination of homologous points between consecutive images and image strips in a conventional aerotriangulation, or the matching between strips of linear array cameras.

Just like conventional photogrammetric image data, airborne laserscanning data of larger areas is also acquired in a stripwise manner. Basically, the  $2^{1/2}$ -D point clouds generated by airborne laserscanning are directly geo-referenced due to the use of GPS/INS systems onboard the aircraft. Because of errors of these instruments or sub-optimality of the GPS/INS integration, however, points of neighbouring laserscanner strips will usually show vertical discrepancies in the order of several centimeters and horizontal discrepancies in the order of a few decimeters (Huising and Gomes Pereira, 1998). The significance of these discrepancies depends on the application and the point density: in datasets with a point density of approximately one point per ten square metres, as are being used for the generation of digital elevation models, one will often only notice the vertical discrepancies; in datasets with several points per square meter used for the generation of 3-D city models, both vertical and horizontal discrepancies are often clearly noticeable.

Several authors have addressed discrepancies between adjacent strips of laserscanner data. Kilian (1994) describes a technique for laserscanner block adjustment based on measurements of horizontal and vertical shifts by least squares matching applied to strip data interpolated to a regular grid. Van Noort (1999) documents vertical strip differences based on the analysis of a very large amount of laserscanner data; de Min et al. (1999) developed techniques for laserscanning strip adjustment based on the measurement of height differences of control fields and along strip boundaries. The latter two techniques are restricted to the height dimension and use flat regions in the datasets to determine height differences. Planimetric and height discrepancies between strips may be measured by LSM applied to laserscanner data interpolated to a regular grid (Behan, 2000). Original laserscanner data points are, however, usually not regularly distributed, and an interpolation may introduce severe degrading effects. Therefore, LSM is better applied to the original irregularly distributed laserscanner data points organised in a TIN-structure.

In the following, the basic algorithm of LSM applied to  $2^{1/2}$ -D point data in a TIN structure will be described (chapter 2), and first results based on a dataset with moderate point density will be shown (chapter 3). Chapter 4 demonstrates problems caused by frequently occurring singularities and presents extensions of the technique to solve these problems. In chapter 5 application fields of the technique are identified and discussed.

## 2. Basic algorithm

The basic algorithm for LSM applied to laserscanner data in a TIN structure is derived from the standard implementation of LSM for raster image data and described in chapter 2.1. Requirements for the patches to allow determinability of all parameters are defined in chapter 2.2. Systematic errors of LSM in the case of discontinuities and refinements of the technique to avoid biased results are presented in chapter 4. Matching strategies indicating suitable points for matching are discussed in chapter 2.4.

### 2.1 Formulation of LSM on a TIN structure

The basic goal of least squares matching applied to laserscanner strip data is to unveil strip errors indicated by local discrepancies between point clouds taken from neighbouring or crossing strips. For that purpose, local patches are cut out of the overlap region. The shape of these patches may be arbitrary, but will often be circular or rectangular. The  $2^{1/2}$ -D data in these patches are shifted in all three coordinate directions in a way that the sum of the squares of height differences reaches a minimum. As the pointsets of the two patches are not identical or arranged on a regular grid, matching is performed between discrete points in one patch and points derived from the corresponding TIN mesh in the other patch.

Observation equations are written for every original data point of both patches. The input for the observation vector is obtained by subtraction of a height computed by linear interpolation at the same location in the corresponding mesh of the TIN structure of the other patch. The gradients for the construction of the design matrix are given by the surface normal of that mesh.

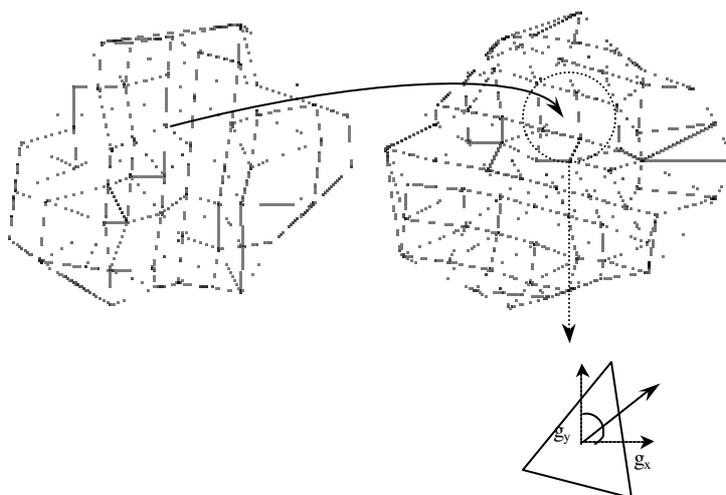


Figure 1: TINs of patches in two laserscanner data strips.

Formulated as a least squares adjustment procedure, the method converges after a few iterations. Obviously, the assignment of a point to a TIN-mesh may change during these iterations. The set of parameters to be determined in this procedure is limited to three shift parameters due to the nature of airborne laserscanner data, but may be extended.

### 2.2 Requirements

The local TINs to be matched must show surface normals in at least three non-coplanar directions to allow for the determination of all three shift parameters by LSM. In many cases, buildings or parts of buildings depict suitable objects for matching. Optionally, constraints may be introduced in situations with insufficient patch contrast, limiting the solution to the determination of a subset of the shift parameters. In fact, laserscanner data will often show large regions, where only a height shift parameter can be determined reliably. The technique may also be formulated as a multi-strip

technique performing matching between more than two strips, e.g. in the case of laserscanner blocks with overlapping and crossing strips.

### 2.3 Data testing

This basic formulation of least squares matching with data in a TIN structure has to be extended in a number of aspects due to the nature of laserscanner data:

- **Exclusion of points**

Data points on vegetation or other objects with an irregular shape in combination with the characteristics of laser pulse penetration, the sampling pattern and the viewing direction may lead to outliers in matching. These outliers will deteriorate the quality of the match and may falsify the result of matching. Therefore they have to be removed by robust estimation techniques. If a prior segmentation of the dataset has been performed, all points on vegetation should be excluded.

- **Planar triangles**

A severe problem is caused by occlusions typically occurring at one side of buildings close to strip boundaries as a consequence of the central perspective of laser scanning across flight direction. These occlusions become visible as gaps in one of the patches. Due to the composition of laser-scanner data blocks of parallel strips, these occlusions will usually have points (on the ground and possibly also on the wall) in the corresponding patch (Figure 2).

As a consequence of this fact and of the minimisation of the sum of height differences between the original data points of both patches and their interpolated corresponding values, the solution of LSM will be shifted by half the width of the occlusion area. In the case of symmetry, including a whole building into the patch will compensate the effect, but in general such occlusions may cause biases of several decimeters in the solution of LSM. Obviously, the same problem occurs when interpolating data points to a regular grid and applying raster-based LSM.

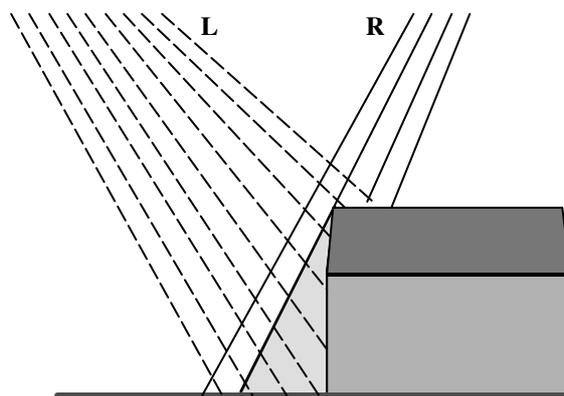


Figure 2: Occlusion at building gable

To avoid these effects, points falling into TIN-meshes with irregular size or shape are excluded from the matching process. Triangles in occlusion regions can be recognised by their longish shape as well as their steepness, which is defined by the geometry of the occlusion as well as the triangle's position in the strip and the flight parameters. This option performs well for large occlusions such as occlusions behind gables, but may be ineffective for occlusions caused by smaller objects. See chapter 4.1 for further refinement of the technique.

### 2.4 Matching strategy

A suitable matching strategy depends on the goal of the application. For the main application field of the developed tool in the strip adjustment of airborne laserscanner data, a matching strategy should provide regions well distributed over the strip overlap area, which do allow for the determination of all three shift parameters.

So far, manually chosen patch locations have been used for controlled testing of the method. Interest operators as used in many photogrammetric applications did not show fully satisfactory results yet. As buildings proved to be suitable objects in many cases, techniques for the segmentation of laserscanner data and the detection of buildings (e.g. Maas 1999a, Oude Elberink and Maas 2000) and techniques for modelling buildings (e.g. Maas 1999b, Maas/Vosselman 1999) form a viable alternative for indicating suitable regions for matching.

## 3. First results

To evaluate the potential of the technique, it was applied to a the laser scanning test block 'Eelde'. The dataset 'Eelde' contains a small city in the Netherlands. It was flown with a 10kHz Optech system at a flying height of 500m with a strip width of 250m and a point density of 0.3 points per square meter, corresponding to an average point spacing of 1.8 meter. The dataset consists of 10 strips flown in north-south direction and 10 strips flown in east-west direction. Both first and last pulse echoes were recorded; only the last pulse data were used for LSM in order to reduce effects caused by vegetation.

As a first test, the implemented technique was applied for matching 100 points which were manually chosen on buildings in the overlap area of two crossing strips (Figure 3). The results, averaged over these 100 circular patches with a radius of 12m, are summarised in the following:

- Setting the acceptance criteria quite rigidly, 74% of the matches were accepted.
- The average standard deviation of unit weight was 12.1 cm.
- The average standard deviation of the shift parameters was 2.1cm/2.0cm/1.0cm in X/Y/Z. Related to the average point density of 0.3 points per square meter, this corresponds to a relative precision of the planimetric shift parameters of  $\sim 1/80$  point spacing.
- The maximum standard deviations were 3.0cm/4.0cm/1.3cm in X/Y/Z.

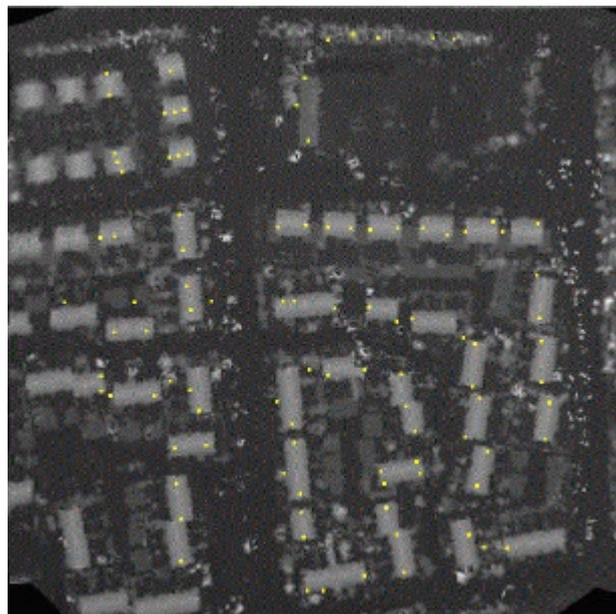


Figure 3: Part of dataset Eelde with chosen patch centers

In particular, the estimated standard deviations of the horizontal shift parameters seem much too optimistic. This is a result of the fact that the design matrix in LSM is derived from observations with stochastic properties; in combination with the noise properties and the point density this causes the values in the covariance matrix to be too large. This problem can only partly be compensated by filtering of the data, as filtering will often create non-existent gradients in the vicinity of range data edges. Edge preserving smoothing would be a solution, but has not been implemented to irregularly distributed data yet.

A more realistic figure on the precision of the determined shift parameters can often be obtained by comparing the results of patches with the patch center point slightly shifted over a small range in X and Y. Averaged over the 74 successfully matched patches, this approach indicated a precision of 4.5cm/3.7cm/0.7cm for the shift parameters in X/Y/Z. This figure may be too optimistic as well, as further outlined in the following chapter.

#### 4. Refinement of the algorithm

A general requirement for the application of LSM techniques is the presence of a bandwidth-limited signal. This is not fulfilled over flat areas and at height discontinuities such as building edges. While flat areas provide information for the determination of the vertical shift parameter and tilted roof planes with gradients in at least two different non-opposite directions provide information for the determination of the two planimetric shift parameters, discontinuities do not contribute to the determinability of parameters and may lead to convergence problems or even biased solutions as shown in chapter 2.3.

In the case of the example shown in chapter 3, this problem becomes obvious as inconsistencies between results for neighbouring patches. Figure 5 (left) shows shift parameters for the successfully matched points of the test area shown in Figure 3; a trend, which should be expected as a consequence of strip errors, is hardly recognisable in these results. These inconsistencies are caused by local singularities, which emerge after the exclusion of irregular TIN-meshes in regions of occlusions. A patch containing a building with a standard gable roof (Figure 4 - left), for example, will show a singularity in the gable direction: the normal vectors on the ground and the two roof planes form a plane, and the shift perpendicular to this plane (i.e. in gable direction) is not reliably determinable.

This problem does not become obvious from the covariance matrix obtained from LSM, or from an analysis of the local variation of the patch center, as outlined in chapter 3. As a consequence of the noise in the laserscanning height data in combination with the fact that the design matrix is generated from observations with stochastic properties, the estimated standard deviations and correlation between parameters are too optimistic and often do not indicate these singularities. In practice, even for patches extracted over perfectly flat terrain, the standard deviations of the planimetric shift parameters were sometimes less than 10cm in a dataset with an average point spacing of 1.8m. The same applies for the standard

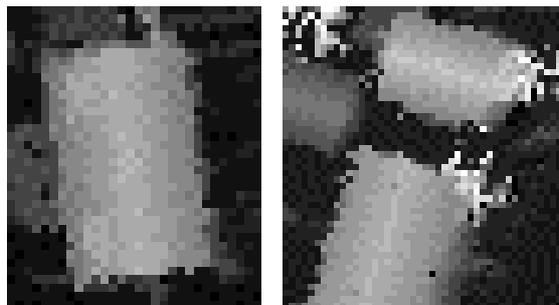
deviation in gable direction of buildings, where standard deviations were usually under 10cm and the correlation between the X- and Y-shift parameters was usually low.

#### 4.1 Matching planar patches

One possible solution to the errors caused by discontinuities is a restriction of matching to planar patches. This can either be achieved by explicitly extracting planar patches from the laserscanner data, as shown by (Vosselman, 1999), or by excluding points close to edges after an analysis of height texture measures (Maas, 1999a). The latter option was implemented by the analysis of a plane-fit through every data point and its neighbours in a Thiessen structure derived from the TIN. This rejection of points which are not situated on planar patches goes clearly beyond the exclusion of points falling into irregular triangles (chapter 2.3), as it will generally exclude all points which are less than one point spacing away from discontinuities.

After the restriction to points on planar patches, the problem of singularities will be accentuated. Sufficient patch contrast is only provided by more complex roofs or in a patch containing two buildings with different gable directions (Figure 4 - right).

Figure 4: Height data with uni-directional and bi-directional roof gradients



In practice, such configurations are not always present, especially when tie points have to be found in high-resolution laserscanner strip data with small lateral overlap. In fact, the number of accepted matches of the pointset shown in Figure 3 decreased drastically when the 'planar patches' option was used. Choosing larger patches could only partly compensate for this.

#### 4.2 Refined analysis of gradients

Also after restriction of the matching process to points falling into planar patches the difficulty of estimating realistic precision measures and detecting singularities remains. This problem originates from the noise of the laserscanner data points themselves, which is estimated in the order of 3-5cm (van der Wolk, 2000), plus some model noise caused by the fine-structure of roofs, the laser spot size, and insufficient sampling. This noise propagates into the gradients of the TIN meshes, which determine the design matrix and covariance matrix used for parameter estimation in LSM.

To solve for this problem, a procedure for a detailed analysis of the design matrix was defined:

- All gradients are analysed on their significance, and the number of triangles with significant gradients is compared with a pre-set threshold.
- The directional distribution of significant gradients was analysed. This distribution should be uniform or show two maxima in orthogonal direction. In the case of a directional histogram with a clear single peak, the match was not accepted.

In addition, the convergence behaviour is analysed, and optionally the results of auto-correlation are examined.

This detailed analysis of the design matrix and the convergence behaviour, combined with the restriction of matching to points on planar patches, leads to a partial success. The results obtained from the dataset described in chapter 3, using the planar patches option (chapter 4.1) in combination with the gradient analysis as described above, are visualised in Figure 5. They show about the same standard deviation as mentioned in chapter 3, but a significantly improved consistency and a clearly recognisable trend of the shift parameters over the region shown in Figure 3. This trend indicates a the presence of a displacement of about 0.3m between the two crossing strips as well as a tilt. These effects are likely to result from errors of the position and orientation determination unit of the laserscanner system.

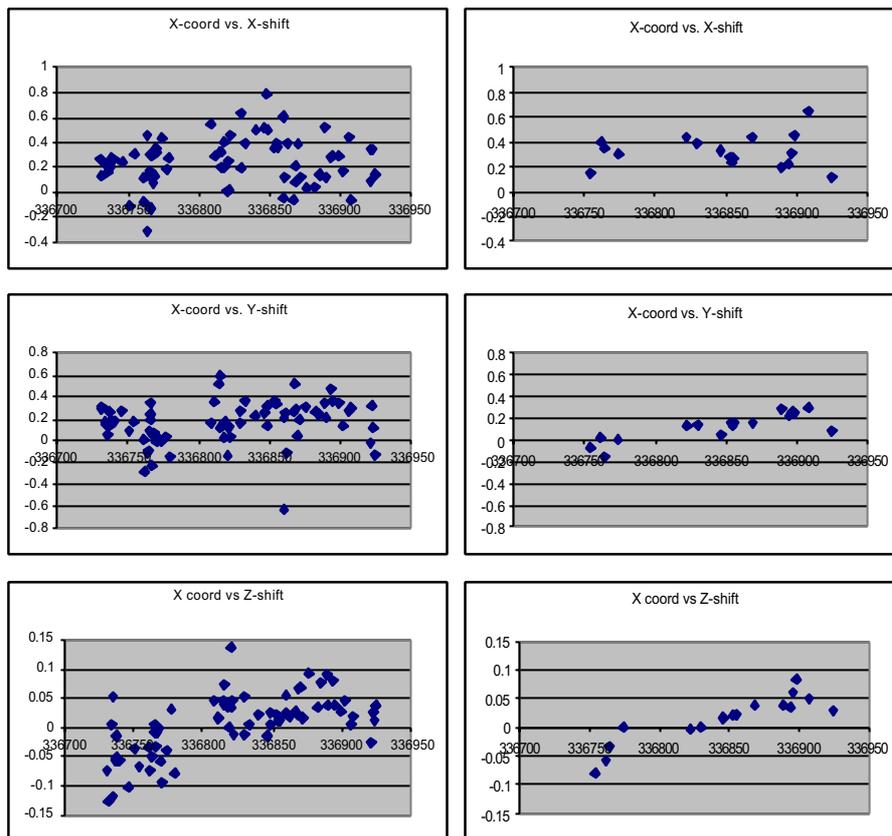
Another outcome of the analysis of the determinability of the shift parameters is a reduction of successful matches from 74% to only 20%. This is unfortunate, but seems realistic considering the structure of the test area, which contains many buildings with simple gable roofs.

A linear regression analysis of these results yields the following standard deviations of the LSM shift parameters in X/Y/Z:

- Basic technique (ch. 2, Figure 5 - left): 19.4/19.4/4.0cm
- Planar patches (ch. 4, Figure 5 - right): 13.3/7.1/1.9cm

The latter figures give a more realistic indication of the achieved precision of the shift parameters. The standard deviation in height direction suffers from planimetric effects; when restricting constrained matching to almost flat patches, sub-centimeter precision can be obtained in height direction.

Figure 5: Trend in shift parameters over test area (left = before, right = after refined analysis of determinability)



## 5. Application fields

Recent developments in the field of direct geo-coding show rather high precision figures for the determination of sensor position and orientation parameters (e.g. Cramer, 1999), which might lead to the conclusion that a tool for the measurement of planimetric shifts between laserscanner strip data obsolete. Nevertheless, there remain a number of reasons justifying the application of least squares matching in airborne laserscanning:

- LSM may be used as a tool for the verification of the proper function of the system.
- In certain regions or under certain circumstances, proper differential GPS service may not be available.
- LSM may be used as a tool for the improvement of the geometric quality of data obtained from low-cost laserscanner systems, which are not equipped with a full high-grade GPS/INS system.
- The accuracy of LSM exceeds the accuracy of direct geo-coding systems at least in the height direction.
- Besides malfunction of the GPS/INS system, significant systematic errors in laserscanner strip data may also be caused by misalignments between the position/orientation determination system and the actual laserscanner (van Noort, 1999), or by other components of a laserscanner system. In this context LSM remains an important tool in the procedure of self-calibrating laserscanning strip adjustment.

Beyond this, the method can be used as a general tool for matching  $2^{1/2}$ -D point clouds. In (Postolov et al., 1999) a similar approach has been used for matching laserscanner data to surfaces generated interactively from stereo imagery. With building models in a proper representation, the technique can also be used for matching given models to laserscanner data. Similarly, laserscanner data can be fitted to given building models, which can be used as ground control. The latter forms an analogy to a technique for fitting digital terrain models to not explicitly identified ground control points presented by (Ebner/Ohlhof, 1994).

## 6. Future work

The main goal of the development of the matching technique is the provision of a tool for the detection and measurement of discrepancies between neighbouring strips of airborne laserscanner data. An optimised matching strategy as well as a tool for self-calibrating laserscanner strip adjustment still have to be developed. Further options of improvement include:

- **Partial results:** So far, the matching tool is designed to determine either all three shift parameters or only the vertical shift. The latter can be selected by the user or enforced automatically after the analysis of the design matrix. In all cases of a singularity in one planimetric direction, the match is rejected, although the shift parameter perpendicular to that direction is still determinable. This option has not been implemented yet, as the current analysis procedure is

capable of indicating singularity situations, but cannot determine the direction of the singularity with sufficient precision.

- **Reduction of singularities:** The problem of the singularities in gable direction as addressed in chapter 2.3 and 4.2 can be reduced by implementing the knowledge that points on a roof cannot be occluded and thus should not be excluded, even if they fall into an occlusion zone. Provided that a patch contains a complete roof, this option may be used to solve the singularity. Nevertheless, the information for the shift in gable direction will be derived from only a few points at the roof edges, so that the determinability of the parameter remains weak. Moreover, this option will conflict with the restriction of matching to planar patches as described in chapter 4.1.
- **Use of features:** As an alternative to the matching based on local point clouds as shown in the previous chapters, strip discrepancies can also be detected via a comparison of parameters of objects or object parts (such as buildings or roof planes) modelled independently in both strips. While this option may offer more flexibility concerning shortcomings of the local contrast situation and effects caused by occlusions, it can realistically only be applied to high density laserscanner data with a density of at least one point per square meter.
- **Use of reflectance data:** Several airborne laserscanner systems deliver a reflectance signal derived from the intensity of the backscattered pulse-echo in addition to the actual height measurement. This reflectance value is sometimes being used in segmentation tasks. As the reflectance value is perfectly referenced with the height data points, it may also be used for the determination of planimetric shifts between neighbouring strips.

This option may especially be relevant in rather flat areas, where the patch contrast situation in height data only allows the determination of the vertical shift parameter. In such situations, the use of reflectance values may complement well for the determination of the two planimetric shift parameters.

Figure 6 shows an example for the potential of simultaneous matching in height and reflectance data, where the height data allows only for the determination of a vertical shift parameter, while the reflectance data allows for the determination of the two planimetric shift parameters.



Figure 6: Road crossing in high-resolution laserscanner data: Height and reflectance image.

## 7. Conclusion

Least squares matching applied to laserscanner data in a TIN structure can be a very valuable tool for the analysis and improvement of the quality of laserscanner data, both in vertical and horizontal direction. Special attention has to be paid to the avoidance of systematic errors caused especially by occlusions and to a realistic estimation of the precision and determinability of parameters.

Taking these necessities into consideration, shift parameters between homologous patches in neighbouring or crossing data strips can be determined with a precision in the order of one centimeter in vertical direction and about one decimeter in horizontal direction, related to a dataset with an average point spacing of 1.8 meter. This is sufficient to determine strip discrepancies significantly, so that least squares matching may be used as a tool for the improvement of the geometric quality of laserscanner data.

## Acknowledgement

The author would like to thank Fotonor (Sandefjord, Norway) and Geodelta (Delft, The Netherlands) as well as the Rijkswaterstaat Survey Department (Delft, The Netherlands) for providing data for the work presented here.

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