A MATHEMATICAL MODEL FOR CORRECTING THE PHOTOGRAPHIC COORDINATES DUE TO ORIENTATION ERRORS (AN APPLICATION IN CLOSE RANGE PHOTOGRAMMETRY)

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ABSTRACT

Photographic coordinates x and y are fundamental data in analytical photogrammetry. At the same time these coordinates are functions of interior and exterior orientation elements, and contain some errors. Therefore some corrections and reductions are applied to the image coordinates measured in comparators before they are used in mathematical models. Some corrections and reductions related to the physical error sources inherent in photogrammetric system are applied to the comparator coordinates, but any correction related to the exterior orientation elements is not applied to these coordinates. In this study, a mathematical model has been proposed to correct the photographic coordinates due to the small errors of exterior orientation elements of camera. The data obtained a close-range test field have been used in the offered mathematical model and the results have been criticized.

1 INTRODUCTION

There are two different techniques in restituting a photograph or photographic pair in photogrammetry. The first one is analog method which produces plain, profils or contour maps. The standart outputs of analog restitution method are in grafical form. Th second one is analytical or numerical method which depends on the analytical reconstruction of the bundles of rays and of the stereomodels. In analytical method the main data used in mathematical models are image coordinates obtained from comparators or digital plotters. These coordinates which called as “raw data” at the beginning, are functions of interior and exterior orientation elements. Therefore, every errors in these orientation elements will affect the image coordinates, and these errors will indirectly reflect to the object space coordinates. In this case, it is apperent that some corrections related with small errors in interior and exterior orientation elements must be applied to the image coordinates which will be used in mathematical models. Image coordinates must be especially corrected due to the small orientation errors for some photogrammetric applications which all of orientation elements or some of them are a priori known and are assumed errorless (Baş, 1985; Veress and Sun, 1978; Brandenberger and Erez, 1972); or for photogrammetric applications in which photos are taken from the same exposure station and are reconstructed with the same interior and exterior orientation parameters (Dauphin and Torlegard, 1977; Altan 1993).

The aim of this study is to offer a mathematical model and its application in close-range photogrammetry to correct the image coordinates due to small orientation elements after transforming the comparator coordinates to the photographic coordinate system.

2 REFINEMENT OF MEASURED IMAGE COORDINATES

2.1 Corrections For Systematic Errors

Image coordinates measured in comparators contain some systematic errors sourced from the photogrammetric system. Some corrections and reductions must be applied to these coordinates before they are used in the mathematical models. These corrections and reductions are comparator calibration effect and emulsion carrier, lens distorsion, atmospheric refraction, earth curvature and image motion (Gosh, 1979; Brown, 1980). These corrections, except lens distorsion correction , are independent from the orientation of exposue system of photographs.

2.2 Corrections Related to Orientation Parameters

Corrections related to the small errors of orientation parameters can be classified as following , with taking account of some applications:

a. In some applications (Veress and Sun, 1978; Brandenberger and Erez, 1972), all of the exterior orientation elements or some of them are determined directly with measurements in field. A phototheodolite or a stereometric camera are used to exposure the photographs in these applications.
b. Interior and exterior orientation elements are a priori known and can be assumed errorless in some photogrammetric applications (Baş, 1985).
c. In some applications of analytical photogrammety (Fuad, 1984; Abdel-Aziz, 1982), orientation of stereometric camera is fixed (such as Wild C40, C120, Zeiss SMK40, SMK120) . Namely orientation of two cameras remains constant for all of taken stereopairs in these applications.
d. In some photogrammetric applications (Altan, 1983; Scott, 1978; Porter and Burns, 1978; Smidrkal, 1968), all photographs are taken from the same exposure station and under the same conditions, with the same interior and exterior orientations, at different times. The dimensional changes of objects are examined according to the principles of parallax photogrammetry, "False Parallax" method or "The Time-Parallax" method, in these applications.

A stereometric camera consists of two identical metric cameras mounted rigidly at the ends of a fixed base so that their optical axes are parallel to one another. Angular orientation of a stereometric camera is set up by levels,and the accuracy of orientation depends on the sensitivity of used levels. This situation is valid for phototheotolites.

Therefore the image coordinates measured in comparators can be corrected for small errors of orientation by using the approach introduced in this study. For applications in step **d,** image coordinates measured in the second photograph can be transformed onto the coordinates of first reference photograph by means of the offered approach.

3 MATHEMATICAL FORMULATIONS

The equations for the influence of small errors of orientation on the image coordinates can be found from literature (Gosh, 1979; Finsterwalder and Hoffman, 1968; Hallert, 1960) in different forms and for different aims. Figure 1 shows the used coordinate systems in this study.

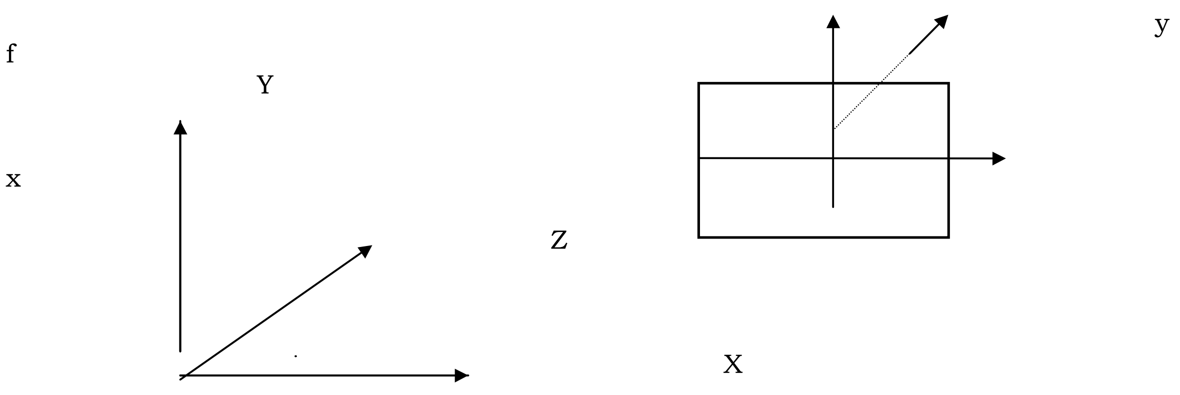

Figure 1. Object Space and Photographic Coordinate System

The diferantial equations for the influence of small errors of inner and outer orientation on the image coordinates x, y are

$$dx = dx_0 - (x/f)df + (f/z)bx + (x/z)bz + (xy/f)d\omega - f\,(1+x^2/f^2)d\phi + y d\kappa \quad (1)$$

$$dy = dy_0 - (y/f)df + (f/z)by + (y/z)bz + f\,(1+y^2/f^2)\,d\omega - (xy/f)\,d\phi - x d\kappa \quad (2)$$

where dx and dy represent the differences, at photo scale, between the corrected and measured image coordinates of any point; x and y are measured comparator coordinates of point; f is principal distance; dx_0 and dy_0 are the error of the determination of position of the principal point from the fiducial marks in the photograph; df is the error in the principal distance; $d\omega$, $d\phi$, $d\kappa$ are small rotations about the X, Y and Z axes respectively; bx, by, bz are translations of the camera in the X, Y and Z directions; and Z is object istance. In a similiar way differantial formulas can be derived for arbitrary cases of photogrammetry. The differantial formulas of convergent case which is fundamental importance for the terrastrial photogrammetry have been derived following:

$$dx = dx_0 - (x/f)df + (f/z)bx + (x/z)bz + y/f\,(x\cos\phi - f\sin\phi)d\omega - f\,(1+x^2/f^2)d\phi + y/f\,(f\cos\phi + x\sin\phi)d\kappa \quad (3)$$

$$dy = dy_0 - (y/f)df + (f/z)by + (y/z)bz + \{(x/f\ \sin\phi) - (1+y^2/f^2\cos\phi)\}f\,d\omega - (xy/f)\,d\phi - \{x\cos\phi - f\sin\phi\,(1+y^2/f^2)\}d\kappa \quad (4)$$

In equations (1) and (2) the unknown orientation errors dx_0, dy_0, df, …… etc. are solved by means of the same equations to correct the measured image coordinates. Control points are used in solving these elements. Then corrected photo coordinates of all points on photo are obtained by using these elements in matrix equations (5). The detail of application is explained in the Practical Application paragraphs.

$$\begin{vmatrix} x^* \\ y^* \\ f \end{vmatrix} = f/f^*\ dD \begin{vmatrix} x \\ y \\ f \end{vmatrix} + f/z \begin{vmatrix} bx \\ by \\ 0 \end{vmatrix} + bz/Z \begin{vmatrix} x \\ y \\ 0 \end{vmatrix} - df/f \begin{vmatrix} x \\ y \\ 0 \end{vmatrix} + \begin{vmatrix} dx_0 \\ dy_0 \\ 0 \end{vmatrix} \quad (5)$$

Where $f^* = dD_3\,(x \quad y \quad f)^T$

$dD_3 = (d\phi \quad -d\omega \quad 1)$

x^*, y^* = corrected image coordinates

x , y = image coordinates measured in comparator and corrected for systematic errors.

4 PRACTICAL APPLICATION

The present mathematical model has been applied to the normal case of close-range photogrammetry. The used data in this application have been obtained from the test field the detail of which was explained by Müftüoğlu (1980). The application has been performed following steps:

a. 25 test points were chosen from the test field.
b. Theoretical correct image coordinates of these points were computed by using the geodetic coordinates of related points and the interior and exterior orientation data of camera. The equations (6) and (7) have been used to compute these coordinates.

$$a_{11}(X - X_0) + a_{12}(Y - Y_0) + a_{13}(Z - Z_0)$$

$$x \frac{\qquad}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)} = f \qquad (6)$$

$$y \frac{a_{21}(X - X_0) + a_{22}(Y - Y_0) + a_{23}(Z - Z_0)}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)} = f \qquad (7)$$

c. The difference between measured and computed image coordinates dx, dy were obtained for all points.
d. Unkonown orientation errors dx_0, dy_0, df,..... etc., were solved by means of the equations (1) and (2).
e. Then corrected photo coordinates of all image points were computed by matrix equation (5).

Table 1 shows the results obtained from the practical application. The results shows that corrected photo coordinates are nearer than measured image coordinates to the theoretical correct photo coordinates obtained from equations (6), (7).

Table 1. The results for the normal case of stereophotogrammetry
(the coordinates belong to left photo of related stereopair)

Test point number	image coordinates obtained from stereocomparator		theoretical correct photo coordinates from equations (6,7)		corrected photo coordinates from matrix equation (5)	
	x	y (mm)	x	y	x*	y*
3-3	-8.100	10.090	-7.626	11.245	-7.527	11.268
3-9	11.970	10.031	12.486	11.169	12.584	11.196
6-6	2.139	0.114	2.590	1.223	2.734	1.217
9-3	-8.054	-10.069	-7.590	-8.914	-7.495	-8.953
9-9	12.064	-10.036	12.504	-8.940	12.603	-8.972
1-4	-4.781	16.793	-4.297	17.935	-4.198	18.009
1-5	-1.434	16.822	-0.948	17.969	-0.849	18.038
1-6	2.319	17.906	2.830	19.100	2.956	18.543
1-7	5.288	16.779	5.782	17.902	5.897	18.004
1-8	8.593	16.810	9.095	17.948	9.216	18.036
3-4	-4.755	10.122	-4.268	11.226	-4.181	11.295
3-8	8.647	10.017	9.148	11.145	9.248	11.183
9-7	5.350	-10.036	5.786	-8.923	5.885	-8.959

9-8	8.713 - 10.060	9.139 - 8.939	9.249 - 8.989

The object space coordinates X and Y of the same test points have been computed by using the the image coordinates obtained from comparator readings and corrected photo coordinates x*,y*. The results are given in table 2. It is obvious from stdying the table 2 that the object space coordinates which have been computed by corrected photo coordinates are more accurate than the object space coordinates which have been computed by the image coordinates obtained from comparator readings.

Table 2. The object space coordinates of some test points

test point number	object space coordinates obtained fromcomparator image coordinates		object space coordinates obtained from geodetic measurements		object space coordinates obtained from corrected photo coordinates	
	X (m)	Y	X	Y	X	Y
3-3	11.079513	10.790114	11.114645	10.876585	11.122076	10.878282
3-9	12.576627	10.785714	12.615403	10.870868	12.622721	10.872843
6-6	11.843287	10.045959	11.862072	10.121621	11.871973	10.121192
9-3	11.082944	9.286360	11.117226	9.372099	11.124333	9.368639
9-9	12.583639	9.288823	12.615839	9.371071	12.623193	9.368639
1-4	11.327093	11.290125	11.363578	11.373795	11.370919	11.379335
1-5	11.576761	11.292287	11.613141	11.376087	11.620811	11.381231
1-7	12.078185	11.289078	12.115041	11.372844	12.123653	11.380488
3-4	11.329033	10.792502	11.365504	10.874441	11.372030	10.879577
3-7	12.0799979	10.784894	12.117430	10.870146	12.124350	10.872797
3-8	12.328749	10.784669	12.366228	10.868909	12.373664	10.871704

The mean square value M_X and M_Y which are computed from differences between the geodetic coordinates and computed photogrammetric coordinates are given following:

M_X = 8.285 mm., M_Y = 10.35 mm. (for the oject space coordinates computed from corrected photo coordinates)

M_X = 34.324 mm., M_Y = 81.69 mm. (for the object space coordinates computed from uncorrected photo coordinates)

It is obvious that the object space coordinates computed by using corrected photo coordinates are nearer to the geodetic coordinates than the object space coordinates obtained form uncorrected photo coordinates.

5 CONCLUSION

The offered mathematical model in this study (equations 1, 2, 3, 4, 5) can be used to correct the photographic coordinates due to small orientation errors of camera. The same mathematical model also can be used to transform the image coordinates of comparision photography to the coordinates of reference photo in “False Parallax” or “Time Parallax” method. The practical experiments show that if the comparator coordinates are corrected by offered mathematical model in this study, after reduction of the comparator coordinates to the photographic coordinate system and the other refinements, photo coordinates and object space coordinates of points can have appreciably more accurate results.

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