TOPOLOGICAL ISSUES OF RASTER SPACE FROM VIEWPOINTS OF PANSYSTEMS

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ABSTRACT

In recent years, models of spatial relations have attracted much attention from GIS community, especially topological relations. In this paper, some basic topologic models for spatial entities in both vector and raster spaces are discussed.

To make the concepts more formalized, some new concepts from a new branch of mathematics -- pansystems -- are adopted. In doing so, the basic concepts of pansystems are introduced, upon, some important concepts on raster space, as a spatial reference system, are discussed. Raster topologies are formalised. Basic topological components of spatial entities in raster space are defined and the topological issues are examined, such as topological paradox and the dependency of raster topology on vector topology.

1. INTRODUCTION

In daily life, people are concerned with information about spatial entities (or objects), such as their positions, attributes and/or spatial relations between them such as direction, distance, adjacency, topology and so on. Topological information is related to the neighbourhood of spatial entities and is of particular interest to GIS community, as topological properties of spatial entities are the most fundamental, compared others. In GIS literature, a body of literature on topological relations could be found. However, discussions are concentrated on vector space.

Spatial entities are represented either in vector format or in raster format. And there has been an almost constant debate about their natures and their importance in the future, through the history of GIS (e.g. Peuquet 1984; Maffini 1987; Maguire *et. al*., 1991). This paper deals with fundamental issues on the topology of raster space. In other wards, some fundamental topological properties of raster spaces will be discussed from the point of view of pansystems.

2. BASIC CONCEPTS OF PANSYSTEMS

Pansystems refer to generalised systems. The theory was formalised by Prof. WU Xuemou in 1976 (Wu, 1990). It deals with generalized relationships and various compositions of pansystems. A generalized system, G, consists of two components, i.e.

$$G = (H, S) \tag{1}$$

Where, **H** is generalized hardware and **S** is generalized software. Generalized hardware **H** is any set of given things including other well-defined pansystems sets and generalized software **S** is some relations and pan-weighted compositions of relations about **H**.

Basic pansystems relations include whole-part relation, body-shadow relation, difference-identity relation, panorder (generalized order), element-system relation, outside-inside relation, panderivatives (generalized variation-variation relation, generalized differentiation, generalized integration, etc.), clustering-discoupling relation, observo-controllability (observation and control), macro-micro relation, shengke (generalized optimization and generalized inferiority), offering-requesting relation, cause and predestined relation, optimization, transformation, pansymmetry (including symmetry, anti-symmetry, broken symmetry, stability-instability, motion-motionless, and relative less variation among changing things), imitation, series-parallel relation, false-true relation, contracting-expanding relation, etc.

From the view of pansystems, a raster space is a generalized system. It is a combination of generalized hardware A and generalized software B, denoted as S=(A,B). Hardware A is a set, while software B is a set made up of some relations on A, called relation set on A. Then question arising is "what is the hardware and software of raster space Z^2?"

3. RASTER TOPOLOGY FROM VIEWPOINT OF PANSYSTEMS

To describe mathematical entities, which are abstracted from spatial entities, one has to employ a referencing system. Cartesian coordinate system is the most widely used one. Based on which vector space (Euclidean space) is constructed. Vector space is a continuous space. It is also possible to construct a discrete space.

A discrete space is a result of partitioning a connected space into small pieces which cover the all space contiguously. This is also called space tessellation. Various approaches are possible such as feature-primary and space-primary tessellation (Lee and Li, 1998) and some examples of space tessellation are illustrated in Figure 1. Grid-based tessellation is only one of them.

(a) Regular
(b) Regular
(c) Irregular
(d) Irregular
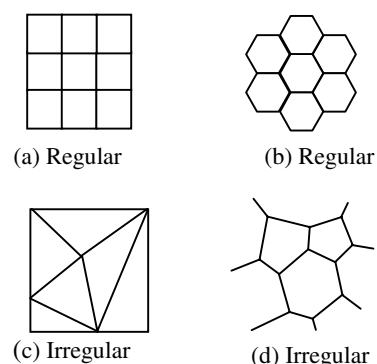
(*Figure 1 Some examples of space tessellation* tessellation:

The tessellation based on squared-grids constitutes raster space Z^2 (i.e. the Cartesian product of Z and Z, where Z is integer set). Each grid is viewed as an element, called 'spel' (short for spatial element). All the spels in the plane can form a new set, which could be named as grid set G. The G can then be regarded as the hardware of raster space and the transitive closure (δ) of the adjacency relation between two spels in G. This system could be expressed as $\langle G, \delta \rangle$, where δ is the binary relations. Such adjacency relation determines the connectedness between spels in G.

In 2-dimensional raster space, the adjacency between two neighbouring spels could be defined in two different ways, i.e., 4-adjacency or 8-adjacency (see Figure 2). With 4-adjacency, a spel is considered as being connected to neighbouring spels along only 4 sides, i.e. up, down, left and right sides (Figure 2a). However, 8-adjacency, a spel is also considered as being connected to neighbouring spels along 4 diagonal directions, i.e. upper/left, upper/right, lower/right and lower/left (Figure 2b).

In 3-dimensional raster space, there is a similar problem. In this case, it is 18-adjacency or 26- adjacency. More detailed discussion regarding this issues can be found in the paper by Kong and Rosenfeld (1989).

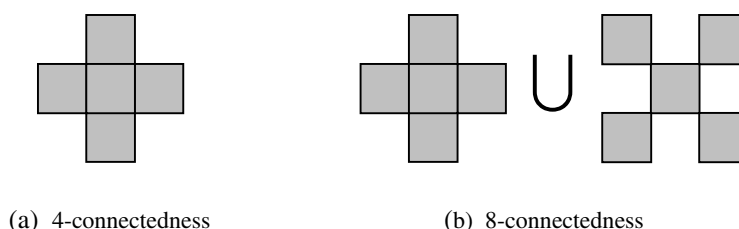(a) 4-connectedness
(b) 8-connectedness

Figure 2 Definitions of 4-connectedness (i.e. connected along 4 sides only) and 8-connectedness (connection along 4 sides and 4 diagonals) and 8-connectedness (connection along 4 sides and 4 diagonals).

In fact, the raster topology $\langle G, \delta \rangle$ is normally referred to as 'digital topology'. This allows us to discuss topological problems related to spatial entities in Z^2, but not in R^2. The raster topology in R^2 is defined as $\langle R^2, \chi \rangle$, where χ is a family of unions of elements in G plus empty set and R^2. With such definitions, the raster topology expressed by $\langle R^2, \chi \rangle$ is referred to as raster quasi-topology in this paper as such as a topology satisfies only following 2 conditions:

Axiom 1: Any union of open sets is open;
Axiom 2: $\varnothing$ and U are open.

4. BASIC TOPOLOGICAL COMPONENTS OF A SPATIAL ENTITY IN Z^2

The basic topological components of a spatial entity (E) in raster space are still interior, boundary and exterior. The interior of E in Z^2, E^o, is the largest connected part of G contained in E and the closure of E. $\bar{E}$, is the smallest

connected part of G containing E. The boundary as defined in vector space, ∂E, is the intersection between the closure

of E and the closure of the complement of E, $\overline{E^-}$.

The relationship between interior, closure and boundary is as follows:

$$E^o \cap \partial E = \Phi$$
$$E^o \cup \partial E = \overline{E} \tag{2}$$
$$\partial E = \overline{E} \cap \overline{E^-}$$

The boundary of a spatial entity in raster space, as shown in Figure 3(a), needs to be presented by a string of spels (Kong and Rosenfeld, 1989), which has an area. According to the Equation (2), most appropriate spels defining the boundary of the entity E are the two strings of spels shaded in Figures 3(b) and 9(c). This boundary string is not difficult to obtain if the 'any pixel intersected' method is used in rasterisation, as shown in Figure 3(e). However, in practice, the raster representation of E could be something like the E' shown in Figure 3(d) if the 'dominant pixel' method is used in rasterisation. This might also be more close to the case of imaging in raster format.

With E' as raster representation of E, the boundary E' might be defined in a few alternative ways, as illustrated in Figures 3(f) to 3(i). In the case when black spels in the outer ring of E' are used to form boundary, the interior of E will become smaller than it should be. On the other hand, if the white spels neighbouring to the outer ring of E' are used to form boundary, the interior of E will become larger than it should be.

(a) A spatial entity E to be observed by a raster system

(b) The true boundary for E in a raster system with 4 adjacency

(c) The true boundary for E in a raster system with 8 adjacency

(e) A possible result (E') of E observed by a raster system

(d) An alternative result (E") of E observed by a raster system

(f) Bounbary definition 1 for E': Black pixels with 8 adjacency

(g) Bounbary definition 2 for E': Black pixels with 4 adjacency

(h) Bounbary definition 3 for E': white pixels with 4 adjacency

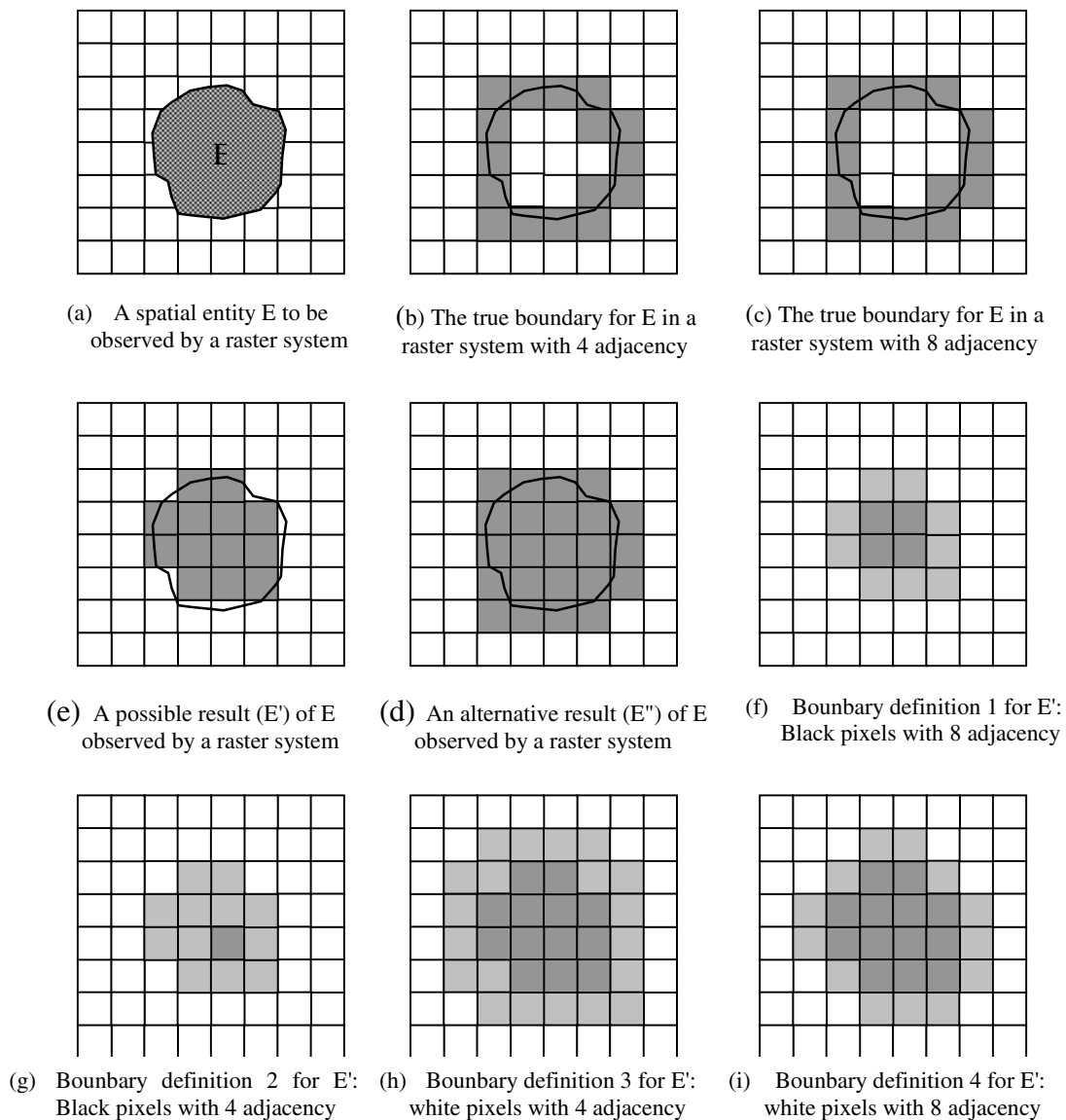(i) Bounbary definition 4 for E': white pixels with 8 adjacency

Figure 9 Boundary of an area entity in raster space

These definitions of boundaries may cause some topological problems. In the case of "white boundary", the two disjoint spatial entity may share a common boundary. On the other hand, in the case of "black boundary", an areal entity may have empty interior. Efforts have also been made to avoid such problems. Indeed, in image processing literature, some researchers even suggest the use of vector representation boundary for E', as illustrated in Figure 4.
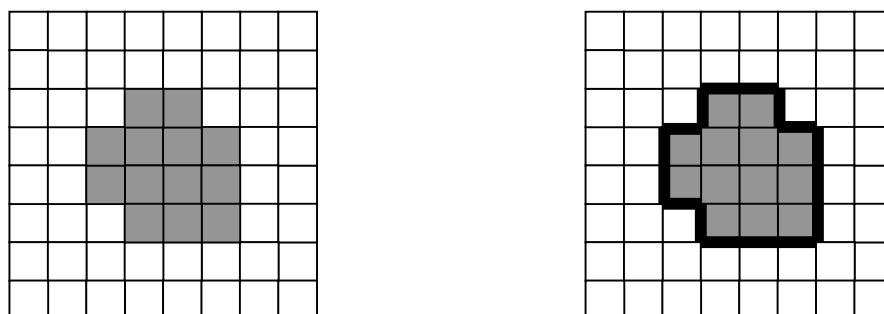
(a) Spatial entity E' observed by a raster system

(b) Vector representation of boundary for E'

Figure 4 Spatial entity E' observed by a raster system and its vector representation of boundary for E'. This boundary can be regarded as the result observed by Euclidean topology on R^2.

5. TOPOLOGICAL PARADOX ASSOCIATED WITH ADJACENCY RASTER SPACE Z^2

In 2-D, a basic topological convention is the so-called "Jordan Curve" which is illustrated in Figure 5. In this figure, L represents the boundary of the area; I and E denote its interior and exterior; x denotes a point in the interior and y denotes a point in its exterior. If one want to travel from x to y, one must cross the boundary at least once. In other words: *"A closed curve separates its interior from its exterior".* Such a curve is called the Jordan Curve. Such a topological property needs to be retained in raster space.

However, this topological property might becomes a problem if the adjacency problem is not well considered. Rosenfeld and Pfalez (1966) have discussed a paradox related to definition of adjacency. It is illustrated in Figure 6. In this figure, there are 4 black spels and some white spels. There is one white spel surrounded by 4 black spels. If 8-adjacency is defined, the black spels are connected and should form a closed line, however, this black line cannot separate the central white spel from the rest. If a 4-adjacency is defined, the black spels do separate the central while spel from the rest, however, these black spels are totally disconnected and thus no closed line has been formed by the black spels in this case. This is another interesting phenomenon but in raster space. It violates the so-called Jordan Curve theorem in vector space. This theorem is the basic topological property in vector space and one must wish to keep in the Z^2 raster space.

It has also been realised by Kong and Rosenfeld (1989) that this problem could be solved if the white spels are defined as being 4-connected and black spels 8-connected, or vice versa. However, the question "why this paradox happens" still needs to examined in more detail before any discussion on the topology in Z^2 raster space can be further discussed.

To explain this paradox more clearly, only 4 spels are used in Figure 7, where point P is the intersection point of these 4 spels. According
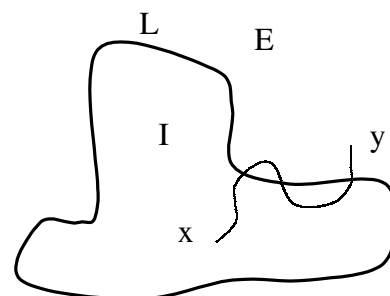

Figure 5 Jordan's Curve in vector space: A closed curve separates its interior from its exterior
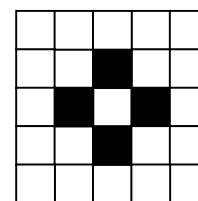

Figure 6 Topological paradox in raster space (Modified from Kong and Rosenfeld, 1989).

to 8-adjacency definition, the black spels, i.e. spels 1 and 4, are connected. Similarly, spels 2 and 3 are also connected. As a consequence, two questions arises:

(a) Does P belong to black line or white line? and
(b) What makes spels 1 and 4 (or spels 2 and 3) connected?

These two questions are inter-related. In reality, when one considers spels 1 and 4 are connected, one has already implicitly assumed that P belongs to the black line. On the other hand, when one considers spels 2 and 3 are connected, one has already implicitly assumed that P belongs to the while spels. That is, the point P belongs to two different things. If the black spels represent spatial entities and the white spels represent the background, then point P belongs to both background and the entity at the same time, thus having dual meanings. This of course leads to paradox - a kind of ambiguity.
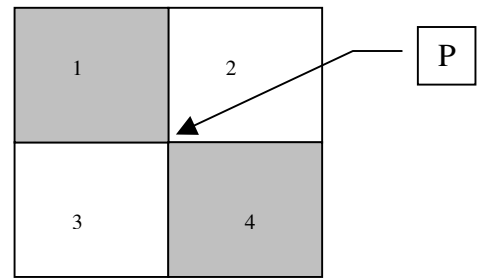
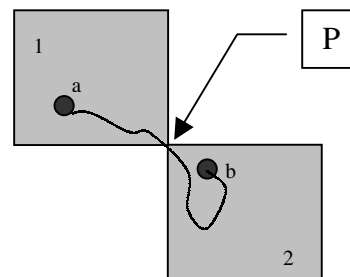Figure 7 The ambiguity at point P, causing topological paradox

To solve the problem, one must eliminate the dual meanings of point P. One should only allow P to belong to either the entity or the background but not both.

6. DISCUSSION

The connectedness of raster space is based on the adjacency of two neighbouring spels. The connectedness of two neighbouring spels in 4-connectedness and 8-connectedness is illustrated in Figure 8. There is a common line (Figure 18a) between the two spels in the case of 4-connectedness. On the other hand, in the case of 8-connectedness, the common part could be either a line (Figure 8a), a point (Figure 8b), or both. In other words, there is at least a point in common if the two spels are to be connected.
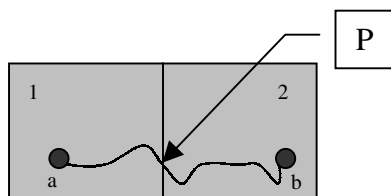
(a) In the case of 4-adjancncy   (b) In the case of 8-adjancncy

Figure 8 Implicitly dependent of topological connectedness
in raster space on that in vector space

If an arbitrary (vector) point is selected from each spel, say "a" and "b", then the path from "a" to "b" intersects the common line at P. Points "a", "b" and P are the points in vector space. Points a and P are connected in the left spel and points P and "b" are also connected in the right spel in vector space. As the connectedness is transitive, points "a" and "b" are therefore connected. As a result, any point in the left spel is connected to any point in the right spel. It means that the connectedness concept in vector space has been implicitly adopted when the connectedness concept in raster space is discussed.

Suppose a subset in R^2 represents a spatial entity, it can be studied in vector space. It can also be studies in raster space. When an entity is studied in raster space, it is implied that there is a relationship between vector space and raster space. The observation of the entity has been transformed from vector space to raster space. In this transformation, some kind of invariability is retained or implicitly inherited, i.e. the connectedness. This connectedness invariant is the fundamental reason why the topological properties and relations of spatial entities can also be studied in raster space.

7. CONCLUSIONS

In this paper, topology is regarded as a generalised system and the software of the system is used to approximate spatial entity. Topological models for multi-dimensional spatial entities in raster space have been presented. The dependency of raster topology on vector topology is examined.

It is discussed that the definition of the boundary of a raster spatial entity is more complicated. The raster boundary in Z^2 is quite different that in R^2. In boundary of raster spatial entity is also dependent on the definition of adjacency or connectedness of raster space. If connectedness is not carefully defined, topological paradox will be caused.

It has also been noted that, in raster space, the basic topological concept in vector space -- connectedness -- is implicitly inherited. This is why topological properties of spatial entities can also be studied in raster space.

ACKNOWLEDGEMENT

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