THREE CONCEPTUAL UNCERTAINTY LEVELS FOR SPATIAL OBJECTS

Martien Molenaar
Professor and Chair of the Division of Geoinformatics and Spatial Data Acquisition
International Institute for Aerospace Survey and Earth Sciences (ITC)
P.O. Box 6, 7500AA Enschede
The Netherlands
e-mail: molenaar@itc.nl
Phone: 31 - 53 - 4874 269 Fax: 31 - 53 - 4874 355

ISPRS IC WG IV/III.1 - GIS Fundamentals and Spatial Databases

KEY WORDS: Spatial data models, uncertainty, fuzzy objects, geometry, vector data, raster data.

ABSTRACT

The geometry of spatial objects is generally determined through their boundaries. Photogrammetry, land surveying and digitising methods are based on this approach. This is however only possible if the objects are crisp so that their boundary can be identified. When objects are fuzzy this becomes problematic, or even impossible because their spatial extent is not fixed. For such objects uncertainty plays a role at three definition levels (Molenaar, 1998):

- *The existential uncertainty* expresses how sure we are that an object represented in a database really exists.
- *The extensional uncertainty* expresses that the area covered by an object can only be determined with limited certainty.
- *The geometric uncertainty* refers to the precision with which the boundary of an object can be measured, if it can be determined.

The geometric uncertainty plays a role in the representation of crisp objects, where the determination of the spatial extent is no problem. The extensional uncertainty plays a dominant role in the spatial representation of fuzzy objects, for these objects the geometric precision is not so relevant. In these cases it might even be doubtful whether the spatio-thematic data collected by a surveyor or obtained through image analysis gives sufficient evidence for the existence of an object, i.e. it might be that this existence is uncertain.

There are many situations where the geometric and extensional uncertainty can hardly be distinguished. This paper will elaborate the concepts for describing object uncertainty at three levels and investigate how they are related. Crisp objects will be shown to be a special case of fuzzy objects.

1 INTRODUCTION

The determination of spatial objects by land surveying and photogrammetry is generally based on the measurement of their boundaries. The geometry of the objects and their topologic relationships are then expressed through these boundaries. We will call this the *geometric approach*. This approach works well for crisp objects, i.e. for objects with a crisp spatial extent. For such objects the uncertainty of their location and shape can be expressed through the uncertainty of their boundaries. The epsilon-band method is well known in this context (Dunn et. al., 1990), (Shi, 1994).

The geometric approach is in many cases not satisfactory though. The reason is that the geometric uncertainty of geo-objects is generally not only a matter of co-ordinate accuracy. In many applications it is rather a problem of object definition and thematic vagueness (see also the discussions on this topic in for example (Chrisman, 1991), (Burrough and Frank, 1995), (Burrough and McDonnell, 1998) and (Goodchild et al., 1992)). This latter aspect cannot be handled by a geometric approach alone. This problem occurs in many applications like the monitoring of natural vegetation and forest areas, the development of land use, coastal development, etc. Such monitoring processes are often based on the use of remote sensing data from which the information about the relevant features is to be extracted. The vagueness of concepts and definitions in these applications has the effect that such features can only be identified with a limited level of certainty so that there is a substantial fuzziness in their spatial description.

This becomes apparent when mapping is not done in a vector-structured geometry as for landsurveying and photogrammetry, but when it is based on feature extraction from digital images that have a raster structure. The uncertainty of remote sensing image classification is primarily considered to be thematic; the certainty that a pixel belongs to a thematic class might be expressed through a likelihood function, which is evaluated in the classification process (Lillesand and Kiefer, 1994) and (Buiten and Clevers, 1993). Image segments can then be formed of contiguous sets of pixels falling under the same class. If these segments represent the spatial extent of objects then the uncertainty of the geometry of these objects is due to the fact that the value of the likelihood function varies per pixel (Canters 1997, Fisher 1996, Wickham et al. 1997, Usery 1996 and Brown 1998). So these type of image interpretation processes result in objects with an uncertain spatial extent. We will call this the *extensional approach.* If objects have an uncertain spatial extent then their geometric uncertainty can not be expressed through their boundaries, because they do not have boundaries in an absolute sense. Such objects can be represented as fuzzy fields, where only conditional boundaries can be defined.

It might even be that the uncertainty of the identification of the spatial extent of an object reaches a level that it is doubtful whether the observational or image data give sufficient ground for the conclusion that a real world object has been identified at all. The fact mentioned before that uncertain spatial objects can be represented as fuzzy fields make a gradual transition possible from field structured phenomena to object structured phenomena. In the grey zone that exists here it may be difficult to distinguish clearly between field fluctuations and fuzzy objects. This situation points us once more to the fact that seeing the world as a complex of objects is just one perspective and seeing it as consisting of fields is an other one. So the identification of objects firstly requires an object structured view on the world and secondly it requires inference procedures to extract the object information from the observational data. The extensional and geometric approach are two strategies in this respect.

The geometric and extensional approaches are characteristic for the different techniques for object determination and the surveying disciplines using these techniques. It might very well be that these two approaches are the reason why there is much difference between survey disciplines in the perception of what spatial objects are. This difference might also be the cause of the fact that geometry-oriented topographic surveyors and the more theme-oriented surveyors of other disciplines have so much difficulties in understanding each other when the discuss spatial accuracy. Therefore this paper tries to unify these two approaches. This unification will be based on the formalism presented in (Molenaar, 1994, 1996 and 1998) for the representation of spatial objects. This formalism can handle both the geometric approach and the extensional approach for the representation of spatial objects.

2 THE SPATIAL EXTENT AND BOUNDARY OF CRISP OBJECTS

Let M be a spatial database containing a terrain description where U_M is the collection of all terrain objects represented in this database, i.e. U_M is the universe of M. We will assume that the geometry of the objects is represented in a vector format with a full topological structure, i.e. the geometry is described in nodes, edges and faces defining a geometric partition of the mapped area (or 0-, 1- and 2-cells). Let *Geom(M)* be the geometric component of M, i.e. it is the collection of all geometric elements describing the geometry of all objects of the universe. Let *Face(M)* be the collection of all faces in *Geom(M)* and similarly *Edge(M)* is the collection of all edges.

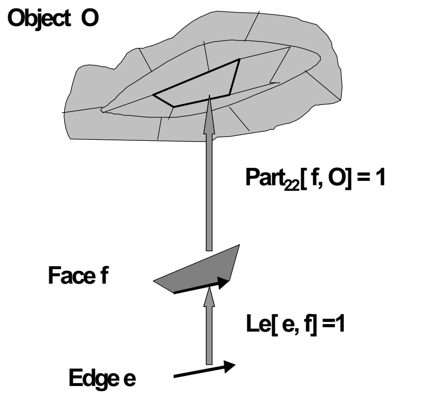


Figure 1. Relationship between Edge, face and Object

The function $Part_{22}[f, O]$ will be introduced to express the relation between a face $f \in Face(M)$ and an object $O \in U_M$. If this function has the value = *1* then the face belongs to the spatial extent of the object, if the value = *0* then that is not the case. We can now define the set:

$Face(O) = \{f \mid Part_{22}[f, O] = 1\}$

This is the extensional statement of the object in the sense that *Face (O)* is the spatial extent of *O*. In this notation the geometric description of the objects is organised per object. This formulation is valid for crisp objects because the extent of the objects has been determined with certainty, i.e. for each face we can establish whether it belongs to the extent of the object or not. For each edge *e* we can express its relationship to a face *f* by the functions:

$Le[\, e, f\,] = 1$ if *e* has *f* at its lefthand side and $= 0$ otherwise, and similarly
$Ri[\, e, f\,] = 1$ if it has the face at its right hand side and $= 0$ otherwise.

With these functions the relationship between an edge *e* and an object *O* can be established:

$Le[\, e, O \mid f\,] = MIN(Le[\, e, f], Part_{22}[\, f, O])$ and
$Ri[\, e, O \mid f\,] = MIN(Ri[\, e, f], Part_{22}[\, f, O])$

If the edge has the face at its left hand side and the face is part of the spatial extent of the object then both functions on the right hand side of the first expression have the value $= 1$ and therefore the first expression gets the value $= 1$. This means that the edge has the object at its left-hand side and thus we get $Le[\, e, O \mid f\,] = 1$, otherwise it will be $= 0$. Similarly if the edge has the object at its right hand side then $Ri[\, e, O \mid f\,] = 1$ otherwise $= 0$. If there is a face for which $Le[\, e, O \mid f\,] = 1$ then that implies that the edge *e* has object *O* at its left-hand side, so that $Le[\, e, O\,] =1$. Otherwise $Le[\, e, O] = 0$. In a similar way we can evaluate the function $Ri[\, e, O\,]$. With these two functions we define:

$B[\, e, O] = Le[\, e, O\,] + Ri[\, e, O\,]$

If the edge has the object at its left and right-hand side this function has the value 2. If the object is only at one side, so that the edge belongs to the boundary of the object, the value will be 1, If the edge is not related to the object then the value will be 0. The boundary of the object is therefore the sets for which this function has the value 1:

$\partial O = \{\, e \mid B[\, e, O\,] = 1\}$

3 THREE UNCERTAINTY LEVELS OF SPATIAL OBJECTS

The formalism developed in (Molenaar, 1994, 1996 and 1998), for handling spatial object information makes it possible to distinguish three types of statement with respect to the existence of these spatial objects:

1. An *existential statement* asserting that there are spatial and thematic conditions which imply the existence of an object *O*.
2. An *extensional statement* identifying the geometric elements which describe the spatial extent of the object.
3. A *geometric statement* identifying the actual shape, size and position of the object in a metric sense.

These three types of statement are intimately related. The extensional and geometric statements imply the existential statement,; if an object does not exist it cannot have a spatial extent and a geometry. The geometric statement also implies the extensional statement. All three types of statement may have a degree of uncertainty and although these statements are related they emphasise different aspects of uncertainty in relation to the description of spatial objects.

3.1 Existential uncertainty

The uncertainty whether an object O exists can be expressed by a function: $Exist(O) \in [\, 0, 1]$. If this function has a value = 1 we are sure that the object exists, if the value = 0 we are sure it does not exist. This latter case leads to a philosophical problem, because how can we make existential statements about objects that do not exist, or rather how can we identify non-existing objects and refer to them as an argument of this function? This problem will not be elaborated here, rather we will follow a pragmatic approach by restricting the range to $Exist(O) \in (\, 0, 1]$, this means that the function can take any value larger than 0 and less or equal to 1. The uncertainty of the existential statement is due to the fact that observational procedures, such as photo-interpretation or satellite image analysis can identify observational conditions suggesting that an object might exist at some location without giving definite certainty that it really exists as an independent object. The ‘observed object' then gets an object identifier but it might, in fact, be a part of another object. The uncertainty of the ‘exist' function expresses in this case the uncertainty of the actual real world state of the observed object. The problem arises due to the fact that in many GIS applications it is only possible to refer indirectly to real world objects through descriptions provided by observational systems. This problem is strongly related to the referential problem identified in philosophy, see, for example, (Evans, 1995), (Neale, 1990) and (Quine, 1960). If the existence of objects is uncertain then the universe of a map becomes a fuzzy universe and the definition of Section 5.3.4 should be modified to

$$U_M = \{\ldots,\{O_i\,, Exist(O_i)\},\ldots\}$$

The members of this fuzzy universe are the objects with the function expressing the uncertainty of their existence. This situation is fundamentally different from the situation dealt with by the theory of fuzzy (sub)sets (Kaufman, 1975), (Klir and Folger, 1988), (Klir and Yuan, 1995) and (Zimmermann, 1985). There the existence of the members of the universal set is not uncertain, only the subsets of the universal set are fuzzy. Therefore the concepts of the theory of fuzzy subsets and of fuzzy reasoning should be applied with care in our situation. If the universe of a map is not certain then what is? The formalism as developed in the previous chapters suggests that the sets of geometric elements of a map can serve a universe with a certain extent. If we look at the set of faces (or the set of raster elements), then subsets can be generated by assigning these faces to the spatial extent of objects, these extents are then the fuzzy subsets of the set of faces. This would explain why many mapping disciplines have such a strong affinity towards the field approach rather than the object approach. We will see in the next section how these two are related.

3.2 Extensional uncertainty

For fuzzy objects the relation between face and object can not be established with certainty so that $Part_{22}[f, O] \in [0,1]$. The spatial extent of a fuzzy object is therefore uncertain, it is given by:

$$Face(O) = \{ f \mid Part_{22}[f, O] > 0 \}$$

The situation of Figure 2 presents an example of this case.

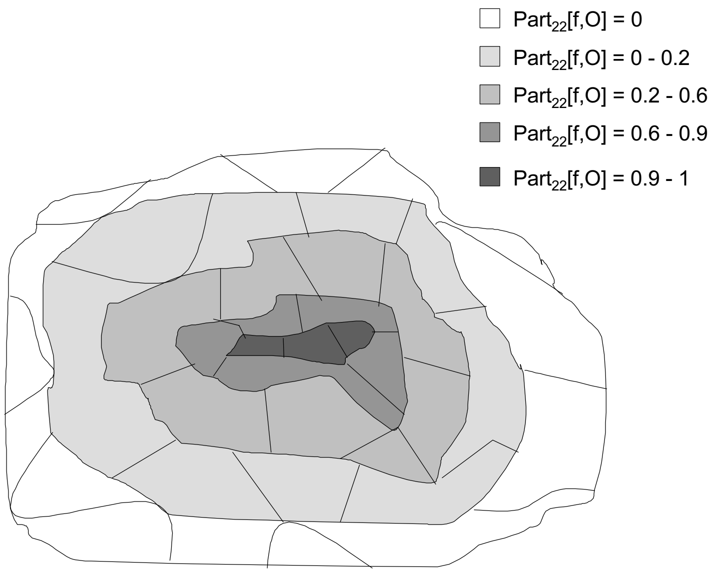


Figure 2. Object with Fuzzy spatial extent

The approach developed by (Molenaar, 1994, 1996 and 1998) shows that the vector and the raster geometry have a similar expressive power, if the cells of a raster are considered as rectangular faces. That implies that the handling of spatial uncertainty should in principle also be the same for both geometric structures, so that it must be possible to combine or even unify the vector and raster oriented approaches found in literature. In case the geometry of the map is represented in a raster format the function $Part_{22}[\,]$ is evaluated for each cell of the raster.
For crisp objects we defined, in Section 2, the function $B[e, O] = Le[e, O] + Ri[e, O]$ with $B \in \{0, 1, 2\}$. With this function we could identify the object boundary as $\partial O = \{ e \mid B[e, O] = 1\}$. In the case of fuzzy objects this interpretation of $B[e, O]$ is not possible because here $B \in [0, 2]$. When $B[e, O] = 1$ then this does not simply mean anymore that the edge has the object at only one side, because for fuzzy objects $Le[e, O]$ and $Ri[e, O]$ can take any value between 0 and 1. So there are many value combinations that may give this result

Conditional spatial extent. Let the set of faces related to object O_a with certainty level c be:

$$Face(O_a \mid c) = \{ f_j \mid Part_{22}[f_j, O_a] \geq c \}$$

This will be called a *conditional spatial extent* of the object, comparable to the (strong) α-cut in (Klir and Yuan, 1995). With this set we can define the conditional functions:

$Part_{22}[\,f_j, O_a \mid c\,] \quad = 1 \quad f_j \in Face(O_a \mid c)$
$\qquad\qquad\qquad\qquad = 0 \quad f_j \notin Face(O_a \mid c)$

Conditional boundaries. The relationships between faces and a conditional spatial extent of an object are crisp. This implies that if we define

$Le[\,e, O \mid f, c\,] = MIN(Le[\,e, f], Part_{22}[\,f, O \mid c\,])$ and
$Ri[\,e, O \mid f, c\,] = MIN(Ri[\,e, f], Part_{22}[\,f, O \mid c\,])$

Then these functions are also crisp. Now we can define the conditional function

$B[\,e, O \mid c\,] = Le[\,e, O \mid c\,] + Ri[\,e, O \mid c\,]$

Which has a similar interpretation as the function *B[e, O]* in Section 2. There it has been used to identify the boundary of a crisp object, here it will be used to identify the boundary of the conditional spatial extent of a fuzzy object. This will be called a conditional boundary, so for a certainty level *c* we find the conditional boundary

$\partial_c O = \{\, e \mid B[\,e, O \mid c\,] = 1\}$

Fuzzy area objects represented as fuzzy fields. (Molenaar, 1998) defined a notation where the geometric description of the area objects was organised per face. That notation can also be modified to handle uncertainty; the set of area objects that have a fuzzy relationship with a face is then:

$AO(f) = \{\, O \mid Part_{22}[\,f, O] > 0\,\}$

With this notation it is possible to interpret a fuzzy area object as a fuzzy field; per face f the function $Part_{22}[\,f, O]$ is then evaluated for each object of the map. If the map has a raster structure then the set *AO( )* will be evaluated for cells instead of faces.
Objects can then be combined into one layer by overlaying their fuzzy fields. Suppose that the fuzzy fields of n objects have been represented in a raster format and that these n rasters are to be combined by an overlay operation. The attributes A_1 to A_n of the raster will represent the functions $Part_{22}[cell, O]$ for the n objects. Per cell the attribute value a_k represents the value of the function $Part_{22}[Cell_{i,j}, O_k]$.
The field representation of fuzzy objects seems to be quite natural. This fact could explain why many mapping disciplines have such a strong affinity towards the field approach rather than the object approach. This approach is often used because of apparent syntactic simplicity; the consequence might then be that the object structure of certain terrain descriptions remains hidden and with it much of the semantic content of these descriptions.

3.3 Geometric uncertainty

Digitising operations, hotogrammetry and landsurveying determine directly the geometry of object boundaries. These operations are generally applied in situations where the extent of the objects is crisp. In such cases it might be though that the boundary is strongly curved so that it can only be approximately represented in a geo-database by means of a discretised description consisting of chains of nodes and edges. If this discretisation is not properly adjusted to the actual shape of the boundary then deviations will occur between actual boundary and the geometry of the representing nodes and edges.

The magnitude of these discrepancies will depend on the resolution of the discretisation; this can be expressed in the number of points (nodes) per line length. The expectation of the squared discrepancies is the variance of the approximation of the real boundary by the discretised representation. This the value of this variance depends largely on three factors, see Figure 3:

- *The resolution of the discretisation in relation to the curvature of the actual boundary.* This causes the discrepancies d between digitised line and curved line (Shi, 1994), *d* varies along the digitised line; the expectation value of the d^2, is $\sigma_d^2 = E\{\,d^2\}$. This could then be used as a measure of dispersion between the original and the discretised boundary,
- *The position of the digitised points with respect to the boundary.* The identification of these points will have a stochastic component *E* so that they will not be located exactly at the boundary; a measure of dispersion for this effect could then be $\sigma_E^2 = E\{\,E^2\,\}$; this effect has been studied thoroughly by geodesists, one of the most profound theoretic discussions can be found in (Baarda, 1973),

- *The process by which the position of the digitised points is determined.* This is the measuring process that leads to the determination of point co-ordinates with respect to some reference system, this process will have a stochastic effect e on the computed position of the digitised points. For this effect we could use as a measure of dispersion $s_e^2 = E\{ e^2 \}$; a thorough theoretic analyses of the importance of these effects on the accuracy of point fields can also be found in (Baarda, 1973).

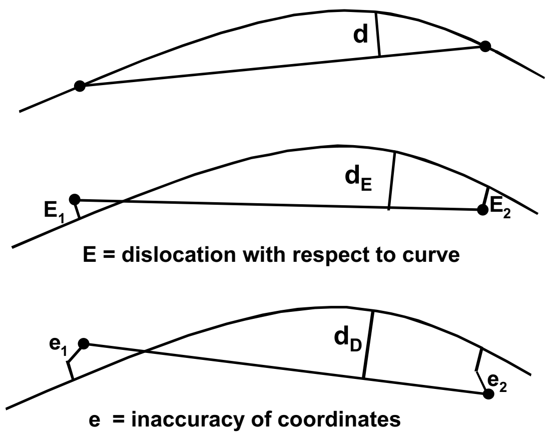


Figure 3. E = misidentification of digitised point,
e = error in co-ordinate determination,
d, d_E, d_D = are discrepancies between digitised line and curved line.

The total displacement of the computed position of a digitised point i with respect to the actual line is then $D_i = E_i + e_i$. The discrepancies d, d_E and d_D give the distance of points on the curved line with respect to the digitised line, these distances are measured along the orthogonal projection of these points onto the digitised line.
If we compute the variance $S^2 = E\{ d_D^2 \}$ then S^2 is a measure for the dispersion of the points of the curved line with respect to the digitised line (compare (Shibasaki, 1994)). This variance is a function of the three parameters that represent the different aspects of the geometric uncertainty of the measurement of position and shape of a boundary, i.e.

$$S^2 = S^2(s_d^2, s_E^2, s_e^2).$$

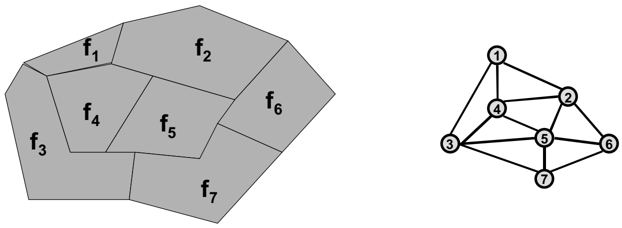
From this variance we can compute *S* which is then the standard deviation of d_D. With this standard deviation confidence regions can be defined for the boundary points with respect to the digitised line. (Shi, 1994) The width of such region can be expressed by means of the distance *b.S* of its outer limits with respect to the digitised boundary; the value of b depends on the required confidence level a; e.g. under the assumption that the dispersion of the boundary points has a normal distribution with respect to the digitised boundary we find for $\alpha = 0.95$ or $\alpha = 0.999$ the values b = 1.96 resp. b = 3.29. For practical reasons these confidence regions are often approximated by *epsilon-bands*.

4 OBJECTS WITH A CONVEX FUZZY SPATIAL EXTENT

An adjacency graph can be defined for the spatial extent of each object as in Figure 4. The adjacency graph of an object O consists of:
- Nodes representing the faces belonging to the spatial extent of O
- Edges so that each edge expresses the adjacency of the faces represented by the nodes it connects.

Elementary area objects can then be defined as objects that have a spatial extent consisting of a contiguous set of faces so that the adjacency graph of these faces is connected. This definition allows for elementary area objects (or even faces) with holes. This definition is valid for crisp objects.
This definition of elementary objects needs some adjustment for fuzzy objects, but the original intention can be maintained. The concept of convex fuzzy sets as explained in (Klir and Yuan, 1995) can be used here, but it will be formulated differently (Molenaar, 1998). First some supporting definitions will be formulated before a definition of elementary fuzzy area objects can be given.

A fuzzy object O has nested conditional spatial extents if: $(" c_i, c_j \mid c_i > c_j) \quad (Face(O \mid c_i) \quad Face(O \mid c_j))$



Node n_i represents face f_i

edge e_{ij} represents ADJACENT[f_i , f_j]=1

Figure 4. The adjacency graph of the face set of an object

This means that the face set representing the spatial extent of an object for a high certainty level should be contained in the face set for a lower certainty level. The second definition requires that for each certainty level the spatial extent of the object is connected, i.e. the face set has a connected adjacency graph.

A fuzzy object is connected if: $(\forall c > 0)$ *Face(O | c) is connected*

When an object complies with this definition then each conditional spatial extent complies with the definition of crisp elementary area objects. With these two definitions objects with convex fuzzy spatial extents can be defined:

A fuzzy object has a convex fuzzy spatial extent if it is connected and if its conditional extents are nested.

Now we are ready to define elementary fuzzy objects:

A fuzzy area object is an elementary object if it has a convex fuzzy spatial extent.

The object in Figure 2 complies with this definition. As stated before the definition also allows (conditional) objects with holes. For elementary fuzzy objects only conditional boundaries can be determined, with these several zones can be identified:

- The area inside $\partial_{c=1}$ O that is the area that certainly belongs to the object,
- The area outside $\partial_{c=0}$ O that is the area that certainly does not belong to the object,
- The area between these two boundaries, the transitional zone where the certainty varies between 0 and 1.

Crisp objects as a special case of fuzzy objects. With these definitions it can simply be illustrated that elementary crisp objects are a special case of elementary fuzzy objects because for the crisp situation the zone between c = 0 and c = 1 collapses to a width equal to 0, i.e. we get

$$\forall c_i, c_j \in (0,1] \qquad Face(O|c_i) = Face(O|c_j).$$

This means that all conditional extents of crisp objects are identical.

5 CONCLUSION

Three aspects of the spatial uncertainty of objects have been discussed these were the existential, the extensional and the geometric uncertainty. The exsitential uncertainty expresses the fact that survey or image data do not give sufficient evidence whether an observed object really exists. This is often a problem when we try to identify an object by its spatial extent and when this extent is fuzzy. That will then have the effect that we can not be sure whether we see the object or not. Such situations occur in small scale land use and land cover mapping through remote sensing.
The extensional uncertainty is generally a spatial effect of fuzziness of the thematic description of the objects; it is therefore strongly related to the thematic class of the objects. The geometric uncertainty is due to a combination of object geometry and measuring procedure. It is obvious that the geometric accuracy will deteriorate when an object has a fuzzy spatial extent so. In that case only conditional boundaries can be determined, but generally the geometry of such

boundaries will not determinable with great precision. But suppose that conditional boundaries can be determined, then three situations can be identified for the relation between extensional and geometric uncertainty:

1) The transition zone between $\partial_{c=1}$ O and $\partial_{c=0}$ O is much larger than the epsilon band representing the geometric precision of the boundary. In that case the uncertainty of the spatial extent of the object overrules the geometric uncertainty. This is the case is many survey disciplines mapping natural objects or land use in small-scale mapping.
2) An intermediate situation occurs when the transitional zone has about the same size as the epsilon bands. Then it is difficult to distinguish between the extensional and the geometric uncertainty, although they are semantically quite different.
3) The objects are crisp so that the transition zone has a width equal to 0. The spatial uncertainty of the object is then only due to the accuracy of the geometric determination of the object. This is often the case in land surveying and photogrammetry

Models for the evaluation of the spatial accuracy of objects should be able to distinguish these two types of uncertainty. The extensional uncertainty is predominantly related content oriented methods for object determination, like satellite image analysis or photo interpretation. The geometric uncertainty is mainly related to geometric approaches for object determination, like land surveying and photogrammetry. The representational model explained here can handle both approaches so that the different uncertainty aspects of spatial objects can be integrated into one unifying quality model. This will take care of the fact that in many cases the uncertainty of the thematic and geometric components of spatial objects are strongly interrelated.

REFERENCES

Baarda, W., 1973, *S-Transformations and Criterion matrices.* (Delft: Netherlands Geodetic Commission), New Series, Vol. 5, Nr. 1,

Brown, D., G., 1998, Classification and boundary vagueness in mapping resettlement forest types, *Int. J. Geographical Information Science*, Vol. 12, No. 2, pp. 105 – 129.

Canters, F., 1997, Evaluating the uncertainty of area estimates derived from fuzzy land-cover classification. *Photogrammetric Engineering and Remote Sensing*, 63, pp. 403-414.

Burrough, P.A. and Frank, A.U., 1995, Concepts and paradigms in spatial information: Are current geographic information systems truly generic? *International Journal of Geographical Information Systems*, 9, pp. 101-116.

Burrough, P.A. and McDonnell, R.A., 1998, *Principles of Geographical Information Systems.* (Oxford: Oxford University Press).

Chrisman, N.R., 1991, The error component in spatial data. In *Geographical Information Systems*, edited by Maguire, D.J., Goodchild, M.F. and Rhind, D.W., (Harlow: Longman), Vol. 1, pp. 165-174.

Dunn, R., Harrison,A. R., and White,J.C., 1990, Positional accuracy and measurement error in digital databases of land use and empirical study, *International Journal of Geographic Information Systems*, Vol.4, No. 4, pp. 385 – 398.

Fisher, P., 1996, Boolean and fuzzy regions. In: *Geographic Objects with Undeterminate Boundaries*, edited by Burrough, P.A. and Frank, A.U., (London: Taylor & Francis), pp. 87-94.

Goodchild, M.F., Guoqing, G. and Shiren, Y., 1992, Development and test of an error model for categorical data. *International Journal of Geographic Information Systems*, 6, pp. 87-104.

Klir, G.J. and Yuan, B., 1995, *Fuzzy Sets and Fuzzy Logic: Theory and Applications.* (Upper Saddle River, N J.: Prentice Hall PTR).

Molenaar, M., 1994, A syntax for the representation of fuzzy spatial objects. In *Advanced Geographic Data Modelling*, edited by Molenaar, M. and Hoop, S. de, (Delft: Netherlands Geodetic Commission), New Series, Nr. 40, pp. 155-169.

Molenaar, M., 1996, A syntactic approach for handling the semantics of fuzzy spatial objects. In *Geographic objects with indeterminate boundaries*, edited by P.A. Burrough and A.U. Frank, (London: Taylor & Francis), pp. 207-224.

Molenaar, M., 1998, An Introduction to the Theory of Spatial Object modelling for GIS. (London: Taylor & Francis), 1998, 246pp.

Shi, W., 1994. *Modelling positional and thematic uncertainties in the integration of remote sensing and GIS.* (Enschede: ITC) Publication Nr. 22.

Shibasaki, R., 1994, Handling Spatio-Temporal Uncertainties of Geo-Objects for Dynamic Update of GIS Databases from Multi-Source Data. In *Advanced Geographic Data Modelling*, edited by Molenaar, M. and Hoop, S. de, (Delft: Netherlands Geodetic Commission), New Series, Nr. 40, pp. 228-242.

Usery, E. L., 1996, A conceptual framework and fuzzy set implementation for geographic feature, In *Geographic Objects with Indeterminate Boundaries*, edited by P. A. Burrough and A. U. Frank, (London: Taylor & Francis), pp. 71 - 85.

Wickham,J.D., O'Neil, R.V., Ritters, K.H., Wade, T.G. and Jones, K.B., 1997, Sensitivity of selected landscape pattern metrics to land-cover misclassification and differences in land-cover composition. *Photogrammetric Enigineering and Remote Sensing*, 63, pp. 397-402.