

# Analysis and implementation of a laser strip adjustment model

Sagi Filin

Department of Geodesy, Faculty of Civil Engineering and Geosciences  
Delft University of Technology, The Netherlands  
s.filin@citg.tudelft.nl

Commision 3, Working Group 3

**KEY WORDS:** Laser altimetry, Error recovery, Strip adjustment, Segmentation

## ABSTRACT

The objectives of a strip laser adjustment are simple to define, namely improving the accuracy of laser data and creating a seamless dataset. Achieving these objectives is however difficult. Complex data acquisition systems and numerous error sources make the formulation of a strip adjustment model a complex problem. Difficulties in manually processing the data and limited information consisting only of the laser point coordinates, but not of the system measurements, imply that a strip adjustment is more than just an analytical model. This paper elaborates on both aspects of the problem. The proposed model is presented first and is followed by a discussion about implementation concerns. The model is system driven and is based on modeling the actual errors in the system. Automatic selection of tie regions in accordance with the proposed model and the implementation of the system driven model when only laser points are provided are two implementation concerns that are discussed in some level of detail. The paper concludes with discussion and analysis.

## 1 Introduction

Existence of noticeable systematic errors in airborne laser data has been acknowledged by now by both users and providers of data, particularly due to the reduced accuracy of the data and the offsets created between the data in the overlapping parts of the laser strips. The consequence of the latter is that generation of a seamless dataset from the independent laser swaths cannot be performed by merely merging the individual datasets but by a more involved process that eliminates the mismatches. Leaving the offset untreated may in turn make extraction of features difficult and the products quality rather questionable. The errors in the data are usually considered only with respect to the height component, yet planimetric errors are often bigger in magnitude than errors in the height. Removal of height offsets only will leave the planimetric offsets untreated so that similar surfaces and objects in the different swaths will not coincide. The removal of systematic errors in both components therefore calls for a 3D adjustment of laser points.

While there is a growing interest in the elimination of systematic error from the laser strips the work that has been done so far is rather limited and only little theory has been developed. One class of solutions that has been proposed can be considered as a data driven strip adjustment. Height differences between the laser points and either control points or points from the overlapping strips are minimized by a transformation of the laser points (Crombaghs et al., 2000; Maas, 2000; Kager and Kraus, 2001). These solutions focus on the effect of the errors but not on their causes. They may fail in removing some error components or introduce new artifacts into the data. A different class of solutions advocates modeling the actual errors in the system, and therefore can be regarded as a system driven approach. Several system based solutions have been proposed in recent years for estimating systematic errors see e.g., Vaughn et al. (1996); Ridgway et al. (1997); Filin and Csathó (1999) and in the framework of laser strip adjustment in Kilian et al. (1996); Burman (2000). The main advantage of a system driven approach is that the actual errors are being modeled and compensated for. No excess of parameters is needed to compensate for various error

effects and it is unlikely that new artifacts will appear in the data.

The elimination of systematic errors from the laser data is, in general, complicated. The main difficulties can be attributed to the variety of potential error sources and the effect they have on the data and the reconstructed surfaces, and to the fact that point correspondence between laser points and ground control is not straightforward to establish. The variety of error sources is the result of errors in the three independent components that constitute an airborne laser system – the laser range finder, GPS, and INS; the integration of those three components that should account for the alignment and positioning of the system components and the synchronization of the data streams; and to the fact that as a dynamic system some errors may vary over time. Another source of errors relates to the interpolation of the data streams. Interpolation becomes necessary as the different components operate in different frequencies; the prediction errors introduce other biases which are difficult to model and to correct for. While numerous sources of errors exist in the system not all of them are recoverable. Some errors are inseparable, and others have, under given conditions, similar effect. Identifying the error sources in the system is therefore only one aspect of the problem that should be followed by an analysis of their recoverability. In addition, the nature of laser data requires the development of adequate algorithms to recover the systematic error. In contrast to traditional reflectance data that are used in photogrammetry laser points sample the shape of the overflown surface. Point correspondence is practically impossible to establish under such conditions, and therefore shape based rather than traditional point based algorithms should be developed.

Most strip adjustment implementations tackle the problem of eliminating errors that occur during the data acquisition mission; errors are usually modeled for the INS and GPS components and account for the “exterior orientation” component of the system. Growing experience shows however that a more detailed error model is required as other types of errors not explained by the currently modeled ones can be noticed in the data (Crombaghs et al., 2000). On a more theoretic

cal level an analysis of the effect of the flight configuration or the shape of the overflow surface on the estimation of the parameters is usually lacking. This information is nevertheless important for understanding the problem, and for developing optimal strategies for estimating the errors. Another major drawback of most system driven implementations is that the existence of the system data streams is usually assumed. So in the regular case where only the laser points are provided, the application of such methods becomes rather limited. Consequently, a strip adjustment solution should address the following issues - i) an appropriate error model, ii) error recovery model and the representation, identification, and incorporation of control and tie information, iii) configurations analysis for optimal estimation of the errors, and finally iv) development of a model that users with no access to all of the system output can use.

This paper presents a strip adjustment model that is based on system error modeling. The paper focuses on the proposed error model and the error recovery model. It addresses implementation related issues, in particular on the analysis of control and tie information and the development of a model that is based on the laser points as input. An analysis of the model properties concludes the presentation.

## 2 The error recovery model

The proposed model is based on constraining the position of the laser points to the surface from which it was reflected. Both natural and man-made surfaces can be used for establishing the constraint. The constraint for an explicit representation of the surface has the form of

$$g(x_l, y_l, z_l) = 0 \quad (1)$$

With  $g$  the surface model and  $x_l, y_l, z_l$ , the laser point coordinates. If the surface is approximated locally by a plane the constraint involving the laser point and the surface has the form

$$s_1 x_l + s_2 y_l + s_3 z_l + s_4 = 0 \quad (2)$$

with  $s_1, s_2, s_3, s_4$  the surface parameters. The coordinates of the laser point can be written as a function of the system observations, namely the GPS, INS, and range finder (range and scan-angle) readings, the systematic errors, and the random errors:

$$[x_l, y_l, z_l]^T = f(Y, \Xi, \bar{e}) \quad (3)$$

with  $Y$  the observations,  $\Xi$  the systematic errors, and  $\bar{e}$  the random errors. Integrating equation (1) and equation (3) yields the form that constraint and laser point position as a function of the systematic errors

$$g(x_l, y_l, z_l) = g(f(Y, \Xi, \bar{e})) = h(Y, \Xi, \bar{e}) = 0 \quad (4)$$

By establishing the least-squares criterion to minimize the distance between the laser point coordinates and the actual surface the systematic errors can be recovered.

The geolocation of the laser points as a function of the observation from the three different components is given, when transformed into a local reference frame, by the form in equation 5

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + R_{INS} \left( \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_m R_s \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} \right) + \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} \quad (5)$$

with  $x_l, y_l, z_l$  the footprint location;  $X_0, Y_0, Z_0$  location of the phase center of the GPS receiver in the local frame;  $R_{INS}$  rotation from body reference frame to reference frame defined by local vertical;  $\delta_x, \delta_y, \delta_z$  offset vector between the phase center of the GPS antenna and laser firing point;  $R_m$  the mounting bias, which designate rotation between the altimeter and the body frame;  $R_s$  laser scanner rotation;  $\rho$  range vector measured by the laser system;  $\bar{e}_x, \bar{e}_y, \bar{e}_z$  random error components.

The following error sources are considered (see e.g. Schenk, 2001; Filin, 2001) – the mounting bias  $R_m$ , which models the misalignment between the laser scanning system and the INS; the range bias  $\delta\rho$ , which models the constant offset in the range determination; a scan angle error  $\lambda_{\omega_s}$  that measures inaccurate scan angle determination that vary linearly; an offset,  $[\delta X_0 \ \delta Y_0 \ \delta Z_0]$ , and a drift,  $[\delta \dot{X}_0 \ \delta \dot{Y}_0 \ \delta \dot{Z}_0]$ , in the GPS system; and an offset (shift) in the INS system,  $\Delta R_{INS}$ , that may occur in the initialization phase and a drift  $\Delta \dot{R}_{INS}$  in the INS angles during the mission. The effect of error sources on the geolocation of the laser point is given in equation 6

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + \begin{bmatrix} \delta \dot{X}_0 \\ \delta \dot{Y}_0 \\ \delta \dot{Z}_0 \end{bmatrix} t + (\Delta R_{INS} + \Delta \dot{R}_{INS} t) R_{INS} \left( \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_m R_{S_f(\lambda_{\omega_s})} \begin{bmatrix} 0 \\ 0 \\ -(\rho + \delta\rho) \end{bmatrix} \right) + \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} \quad (6)$$

These errors account, in general, to calibration errors (ranging, scanning, and mounting biases) and to system errors (INS, GPS). The effect of these errors on the reconstructed laser surface is not studied here in detail. However, the angular biases (INS and mounting) and positional biases exhibit to some extent linear transformations of the reconstructed laser surface, while the biases in the range and the scan angle exhibit non-linear transformations that may account for the phenomenon of the bending of the reconstructed laser surface at the end of the swath in a way similar to the one reported in, e.g., Crombaghs et al. (2000).

Other error sources were studied as well, for example the inclusion of time synchronization biases for the INS system and the GPS system –  $R_{INS} := R[\Omega + \dot{\Omega} \Delta t_{INS}, \Phi + \dot{\Phi} \Delta t_{INS}, K + \dot{K} \Delta t_{INS}]$  and  $R_{INS} v_x \Delta t_{GPS}$ , respectively, with  $v_x$  the aircraft velocity. An analysis of their contribution shows either similar effect to other error sources, or negligible contribution to the error budget.

Not all errors can be assumed constant throughout the mission, for example shifts and drifts in positioning may vary over time. As a result the errors are partitioned into two groups. System errors such as the mounting bias, range correction, and the scan angle correction are considered constant for the

whole laser block, while others are assigned lower level entities such as strips, according to the availability of control information.

Using the relation in eq. 6 to substitute laser points coordinates by the observations and the systematic and random errors, the error recovery model is obtained. The formula for solving the parameters is given, after linearization, in eq. 7.

$$\begin{aligned}
w_i = \mathbf{s} & \left( \begin{aligned} & \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + \begin{bmatrix} \delta \dot{X}_0 \\ \delta \dot{Y}_0 \\ \delta \dot{Z}_0 \end{bmatrix} t + \frac{\partial \Delta R_{INS}}{\partial \omega_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\omega_I \\ & + \frac{\partial \Delta R_{INS}}{\partial \phi_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\phi_I + \frac{\partial \Delta R_{INS}}{\partial \kappa_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\kappa_I \\ & + \frac{\partial \dot{\Delta} R_{INS}}{\partial \dot{\omega}_I} t \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\dot{\omega}_I + \frac{\partial \dot{\Delta} R_{INS}}{\partial \dot{\phi}_I} t \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\dot{\phi}_I \\ & + \frac{\partial \dot{\Delta} R_{INS}}{\partial \dot{\kappa}_I} t \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\dot{\kappa}_I + R_{INS} \frac{\partial R_m}{\partial \omega} R_s \begin{bmatrix} 0 \\ 0 \\ -\rho_i \end{bmatrix} d\omega \\ & + R_{INS} \frac{\partial R_m}{\partial \phi} R_s \begin{bmatrix} 0 \\ 0 \\ -\rho_i \end{bmatrix} d\phi + R_{INS} \frac{\partial R_m}{\partial \kappa} R_s \begin{bmatrix} 0 \\ 0 \\ -\rho_i \end{bmatrix} d\kappa \\ & + R_{INS} R_m R_s \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \delta \rho + R_{INS} R_m \frac{\partial R_s}{\partial \lambda_{\omega_s}} \begin{bmatrix} 0 \\ 0 \\ -\rho_i \end{bmatrix} d\lambda_{\omega_s} \end{aligned} \right) \\ & + \mathbf{s} \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} \quad (7)
\end{aligned}$$

with

$$\mathbf{s} := [s_1 \quad s_2 \quad s_3]$$

$d\omega_I, d\phi_I, d\kappa_I$  – change in the angles of the INS bias.

$d\dot{\omega}_I, d\dot{\phi}_I, d\dot{\kappa}_I$  – change in the angles of the INS drift angles.

and

$$\begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} = R_{INS} R_m R_s \begin{bmatrix} 0 \\ 0 \\ -\rho_i \end{bmatrix} \quad (8)$$

The extended form provides one row in the Gauss-Helmert model (equation 9 whose parameters are solved by eq. 10).

$$w_n = A_{n \times m} \xi_m + B_{n \times 3n} e_{3n} \quad , \quad e \sim \{0, \sigma_0^2 P^{-1}\} \quad (9)$$

with  $w$  the transformed observation vector;  $A$  the coefficient matrix;  $B$  the conditions matrix;  $\xi$  the vector of unknowns;  $e$  the observational noise;  $P$  the weight matrix;  $\sigma_0^2$  the variance component;  $n$  the number of laser points; and  $m$  the number of unknowns.

$$\hat{\xi} = (A^T (BP^{-1}B^T)^{-1}A)^{-1} A^T (BP^{-1}B^T)^{-1} w \quad (10)$$

with:

$$\hat{D}\{\hat{\xi}\} = \hat{\sigma}_0^2 (A^T (BP^{-1}B^T)^{-1}A)^{-1} \quad (11)$$

$$\hat{\sigma}_0^2 = \frac{(B\tilde{e})^T (BP^{-1}B^T)^{-1} (B\tilde{e})}{n - m} \quad , \quad B\tilde{e} = w - A\hat{\xi}. \quad (12)$$

## 2.1 Control information

As a surface based model the most natural type of control information to are control surfaces, sometimes referred to as control fields. These can be the result of field survey or photogrammetric measurements of some designated surfaces in the surveyed area. Transformation of the control fields (mostly provided as a set of ground measurements) into control information is rather straight-forward and involves the computation of the surface parameters for the laser points. Either if the control field is composed of only one surface (e.g., horizontal plane) or a more elaborate one, associating the laser points with the adequate surface parameters is relatively simple. Nevertheless, not always field surveys or other “surface” measurements are available or possible to conduct, therefore the use of other types and more available types of control information are considered. These entities are introduced into the model as additional surface constraints.

**Control points** Control points constrain the surface to the given point position, namely

$$s_1(X + e_X) + s_2(Y + e_Y) + s_3(Z + e_Z) + s_4 = 0 \quad (13)$$

with  $X, Y, Z$  the control point coordinates, and  $e_X, e_Y, e_Z$  are their random error components.

**Linear features** Line feature can either constrain a single surface when they lie in it or two surfaces when they define breaklines like a roof gable.

If a line is modeled as a vector  $\mathbf{l}$  and an origin  $x_0, y_0, z_0$  point two constrains can be written down.

$$\mathbf{s} \cdot \mathbf{l} = 0$$

$$s_1 x_0 + s_2 y_0 + s_3 z_0 + s_4 = 0 \quad (14)$$

One for constraining the surface normal to be orthogonal to the line and the other constrain the surface to pass through the origin.

Surfaces with no a priori surface model that are introduced into the adjustment are considered as tie objects (tie surfaces). Their approximate parameters are computed from the data and refined within the process of strip adjustment. The identification and selection of tie surfaces is part of the strip adjustment algorithm and is discussed in Section 3.2.

## 3 Implementation

As airborne laser data do not easily lend themselves to manual processing of the data the implementation of the strip-adjustment is not confined to the analytical model that was described in Section 2. Furthermore, the information that is usually provided to the users is not the complete set of system measurements but rather the laser points themselves. Both elements show that the scope of a strip adjustment implementation exceeds the derivation and application of the analytical model and imply that a strip adjustment is more of an algorithm that requires some details to be studied further.

### 3.1 Approximation of the observations

A common scenario is that the only information available to the users are the  $x, y,$  and  $z$  coordinates of the laser points and not the actual observation. While a more complete set

of system observations and laser points would better suite the error removal and possibly other applications, the proposed method attempts to overcome the lack of information by approximating some of the observations.

Equation 5 shows that the geolocation of a laser point involves 14 observations – eight system measurements (GPS, INS, and the laser scanner measurements), and six more for the offset vector and the mounting bias. An analysis of equation 5 shows that the INS and the mounting bias angles are unseparable and so are the offset vector and the position of the GPS receiver. The approximation is therefore reduced to position, attitude angles and the two range finder parameters for each laser point. This under-determined problem requires some further assumptions to be made. The main requirement is that the system specifications (e.g., field of view, scanner frequencies etc.) will be known, and to some extent that the scanning angle of a point will be recoverable based on its position in the scan line. Another assumption relates to the position and attitude of the scanning system over one scan, these values are considered constants for the whole scanline. To analyze which parameters are recoverable under these assumptions let us assume without loss of generality a small heading angle; the geo-location equation can then be written as

$$\begin{aligned} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} &= \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} 1 & K & -\Phi \\ -K & 1 & \Omega \\ \Phi & -\Omega & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\omega_s \rho \\ -\rho \end{bmatrix} \\ &= \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} -(\Phi + K\omega_s)\rho \\ -(\omega_s + \Omega)\rho \\ (\omega_s\Omega - 1)\rho \end{bmatrix} \end{aligned} \quad (15)$$

with  $K, \Phi, \Omega$  as the INS angles. With fixed attitude angles per scanline and range that does not vary significantly compared to the flying altitude some parameter will have similar contribution as others. In particular the effect of the pitch angle,  $\Phi$  and the system position in the  $x$  direction,  $X_0$ ; this similarity is quite expected considering the similar effect that shifting the system and tilting it have. Therefore the pitch angle is assumed zero. If a similar argument is used for the roll angle and the  $Y_0$  coordinate there are four constant parameters left to approximate per scanline and one more (range) for each laser point. Using two laser points, mostly the two ends of the scanline, six parameters can be approximated including the system position per scanline, the heading angle, and the two ranges. When applying this method and then comparing the computed scan angle of a point to the expected one, it became clear that for a wide field of view ( $\sim 20^\circ$ ) the lack of approximation of the roll angle  $\Omega$  has a noticeable effect. When solving for the roll angle by adding two more points from a scanline and formulating the approximation as a least-squares problem these differences disappear.

An analysis of the approximation effect on the strip adjustment shows that for the computation of the coefficients in equation 7, the range, attitude and the scanning angle affect the values, while the approximations of the position has no effect. Therefore the assumption of a constant position per scanline has only minor effect on the coefficient values. The system position would affect the left hand side (*lhs*) of system in equation 7. However, the *lhs* practically models the distance from the laser point to the surface. As both the

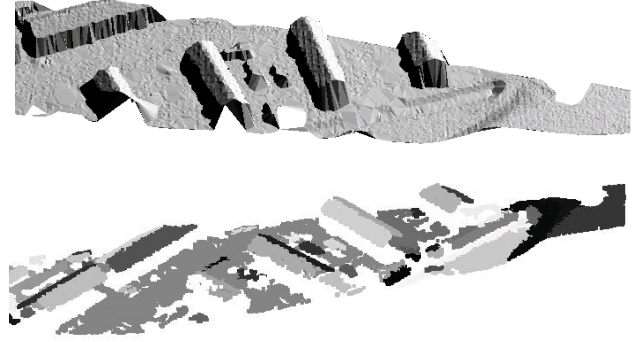


Figure 1: Clustering result in the overlapping zone of two strips. Upper Figure – a shaded relief of the laser data, lower Figure – Segmentation results (gray values  $\equiv$  segments).

laser point and the surface parameters are given, the *lhs* is not influenced by the approximation of the position.

The above assumptions hold as long as a scanline is complete, or almost complete (a few points are missing). However, experience shows that missing points in a scanline is a more common phenomenon than anticipated. So a predefined threshold on the number of missing points may eliminate a significant amount of data from the adjustment. As an example, the existence of water bodies like canals, which are few meters wide, along the laser strip may result in a “loss” of significant amount of points. So, in case of missing information the scanline is analyzed first, and if gaps that explain the missing data exist within the scanline this scanline is also considered for the adjustment.

### 3.2 Tie information

While control information is provided externally, the definition and detection of tie information is an implementation concern. For a surface based strip adjustment, tie surfaces in the overlapping areas are the natural candidates to be used. Candidate surfaces and regions can be determined by analyzing the laser surface and selecting areas that are suitable for the adjustment and that may improve the estimation of the parameters.

Surface information consists of several levels of descriptors in which the fundamental one is the partition of the point-cloud into regions with common attributes. Each region is characterized further by its type, e.g., smooth or planar surfaces or forest or low vegetation area. While the latter regions are not suitable for adjustment the smooth and planar regions require further characterization. The effect of noise on the local surface parameters, in particular when the scanning is relatively dense, may introduce artifacts into the surface parameters and thus inaccurate information into the adjustment. Avoiding this phenomenon calls for attenuation of the noise effect on the data or in other words regularization of the laser surfaces. Points on smooth and planar regions are therefore grouped into segments that can be described mathematically by a surface model. The surface model is used in the form a surface constraint in equation 1. The extraction of information calls for implementation in the framework of data segmentation.

The segmentation procedure that is used here is based on clustering the laser points. Points that share similar features are grouped together and their "segment" properties are validated further. The implementation is based on computing a feature vector consisting of the tangent plane parameters and height differences to the neighboring points for each laser point, followed by an unsupervised classification of the attributes in a feature space. In the feature space each point is represented by its feature vector, where the values of the feature vector determine the laser point coordinates in this space. Clusters are then identified according to proximity of points in feature space. Validation and refinement phases follow the extraction of clusters from the feature space. The validation phase concerns verification that indeed all of the cluster points are part of the surface. The refinement phase tests the extension of the cluster to neighboring points or merging neighboring clusters. Both processes are relevant to smooth and planar surfaces. Results of the point clustering algorithm are presented in Figure 1.

The segmentation approach that is chosen here have some advantages for the implementation of the strip-adjustment model. Point clusters in feature space offer, even before validated, a proposal for surface segments in the point cloud. Surface slope magnitude and orientation have a significant effect on the recoverability of the parameters and the quality of the results in the strip adjustment model. Clustering via the feature space enables therefore "profiling" the segmentation for given slope magnitudes and orientation. The feature space also summarizes the slope distribution of surfaces in the region. This information is valuable for predicting the recoverability of the different parameters with the current surface.

#### 4 Discussion and analysis

The strip adjustment model enables the estimation of errors over general surfaces. No distinct landmarks are needed to perform the adjustment either as control or tie points. Consequently, there are only little restrictions on its application, as the adjustment model is based on modeling the actual effect of the error sources on the geolocation of the laser point on the ground. A system based approach enables modeling and consequently removing the actual effect of the error sources. Furthermore, inclusion or elimination of error sources as more experience is gained becomes easier to implement. Error modeling concerns identifying the system errors and modeling their effect on the geolocation of the laser point.

As being a surface based model the error recovery depends of the surface characteristics, where the dominant component is the surface slope. In general sloped surfaces enable better estimation of the systematic errors. Surfaces can either be terrain or man-made objects, so in this regards sloped roofs are very useful as surfaces for the algorithm. A property that enhances the estimation of the systematic errors is the distribution of the surface slopes that are being used either as control or tie entities. An even distribution of slopes helps in reducing the similarity in effect, and consequently the correlation, between the different parameters. For the system errors that are assumed constant throughout the mission such as the mounting bias and the laser system related errors the distribution of surfaces that are being considered for adjustment is of less importance. Estimation of time dependent errors such as the drifts requires the distribution of



Figure 2: Analysis of slopes – Dark surfaces have slopes  $> 20\%$ , gray slopes  $< 20\%$

the surfaces along the strips in order to obtain sufficient time variations. The flight configuration also plays an important role as not all errors can be recovered under any flight configuration. As an example consider recovering the heading bias, under a roundtrip pattern and no control surfaces an error in the heading direction is unnoticeable, to recover this error flight patterns that include cross strip(s) are required.

The analysis of the type of control that is adequate to a surface based implementation is of great interest as not all type of control can be achieved by simple means. The surprising result is that the algorithm is not very sensitive to prior knowledge of the surface slopes, therefore, while control surfaces or lines are useful entities for the strip adjustment control points that fix the surfaces in space are the more important entities. Experiments have shown that even when the process is begun with an inaccurate estimation of the surface slopes convergence to their actual values is achieved. Control points should be distributed over different surfaces and at least three points are needed in order to register the strip to the reference coordinate system.

##### 4.1 Analysis

As was mentioned in Section 3.2 an evaluation of the surface slopes distribution prior to applying a segmentation allows to apply a rather selective segmentation of the data. Figure 2 presents the results of a simple slope based thresholding of the laser points with slope bigger than 20%. For the elimination of non-surface points only points that met a predefined fitting criterion to the local surface fitted to their surrounding are considered. Segments are then formed by the clustering based segmentation. The existence of steep objects enhances the error estimation, yet even milder slopes provide good estimation of the systematic errors. Notice that the steep surface slopes arrive not only from roof faces but also from the canal banks (passes by the houses) and the sloping terrain on the left. (Filin, 2001) shows that the distribution of slopes in different direction reduce the correlation among the estimated parameters. In the current example the orientation of the houses in different directions as well as the orientation of the canal banks satisfy this condition.

In forming the segments (see result in Figure 1) two parameters are set to control the process, both are accuracy criteria. One parameter defines the upper threshold for the standard deviation (std.) of a segment and the other defines the lower std. threshold. While segmentation is usually associated with detection of structure in the data, here the accuracy of the segment, measured in terms of proximity of the laser points to the extracted surface, is the major criterion. As a lower bound a  $\pm 5\text{cm}$  std. value was chosen and as an upper bound  $15\text{cm}$ . The lower bound is set with the approximated ranging accuracy in mind and is aimed at avoiding undersegmentation. The upper bound is set so that the residuals of the

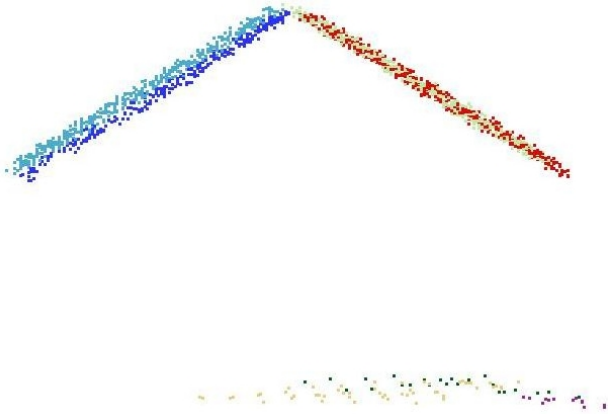


Figure 3: Effect of systematic error on shape of buildings along the flight path (with roof faces from both datasets presented)

points from the surface will still be within a reasonable range allowing one to assume that they are indeed reflected from that actual surface. Experience shows that most of significant surfaces have a std. smaller than 10cm, and segments with a few thousands of points are not a rare scene, both aspects imply that the bounds that are set are not too strict. The results in Figure 1 show that with the criteria set the segments managed to capture the significant structures in the data (e.g., roof faces) and that the size of the segments is relatively big. Bigger segments introduce a much stronger constraint for the laser points and enable associating points from overlapping strips with a higher degree of confidence. This feature reduces complexity of matching points from other strips with the segments (the correspondence problem.)

Evaluation of differences shows height differences in the flat areas in the order of 10cm, however the more significant type of effects appear with the building across the flight path (see Figure 1.) The systematic errors, presented in Figure 3, show a positional offset between the roofs of the two buildings. If errors are measured in a naive fashion by comparing height differences this offset will be translated into height error with a different magnitude than the ones on the flat terrain. A 1D strip adjustment solutions, (e.g. Crombaghs et al., 2000) will fail removing these type of artifacts. A closer inspection of Figure 3 shows that the slopes of left roof face in both dataset seem different. An analytical computation of the slopes of the roof faces in both datasets shows that in fact the roof faces in one set are rotated by about  $0.4^\circ$  from the ones in the other set. One potential explanation for that is the existence of a bias in the pitch direction. Filin et al. (2001) have shown that a mounting bias in the pitch direction has the effect of changing the slope of the reconstructed surfaces for sloped surfaces but not changing the slope for horizontal surfaces. Error recovery solutions that are not based on system error modeling will fail recovering this type of artifact.

## 5 Concluding remarks

Reaching the potential accuracy of laser data and eliminating artifacts requires the removal of systematic errors from the data. A strip adjustment formulation enables removing errors that were not properly eliminated before takeoff and others

that occurred during the mission. A system driven solution enables modeling and removing the actual errors in the system. The proposed model offers a natural way for eliminating the errors as it constrains the laser points to the surface. A surface based model enables using general topography and natural and man-made surfaces for the adjustment and do not require distinct objects in the overflow region. An analysis shows that the estimation of the errors by this model and under very general configurations can be accurate and reliable.

## REFERENCES

- Burman, H. (2000). Adjustment of laser scanner data for correction of orientation errors. *International Archives of Photogrammetry and Remote Sensing*, **33**(B3/1): 125–132.
- Crombaghs, M., E. De Min and R. Bruegelmann (2000). On the Adjustment of Overlapping Strips of Laser Altimeter Height Data. *International Archives of Photogrammetry and Remote Sensing*, **33**(B3/1): 230–237.
- Filin, S., (2001). Recovery of systematic biases in laser altimeters using natural surfaces. *International Archives of Photogrammetry and Remote Sensing*, **34**(3–W4). Annapolis, MD, 85–91.
- Filin S., Bea Csatho, Toni Schenk, (2001). An analytical model for in-flight calibration of laser altimeter systems using natural surfaces, Technical Papers of the 2001 convention of the American Society of Photogrammetry and Remote Sensing, St. Louis MS. Published on CD-ROM.
- Filin, S. and B. Csathó (1999). A Novel Approach for Calibrating Satellite Laser Altimeters. *International Archives of Photogrammetry and Remote Sensing*, **32**(3–W14): 47–54.
- Kager, H. and Kraus, K. (2001). Height discrepancies between overlapping laser scanner strips. Proceedings of Optical 3D Measurement Techniques V, October, Vienna, Austria: 103–110.
- Kilian, J., N. Haala and M. English (1996). Capture of Elevation of Airborne Laser Scanner Data. *International Archives of Photogrammetry and Remote Sensing*, **31**(B3): 383–388.
- Maas, H. G. (2000). Least-Squares Matching with Airborne Laserscanning Data in a TIN Structure. *International Archives of Photogrammetry and Remote Sensing*, **33**(B3/1): 548–555.
- Ridgway, J. R., J. B. Minster, N. Williams, J. L. Bufton and W. B. Krabill (1997). Airborne Laser Altimeter Survey of Long Valley California. *Int. J. Geophysics*, **131**: 267–280.
- Schenk, T. (2001). Modeling and analyzing systematic errors of airborne laser scanners. *Technical Notes in Photogrammetry* No. 19, Department of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus, OH., 40 pages.
- Vaughn, C. R., J. L. Bufton, W. B. Krabill and D. L. Rabine (1996). Georeferencing of Airborne Laser Altimeter Measurements. *Int. J. Remote Sensing*, **17**(11): 2185–2200.