

ON THE USE OF SPREADSHEETS FOR TEACHING ANALYTICAL PHOTOGRAMMETRY

Javier CARDENAL, Jorge DELGADO, Manuel TORRES

University of Jaén, Spain

Departamento de Ingeniería Cartográfica, Geodésica y Fotogrametría
23071-Jaén. SPAIN

jcardena@ujaen.es , jdelgado@ujaen.es , mtorres@ujaen.es

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ABSTRACT

This paper describes how some mathematical models used in analytical photogrammetry can be introduced to students in an easy and comprehensive way. Most of mathematical models used in photogrammetry are solved by application of least squares on linearized equation sets through iterative processes. With this purpose some spreadsheet templates have been prepared and adapted. This work presents some examples developed by the authors and successfully used in the teaching of analytical photogrammetry in the University of Jaén (Spain) to students of Surveying, Geodesy and Cartography. Three examples have been selected: a model for outer orientation (by two step method) of stereo-pairs; a camera calibration by the plumb line method; and an example of Direct Linear Transformation (DLT). But any other photogrammetric model based on the solution of linear equation sets could be applied. The use of a spreadsheet allows students to see physically the involved equations and how these equations are updated with the new initial approximations. Present spreadsheets include fast and powerful computation capacities (especially routines for matrix algebra) and a comfortable presentation (both numerical and graphical) of all involved variables and processes. Equations can be written with the formula editor and the solution is easily carried out by the macro language of any commercial spreadsheet (for example, COREL[®] Quattro Pro or Microsoft[®] Excel with PerfectScript and Visual Basic, respectively, as macro languages).

1 INTRODUCTION

A spreadsheet is a table made up of columns and rows, where each intersection constitutes a cell. Each cell stores a data value or a formula (logical/mathematical). A database with observations and initial approximations to some parameters is built by putting each record in a different row, where fields correspond to different columns. Spreadsheet templates can be prepared in order to solve some mathematical models based on least squares solution. Thus an equation per observation/s can be written in a row and then copied down to form the array for all the observations. These equations, as usual in photogrammetry, are linearized and they can be solved in a spreadsheet context. Programming under the macro language of the spreadsheet is an interesting and effective solution that can be made even with very basic knowledge on the use of this kind of software. This work presents some examples developed by the authors and successfully used in the teaching of analytical photogrammetry in the University of Jaén (Spain) to students of Surveying, Geodesy and Cartography. Three examples have been selected: a model for outer orientation (by two step method) of stereo-pairs; a camera calibration by the plumb line method; and an example of Direct Linear Transformation (DLT).

2 TWO STEP OUTER ORIENTATION

This first example is a spreadsheet template for the outer orientation of a photo-pair (aerial or terrestrial) by the two step method. This template comprises several related sheets in the same archive. Sheets are divided in several parts where data can be entered and formulas and equations are automatically written. All sheets are linked in order to share the same data. The structure of this archive appears in the Figure 1. The process follows the relative orientation (RO) of the pair by means of the coplanarity condition, then model coordinates are calculated by colinearity and, finally the absolute orientation (AO) of the model is computed by a three dimensional rigid transformation. Macros are created for iterative calculation of the coplanarity condition and 3D transformation, but also for compute initial approximations. The templates are prepared for calculations with six parallax points for the RO and four control points for the AO. Next subsections explain the different sheet templates.

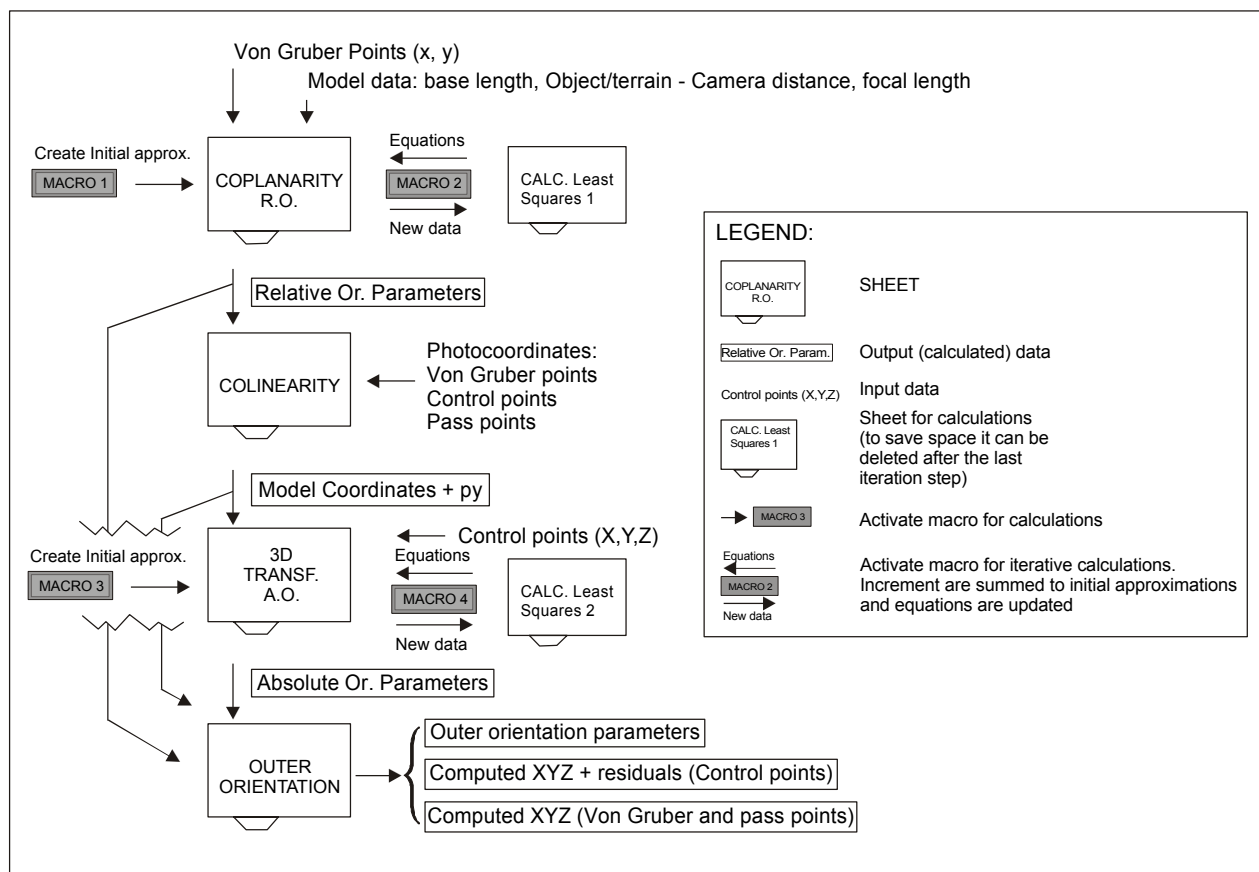


Figure 1. Flowchart of the spreadsheet template for the outer orientation.

2.1 Relative orientation

Relative orientation of the pair is achieved by coplanarity with the dependent method (Ghosh, 1987). This method fixes the left photo (in position and attitude) and the b_x (scale) of the model. The five free parameters are the right rotations (in terms of T , N and ϵ) and the Y and Z base components. In the upper part of the sheet there is a field for entering the refined Von Gruber photocoordinates for the left and right photos. Also some model data will be necessary as, for example, the focal length values for left and right photo. By considering different values of focal length for both left and right photos it is possible the application of this template for stereo-cameras. Base model (b_x) can be introduced in three ways: a) photobase; b) the unity (base components will be angular components) and c) ground model base (aerial or terrestrial). Finally, flying height, zero (for a model oriented at photo scale) or object-camera distance (for terrestrial case) can also be entered.

Prior to calculations a macro has been created to initialize the template. This macro sets the five parameters (T_2 , N_2 , ϵ_2 , b_Y and b_Z) to zero as initial approximations. Thus previous calculations are deleted and equations are initialized. Then relative orientation calculations start. The linearized coplanarity equations for six Von Gruber points are in the general form of:

$$[A]_{(6,24)}(V)_{(24,1)} + [B]_{(6,5)}(\Delta)_{(5,1)} = (-F_0)_{(6,1)} \tag{1}$$

- Where: $[A]$ = $\ast(F)/\ast(\text{observed photocoordinates})$. Residual coefficient matrix.
- $[B]$ = $\ast(F)/\ast(\text{parameters})$. Coefficient matrix.
- $(-F_0)$ = F (function evaluated with observations and approximate parameters)
- (V) = Vector of photocoordinate residuals
- $(\Delta)^T$ = $(\ast T_2, \ast N_2, \ast \epsilon_2, \ast b_Y, \ast b_Z)$. Vector of corrections to parameters

The general technique of least squares (Mikhail, 1976) has been employed to solve the equations. A macro (activated by a button) is programmed to make all the calculations in a separate sheet. Once equations have been solved, the

corrections to parameters are summed to the initial approximations. Then the sum of the five elements are copied and pasted (as values) in the same cells where initial approximations were entered, so equations with references to those cells will be updated. The macro also computes the variances of all adjusted parameters, and these ones with the last computed corrections are pasted in the sheet. Thus students can decide themselves when iterations should stop (each time a new iteration is needed, the macro button must be pressed) by analyzing the new increments or the variances.

2.2 Model coordinates calculation

Next to relative orientation, colinearity is applied in order to compute the model coordinates for all points (Von Gruber, control and pass points). Calculations can be made in a different sheet, where the relative orientation parameters are directly taken from the previous sheet. In equation (2), both left and right colinearity equations for a point *i* are written and become equal to the corresponding model coordinates (X_{Mi} , Y_{Mi} , Z_{Mi}). From the first (X_{Mi}) and third (Z_{Mi}) equations, k' and k'' are found using just two cells for the formulas.

$$\begin{bmatrix} X_{Mi} \\ Y_{Mi} \\ Z_{Mi} \end{bmatrix} = \frac{1}{k'} M_R'^T \begin{bmatrix} x_i' \\ y_i' \\ -f' \end{bmatrix} + \begin{bmatrix} X_0' \\ Y_0' \\ Z_0' \end{bmatrix}_{RO} = \frac{1}{k''} M_R''^T \begin{bmatrix} x_i'' \\ y_i'' \\ -f'' \end{bmatrix} + \begin{bmatrix} X_0'' \\ Y_0'' \\ Z_0'' \end{bmatrix}_{RO} \quad (2)$$

Known the scale factors, relative orientation parameters and left/right photocoordinates, model coordinates are computed. Also, y-parallaxes (py) both at model and photo-scale for every point are automatically computed. For a point *i*, which photocoordinates occupy four cells (in the same row), only six cells are necessary to compute k' , k'' , X_{Mi} , Y_{Mi} , Z_{Mi} , $py_{(MODEL)}$ and $py_{(PHOTO)}$. Once the first point is calculated, the six cells can be copied and pasted down to next rows to compute data for all points. In fact, this sheet does not need any macro for calculations, only it needs the six formulas previously written for the first point and the references to take the relative orientation parameters from the first sheet.

2.3 Absolute orientation

The structure of the absolute orientation sheet is very similar to the previous one for relative orientation. Thus this sheet also needs two macros (associated with two buttons) for calculations of initial approximations and least squares solution. The input consists in the model coordinates (taken from the previous sheet) and the XYZ coordinates of 4 full control points (input from the keyboard). For the control points twelve equations are written (three per point) with seven unknowns (scale factor, three rotations and three translations). The model for the three dimensional rigid transformation comes from Wolf (1983). And the linearized form as indirect observations equations system (Mikhail, 1976) is:

$$[B]_{(12,7)}(\Delta)_{(7,1)} = (L)_{(12,1)} + (V)_{(12,1)} \quad (3)$$

Where: $[B] = *(F)/*(parameters)$. Coefficient matrix.

$(L) = \text{Discrepancy vector (Measured coordinates – Calculated coordinates with approximate param.)}$

$(V) = \text{Vector of residuals}$

$(\Delta)^T = (*s, *S, *M, *5, *T_X, *T_Y, *T_Z)$. Vector of corrections to parameters.

The macro, that computes the initial approximations for the seven parameters, deletes also previous calculations and starts the equations. The initial scale factor is estimated from the relation of the distances between the first two points in terrain/object and model. Values for initials S and M are set to zero. Initial 5 (5_0) is estimated from the difference of the arctangents of the first two points in the terrain/object (X_t, Y_t) and model (X_m, Y_m):

$$5_0 = \arctan((Y_t - Y_t'') / (X_t - X_t'')) - \arctan((Y_m - Y_m'') / (X_m - X_m'')) \quad (4)$$

The three translation vectors (T_X, T_Y, T_Z) are estimated from the differences of the mean values for X, Y and Z coordinates (for all points) in object/terrain and model. The seven initial values are pasted in their correct positions and the twelve equations update. Now the second macro runs the least squares solution. Again the calculations are made in a

different sheet and the computed corrections are summed to the initial parameters. The new parameters are copied and pasted in the active sheet in the proper position so equations are newly updated. As the last increments, calculations from the variance-covariance matrix, and residuals are also pasted in the active sheet, the user will decide when the process must stop. This process finishes the absolute orientation of the model.

2.4 Outer orientation

The last sheet resumes the final process of the outer orientation. This means that the active sheet takes all data from the previous sheets (relative and absolute orientation parameters and model coordinates for all points) and computes the outer orientation parameters (position and attitude for both photographs) and the object/terrain coordinates for all measured points (Figure 1).

3 PLUMB LINE CALIBRATION

3.1 Introduction

The plumb line calibration method (Brown, 1971) has also been adapted to a template. This method states that a straight line in object space that is pictured by an ideal lens appears as another straight line in image space if no distortion exists (the central projection of a straight line is itself a straight line). But actual lenses deflect the light paths. Departures from linearity are attributed to distortion (if another error sources are properly reduced or not considered). The method states that any straight line in a 2D system can be expressed as:

$$x' \sin(\Theta_i) + y' \cos(\Theta_i) = \rho_i \tag{4}$$

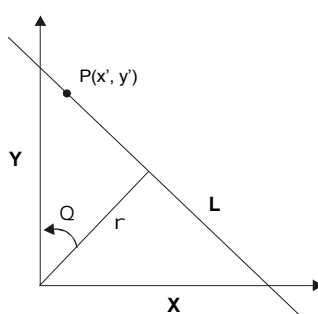


Figure 2. Explanation of the plumb line method (after Brown, 1971)

Where ρ_i is the distance from a line i to the origin (measured perpendicular to the line), and Θ_i is the angle subtended between the Y-axis and the normal to the line i passing through the origin. Any point j lying in the line i (x'_{ij}, y'_{ij}) must conform equation 4 (Figure 2). Actual points measured in photographs of lines in object space do not conform that expression because lines are affected by distortion (both radial symmetric and decentering), but if proper corrections (by means of distortion coefficients) are made to measured photocordinates (x_{ij}, y_{ij}), they will do (Equations 5a, 5b).

$$\left. \begin{aligned} x'_{ij} &= x_{ij}(1 - K_1 r^2) + \Delta x \\ y'_{ij} &= y_{ij}(1 - K_1 r^2) + \Delta y \end{aligned} \right\} \tag{5a}$$

$$\left. \begin{aligned} \Delta x &= P_1(r^2 + 2x_{ij}^2) + 2P_2 x_{ij} y_{ij} \\ \Delta y &= P_2(r^2 + 2y_{ij}^2) + 2P_1 x_{ij} y_{ij} \end{aligned} \right\} \tag{5b}$$

Where K_1 is the radial symmetric distortion coefficient and P_1 and P_2 are the coefficients for decentering distortion (Ghosh, 1987). The model for distortion employed in this work does not include higher terms for distortion coefficients

(as usual for small format non-metric cameras) or the reference for the principal point which is difficult to recover with this method, as it was clearly stated by Brown (1971). All measured photocoordinates must be referred to the indicated principal point (center of the format; for more information see Cruz et al., in this Congress). If equations 5 are substituted in the expression 4, it is obtained an observational equation of the form:

$$f(x_{ij}, y_{ij}, K_1, P_1, P_2, \Theta_i, \rho_i) = 0 \tag{6}$$

An equation No. 6 can be developed for each measured point. The unknowns are the distortion coefficients (K_1 , P_1 and P_2 , common for all equations) and the two geometric parameters of each line (Θ_i and ρ_i , common for the equations of points in the same line). If more observational equations than unknowns are written, a least square adjustment can be achieved. But expression 7 is not linear so linearization by Taylor's series is necessary.

This method works fine when enough lines have been photographed (in the same frame or in different frames). It is enough shooting two families of lines with a strong angle subtended between them. Usually, at laboratory conditions, two photographs are made. The first one with lines vertical or near vertical (plumbed strings immersed in oil to avoid vibrations or elastic ribbons tied to floor and ceiling, for example) and the second one with the camera rotated near 90° (the previous near vertical lines will be near horizontal). Once photographs are made, frames can be measured manually with a comparator or automatically with algorithms for edge detection (Fryer and Mason, 1989). The authors' experiences show that these algorithms are easy to elaborate and can be incorporated in the practical lessons in order to facilitate them.

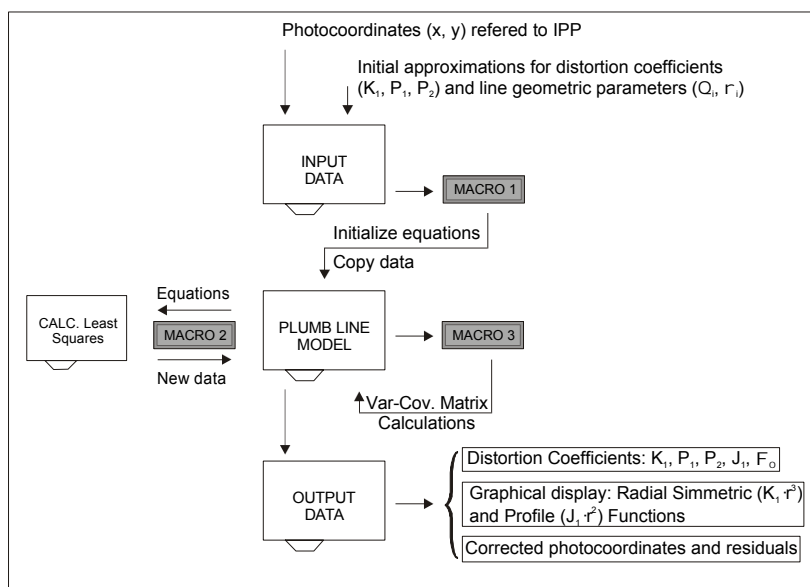


Figure 3. Flowchart of plumb line spreadsheet template. Legend is same that in Figure 2

3.2 Plumb line template

The spreadsheet template for the plumb line model is divided in three different (but linked) sheets (Figure 3). The first one (the input data sheet) is prepared to accept (from file or keyboard) the photocoordinates of 50 points measured in 10 lines (5 points each line). At this stage, initial approximations for distortion coefficients and line geometric parameters are entered. For the most cases, it will suffice rough approximations such as zero for coefficients, 90° and 0° (in radians) for Θ_i in near vertical and horizontal lines, respectively. And the mean value of abscises (vertical lines) or ordinates (horizontal lines) of points that belong to the same line for ρ_i . A button activates a macro in order to delete previous calculations in the main sheet (plumb line model, Figure 3) and paste the initial approximations on it. Thus equations are updated. There are 50 equations and 23 unknowns (3 inner and 20 line geometric parameters). The linearized equations are in the form:

$$[A]_{(50,100)}(V)_{(100,1)} + [B_1]_{(50,3)}(\Delta_1)_{(3,1)} + [B_2]_{(50,20)}(\Delta_2)_{(20,1)} = (-F_0)_{(50,1)} \quad (7)$$

Where: [A] = *(F)/*(observed photocoordinates); Two observations each equation.

(V) = Vector of photo-coordinate residuals

[B] = *(F)/*(parameters). Matrix coefficients (Figure 4)

()^T = (*K₁, *P₁, *P₂) and (*Θ₁, *ρ₁). Vector of corrections to parameters

(Matrixes and vectors with subscript 1 refer to inner parameters and subscript 2 refer to line geometric parameters)

(-F₀) = Function evaluated with observations and approximate parameters.

Again the general technique of least squares is employed to solve the linear system No. 7. As distortion coefficients are being updated through several iterations (usually three or four iterations will suffice) a graphical display (Figure 4) shows the radial symmetric ($dr_s = K_1 r^3$) and the profile ($d_D = J_1 r^2$, where $J_1 = [P_1^2 + P_2^2]^{0.5}$) functions. In this sheet two macros have been programmed. The first one solves the equations by least squares and once the system has reached the convergence, another macro makes the calculations for residuals and standard deviations of the computed parameters. The last sheet (output data, Figure 3) includes the measured points, corrected photocoordinates and the values for radial symmetric, radial asymmetric and tangential distortion for each point (computed as shown in Karara, 1989).

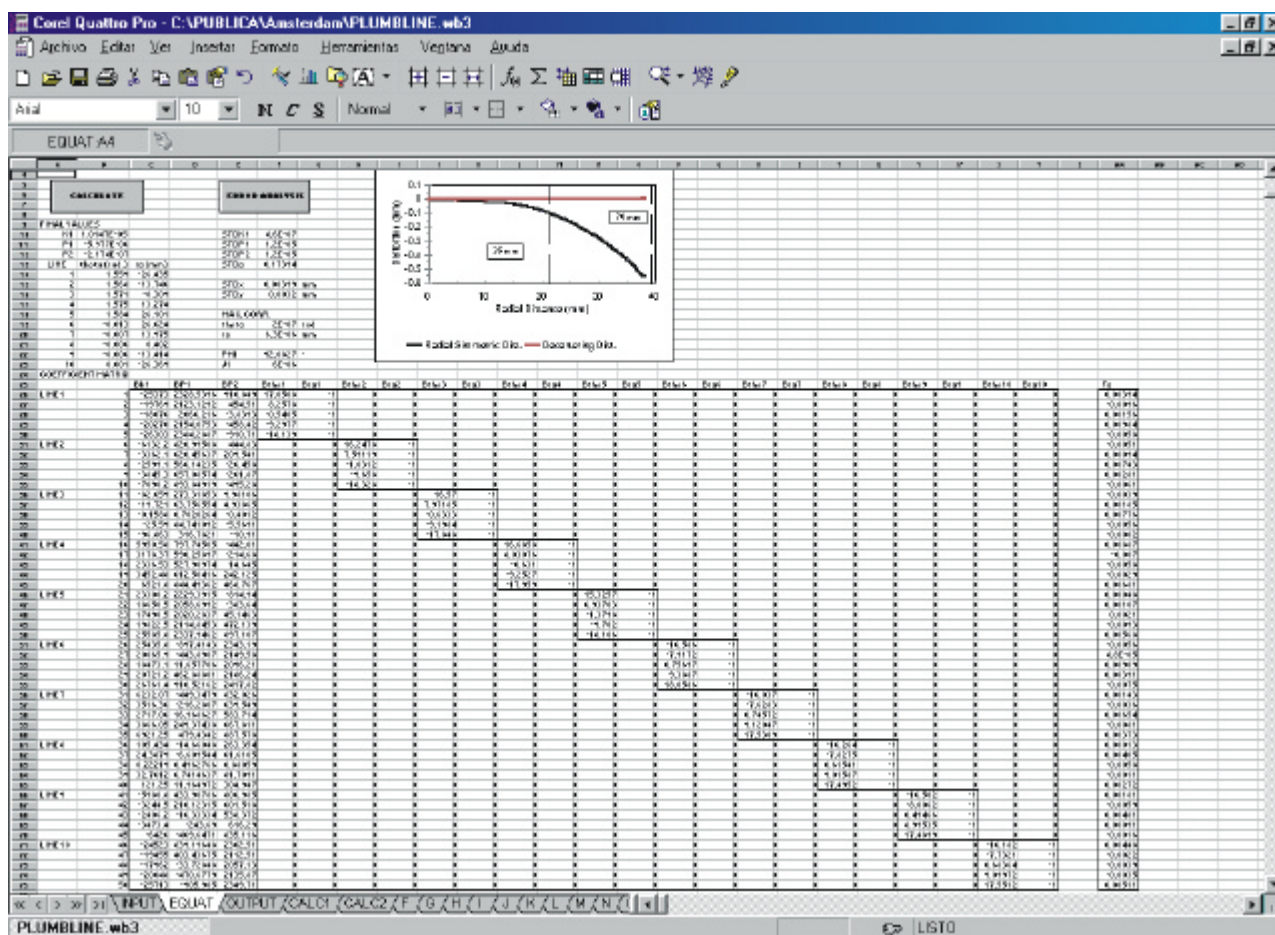


Figure 4. Screen capture (from COREL® Quattro pro) of the plumb line template. Note both matrixes B (coefficients) for inner parameters (the first three columns) and for geometric line parameters (the ten submatrixes for the 10 lines).

4 DIRECT LINEAR TRANSFORMATION

4.1 Introduction

Direct Linear Transformation (DLT) is a method especially suited for use with non-metric cameras, which lack fiducial marks. This method allows a direct transformation between comparator coordinates and object-space coordinates, bypassing the transformation to photocoordinates. Developed in origin by Abdel Aziz and Karara (1971: in Karara, 1989) was extended to incorporate coefficients for accounting systematic errors. The model has been adapted to a template in the form:

$$\left. \begin{aligned} x_i + (x_i - x_0)K_1r^2 &= \frac{L_1X_i + L_2Y_i + L_3Z_i + L_4}{L_9X_i + L_{10}Y_i + L_{11}Z_i + 1} \\ y_i + (y_i - y_0)K_1r^2 &= \frac{L_5X_i + L_6Y_i + L_7Z_i + L_8}{L_9X_i + L_{10}Y_i + L_{11}Z_i + 1} \end{aligned} \right\} \quad (8)$$

These two basic equations are written for each control point and because there are twelve unknowns, at least 6 full spatial control points will be necessary to solve the system. When using the approach stated in equations No. 8, caution has to be taken because the plate coordinates of principal point (x_0, y_0) are incorporated in the model (since the coefficient K_1 is also in the model). Usually if corner frames are present and can be measured, by computing the intersection of the diagonals between opposite corners will suffice. Although DLT can be solved in a more rigorous way (the iterative "11-parameter solution", as derived after Bopp and Krauss, 1978, in Karara, 1989), we have preferred the simplest direct solution. This one is enough accurate for most practical purposes and it is easier to be developed by the own students.

4.2 Direct Linear Transformation template

A spreadsheet was prepared to solve the DLT. This template can accept between 6 (the minimum to solve the system) and 25 full control points. This number of points will suffice the most cases since a higher number of them do not improve the solution. But it is important that those points must be widely distributed in the object-space both in depth and in surface, otherwise the recovery of the DLT parameters and the further space intersection of points can be very weak. As the previous templates, there is an area in this sheet for input data (the image and spatial coordinates of points) and another one for the equations, which are automatically written in the form:

$$[B]_{(50,12)} + (L)_{(12,1)} = (F)_{(50,1)} + (V)_{(50,1)} \quad (9)$$

Where, for a point i:

$$[B_i]_{(2,12)} = \left[\begin{array}{cccccccccccc} -X_i & -Y_i & -Z_i & -1 & 0 & 0 & 0 & 0 & x_iX_i & x_iY_i & x_iZ_i & (x_i - x_0)r^2 \\ 0 & 0 & 0 & 0 & -X_i & -Y_i & -Z_i & -1 & y_iX_i & y_iY_i & y_iZ_i & (y_i - y_0)r^2 \end{array} \right] \quad (10)$$

$$r = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$

$$\left. \begin{aligned} (L)^T &= (L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}, L_{11}, K_1') \\ K_1' &= A \cdot K_1 \\ A &= L_9X_i + L_{10}Y_i + L_{11}Z_i + 1 \end{aligned} \right\} \quad (11)$$

$$(F_i)^T = (-x_i, -y_i) \text{ and } (V) = \text{Residual errors on condition equations}$$

If there are less than 25 measured points, the corresponding position in the cells are not occupied and equations for points not measured are automatically set to zero (logical formulas, such as @IF(..) in Quattro Pro, are useful for this purpose). DLT is solved for several camera stations. Once the DLT parameters have been evaluated, the object space

coordinates of any unknown points are determined by means of multiray spatial intersection. Thus for any point that appears in several photographs, two DLT equations (No. 8) per photograph are written again. Regrouping terms the unknown XYZ coordinates of pass points are computed. Similar spreadsheet templates to those ones explained in this work can be developed with this purpose.

5 CONCLUSIONS

This work has shown how some photogrammetric models can be adapted to spreadsheet environment. This approach has several advantages. The mathematical models are introduced in an easy and comprehensive way. The spreadsheet allows to student to see physically the involved equations and how the iterative process works. Moreover programming under macro language is simple and easy (even for non-trained users) so the students can develop their own templates before they are introduced in more advanced programming languages (such as C or FORTRAN).

Three templates have been prepared. The first one is a routine for the outer orientation of a pair (aerial or terrestrial) by the two step method using 6 parallax and 4 full control points. The object/terrain coordinates of pass points can also be calculated. The input data are the refined photocoordinates of the points. Also a template for a camera calibration by the plumb line method has been presented. This template has specially been developed for calibrating small format non-metric cameras for radial symmetric and decentering distortion. Both templates allow a complete least squares adjustment also with calculations for the variance-covariance matrix. The last template consists in an example for a direct linear transformation (DLT) calculation. Other examples can be easily adapted, such as one step orientation, "in situ" calibration of cameras by collinearity or Church method, etc. The authors use these templates in practical lessons of photogrammetry to students of Surveying, Cartography and Geodesy in the University of Jaén (Spain). These lessons start with the data acquisition (the photographs) and the data capture with a comparator or with the photographs digitized. Then application of mathematical models can be easily made with these templates.

The templates used in this work have been prepared under Corel Quattro Pro (releases 5.0 and 8.0, *.wb1 and *.wb3 extensions), but another commercial spreadsheets can be used. Files are available (by e-mail, see affiliation and address in the first page) by request to authors.

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