A METHOD TO PREDICT ACCURACY OF LEAST SQUARES SURFACE MATCHING FOR AIRBORNE LASER SCANNING DATA SETS

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ABSTRACT:

The registration of data sets in the same coordinate system is an essential task in the fusion of data obtained from similar or different sources. The determination of the registration accuracy by commercial firms is often undertaken by comparing points of one surface to heights interpolated between points of the other surface. The last few years have seen research undertaken in the development of surface matching algorithms, which automate the registration. High redundancy is achieved with these algorithms as each point of one surface can potentially participate in the formation of a normal equation for the least squares adjustment. These algorithms may be expected to become an important tool of data fusion, especially with data acquisition techniques such as ALS which lack thematic information. This paper demonstrates that the accuracy of the registration, both in height and plan positioning, as well as the accuracy of the matching (the residuals of the least squares matching) are related to the density of the reference data. A cross-validation method is used to find the accuracy of the registration and the mean of the matching residuals for different densities of the reference surface. The paper presents two experiments: with artificial data and with ALS data. The reference data is similar in the two instances: the data is evenly distributed and is triangulated using a Delaunay triangulation. Furthermore the paper shows that the curves generated by a cross-validation method can be modelled on hyperbolic functions. These functions are used to predict accuracy for densities higher than that used for the validation. The accuracy of the predictions ranged from sub-millimetre values to 4mm for both experiments. The accuracy of the registration of the ALS data where the differences ranged from 9mm to 134mm was the exception. In this case however the accuracy improved with increasing density.

1 INTRODUCTION

One of the tasks in the processing of spatial data is its registration in an appropriate coordinate system. The change of coordinate system requires a conformal transformation. In photogrammetry, which provides thematic information on the data, the (6 or 7) parameters of the transformation are estimated in a least squares adjustment. The adjustment fits the captured data onto a set of ground control points (GCPs), which are identifiable on the stereopair. The two functions of the GCPs are to allow the orientation of the stereopairs and to facilitate the registration of the data in a chosen coordinate system (Wolf and Dewitt, 2000). The production of GCPs can add significantly to the cost of a survey, especially in rugged country, as surveyors have physically to go on site to mark the ground and assess the coordinates of the marks.

An alternative method is to use a surface registered in the wanted coordinate system to realise the absolute orientation of the stereopair (Mills et al., 2003). This data set is referred to as the reference surface in the rest of this article. The method, called surface matching, involves fitting the two sets by means of a least squares fit which minimises the separation between the surfaces. High redundancy is achieved as a normal equation of the least squares adjustment can potentially be generated for each point of the set.

Similar methods have been used in strip adjustment of data acquired by airborne laser scanning (ALS) (Maas, 2000), and in the fusion of photogrammetric and ALS data (McIntosh et al., 2000). Surface matching without control points is also used in change detection (Mitchell and Chadwick, 1999).

This paper examines the possibility of anticipating the accuracy of surface matching given the density of the reference surface. The method also allows prediction of density requirements of the reference data to achieve a given accuracy.

The project was originally inspired by the Newcastle City Council (NSW, Australia). The Council acquired large amounts of data from 1999, approximately 2 million data points in total, to produce a DEM for a flood study of the area. The accuracy of the data was assessed by comparing the photogrammetric data to approximately 10000 points of existing ground survey data. It was foreseen that surface matching without control could be used to assess the accuracy of the photogrammetric data (Mitchell et al., 2002), by matching the photogrammetric data to the ground survey data and checking the r.m.s. of the matching.

The matching algorithm fits a cloud of points to a facetted surface, and not to the real surface. The facetted surface is a triangulated model. Its faithfulness to the real surface is related to the sampling spacing of the data and to the roughness of the terrain. The use of synthetic data allows verification of the accuracy of the matching to the real

surface by calculating the difference in height between the matched points and that real surface. It also permits testing of the ability of the algorithm to position the cloud of points accurately. This paper develops a technique for applying these checks to real data. The method uses a validation technique to model the accuracy of the matching to the density of the data.

2 THE TOOLS OF THE EXPERIMENT

2.1 The matching algorithm
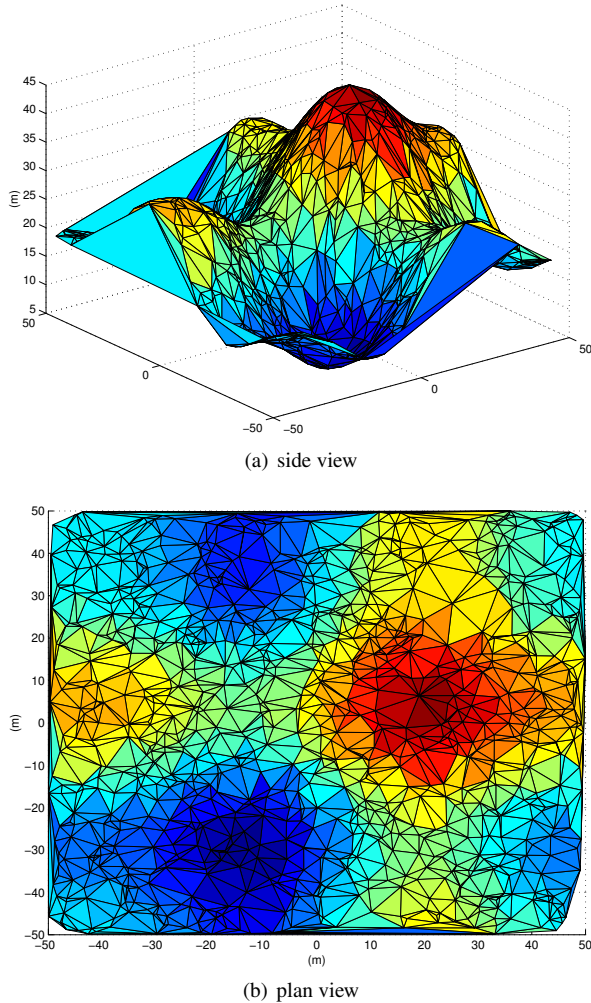

(a) side view

(b) plan view

Figure 1: Reference surface

The matching algorithm adopted for this experiment has been developed and is still being improved at the University of Newcastle, Australia. The reference data is triangulated using a Delaunay triangulation. The advantage of the triangulation is that all the points of the surface are honoured and second generation interpolation is avoided. The only interpolation occurs during the matching when the points of the second surface are interpolated in between the points of the reference surface. The Delaunay algorithm, amongst other methods, has the property that no points in the data set fall inside the circumcircle of any of the triangles formed. The circumcircle is the circle passing through the three vertices of a triangle (Sloan, 1993). The method produces a unique triangulation and the triangles formed

are as equilateral as possible, avoiding long-shaped triangles, whenever possible. The iterative algorithm minimises the normal distances of the elements of the point cloud to the patches of the reference surface, in an algorithm similar to that described by Schenk et al. (2000).

2.2 Data generation and controlled matching

Two experiments were conducted: one involving synthetic data, the other ALS data. The set of tools used in the first experiment is described below.

2.2.1 Data generation The surfaces generated represent a surface of area 100m by 100m. For simplicity, the reference surface and the cloud of points are named respectively S1 and S2 in the paper.

Both data sets are generated in the same manner: the heights of the points are computed as a function of the planimetric coordinates obtained randomly and uniformly distributed. The points of the two data sets lie on the same surface, but do not correspond. The reference surface S1 is then triangulated. A side and plan view of S1 is shown in Figure 1. The second set S2 is then transformed to create some separation between the two surfaces. The matching process can then take place.

2.2.2 Controlled matching The term "controlled matching" refers to the fact that the original coordinates of S2 are known, as well as the transformation parameters used to create a separation between the surfaces (referred to as "initial transformation" in the rest of the text, and as opposed to "matching transformation", the reverse operation). Effectively the differences between the parameters of the initial transformation (chosen by the user) and the parameters of the matching (resulting from the least squares fit and bringing back the surface to its original position before the initial transformation) are only an approximate indicator of the matching success, and are not a reliable measure of the closeness of the fit: the effect of the differences between three rotations, three translations and a scaling factor (i.e.: $\omega_{initialtransf} - \omega_{matchingtransf}$, $\phi_{initialtransf} - \phi_{matchingtransf}$, etc...) happening simultaneously are difficult to assess and interpret.

The significance and extent of this mismatch can be more readily assessed by calculating the distance between the points in their original position (before the initial transformation) and the points after matching. This is therefore a measure of the ability of the matching to bring back the points to where they belong, or where they originally came from. A mean and standard deviation of these distances can then be calculated. This check is referred to as the mean mismatch in the results shown in next section.

It must be mentioned here that the least squares matching algorithm does not actually bring back S2 to its original position exactly, as it matches S2 to a facetted model of the real surface. The difference between the parameters of the separation transformation and of the matching could thus be interpreted as a measure of the validity of the triangulated model, or in other words as a measure of the faithfulness of the triangulated model to the true surface.

3 EXPERIMENT WITH SYNTHETIC DATA

3.1 Determination of accuracy of matching as a function of density

A reference surface S1 of 7000 points and a cloud of points S2 were generated for the experiment. In order to generate an experimental curve of the matching results versus the density (actually the number of points), a cross-validation method was used. A number of points corresponding to one twentieth (350 points) of the total number of points of the reference surface (7000 points) was randomly picked out of the reference set. Matching of the surface S2 to the thinned-out reference surface S1 (now 6650 points) was then undertaken. The operation was repeated a number of times (20 in this experiment) with the same number of points (350 points) deleted from S1, but the points deleted were different and picked out randomly each time.

The average of the 20 results and the standard deviation of the results were then plotted against the number of points (6650 points), as the mean of the mean residuals and an error bar representing 67% confidence.

The same number of matches (20) was then undertaken with a number of points deleted from S1 corresponding to twice the preceding amount, or two twentieths of the total number of points (700 points). The results were plotted as previously. The validation was stopped after the 20th matching of S1 thinned out by nineteen twentieths of its points (350 points remaining).

The same computations were undertaken to obtain a mean of the mean mismatches for different densities. The curves obtained through the validation are shown in Figure 2.

Both curves are smooth and indicate that the mean of the residuals and the mean mismatch diminish as the density was increased. Assuming that the surface S2 was free of outliers and noise, it can be assumed that the means tended to zero as the density tends to infinity.

3.2 Experiment results

A mathematical model was fitted on each experimental curve. The models belong to the hyperbola family:

$$Accuracy = \frac{1}{Density^k}$$

Additional parameters a, b, c, d and e to allow for scaling and translations of experimental values are found by least squares:

$$(a.accuracy + b) = \frac{c}{(d.density + e)^k}$$

Fitting results are shown in Figure 3. The experimental curves with fitted models are shown in Figure 3(a), while the residuals of the fittings are shown in Figure 3(b).

The models were used to predict the accuracy of the residuals and the mismatch for reference surfaces with numbers of points ranging from 8000 points to 15000 points.
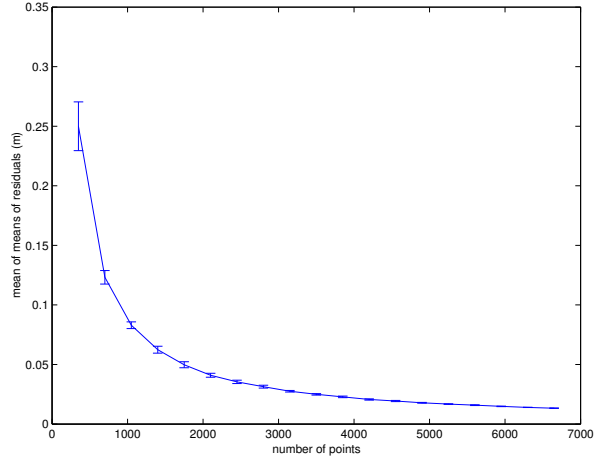
(a) mean of mean residuals vs number of points
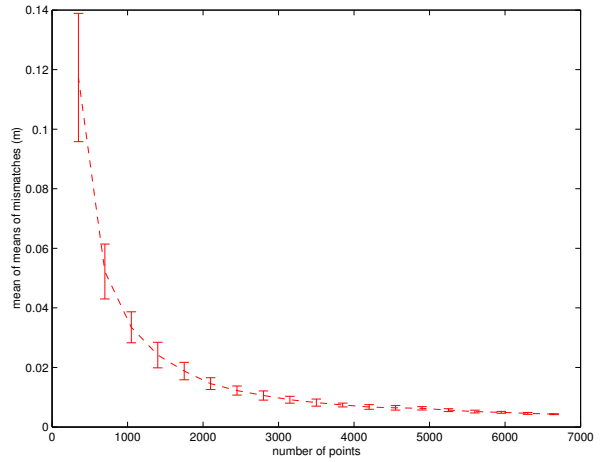
(b) mean of mean mismatches vs number of points

Figure 2: Synthetic data cross-validation

	Mean matching residuals (m)		
No of points	predicted	obtained	Δ
8000	0.0098	0.0111	-0.0013
9000	0.0083	0.0098	-0.0015
10000	0.0071	0.0086	-0.0015
12500	0.0049	0.0065	-0.0016
15000	0.0034	0.0053	-0.0019

Table 1: Predicted and experimental means of residuals obtained with synthetic data

	Mean mismatches (m)		
No of points	predicted	obtained	Δ
8000	0.0031	0.0039	-0.0008
9000	0.0026	0.0034	-0.0008
10000	0.0022	0.0026	-0.0004
12500	0.0015	0.0020	-0.0005
15000	0.0011	0.0017	-0.0006

Table 2: Predicted and experimental mismatches obtained with synthetic data
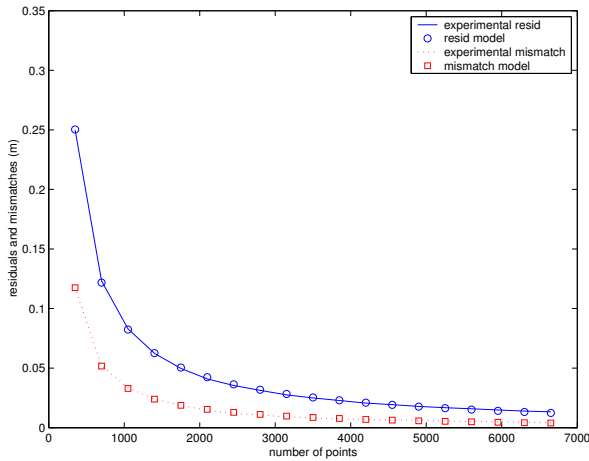
(a) experimental and fitted curves
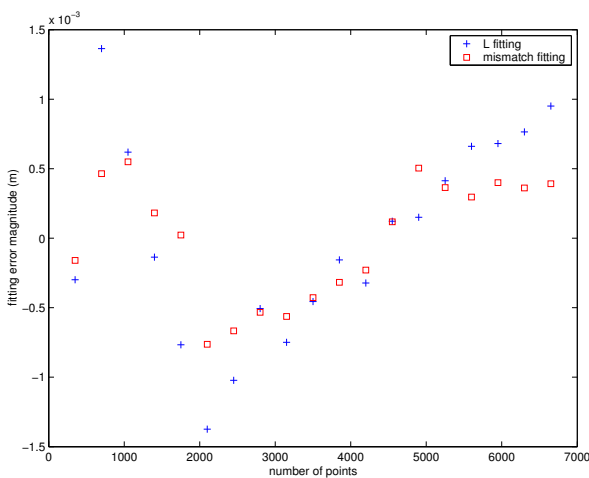
(b) residuals of fittings

Figure 3: Experimental and fitted curves for synthetic data

The comparison between predictions and matching results is shown in Table 1 and Table 2.

From both sets of predictions, it can be concluded that the matching accuracy to both the facetted surface and the real surface can be modelled on hyperbolic functions. The differences between predicted and matched means of the residuals do not exceed 2mm for the densities tested, while the differences in the mean mismatches are all in the sub-millimetre range.

4 APPLICATION TO REAL DATA

4.1 Adaption of validation technique to real data

The data used was part of an ALS survey of the city of Newcastle undertaken by a private surveying firm. An area of 250m by 250m was selected from a larger data set. The data had already been filtered to show only ground points. The set had distribution characteristics similar to those of the synthetic data: the points were not on a regular grid but were evenly distributed across the terrain. The set is shown in Figure 4. The presence of larger triangles indicates the presence of buildings before filtering.
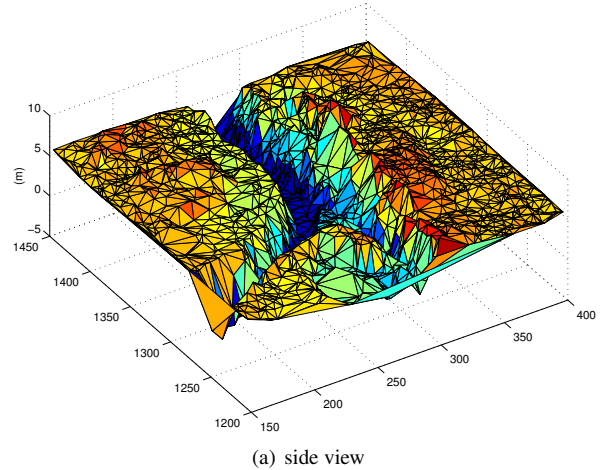

(a) side view
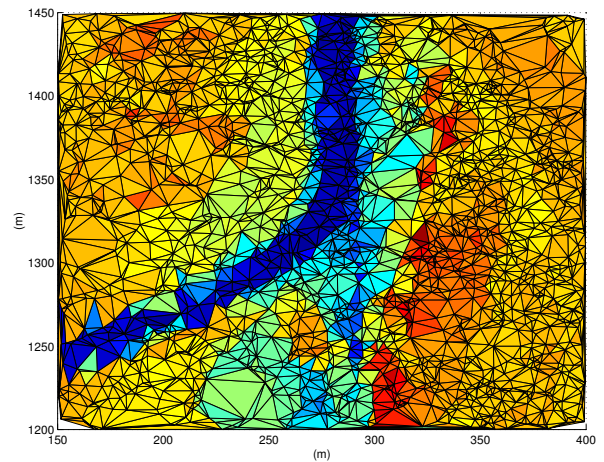
(b) plan view

Figure 4: ALS data

Using real data presented several challenges to the validation method. Firstly, two surfaces are needed for matching. Secondly, and unlike the case of synthetic data, the coordinates of matched points cannot be assessed for the accuracy of their positions. The sample data chosen comprised 15836 points.

A solution to the problem was to randomly extract approximately 2000 points (1976 points exactly) from the original set to form the surface S2. The points of S2 are evenly distributed across the entire surface. The rest of the points (13860 points) were thinned out to create sets of 12000, 11000, 10000, 9000, 8000 and 7000 points. The 7000-point surface was then used to generate an experimental curve of the means of the matching residuals versus the number of points. To undertake this task, the 1976-point surface S2 was first transformed to create a small separation between the two surfaces.

This in turn allowed for a curve of the means of the mismatches to be generated. Effectively, and as with the synthetic data, the coordinates of the matched points of S2 could be compared to that of the same surface before the initial transformation. The generation of the curves involved running the matching program 40 times for each density of the surface S1, as opposed to 20 times in the

previous experiment. The validation of the ALS data is shown in Figure 5.
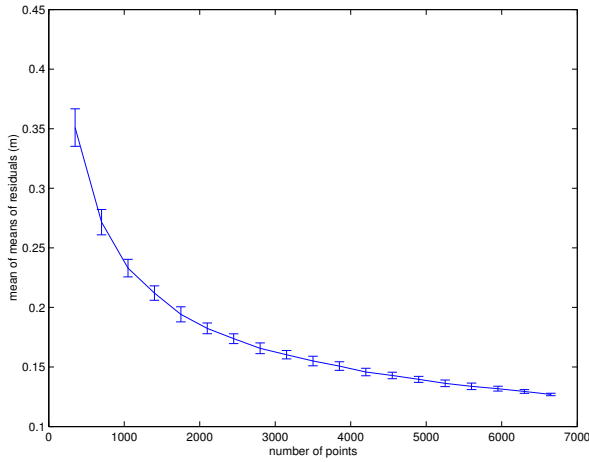
(a) mean of mean residuals vs number of points
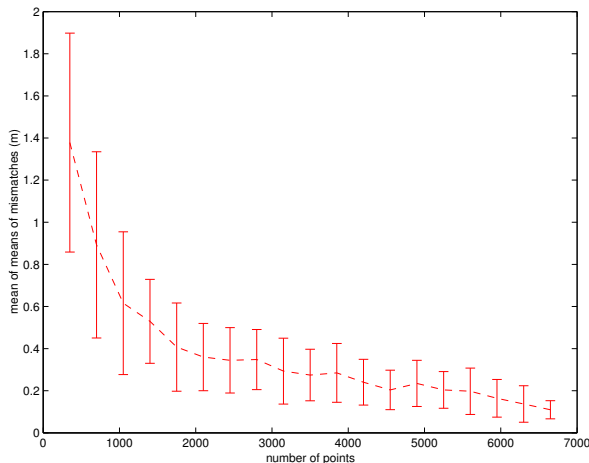
(b) mean of mean mismatches vs number of points

Figure 5: ALS data cross-validation

4.2 Experiment results

Figure 5(a) shows that the curve of the means of the matching residuals is similar to the one obtained with synthetic data (Figure 2(a)). However, the curve of the means of the mismatches is not as smooth as with the synthetic data, and displays large error bars (Figure 5(b)).

As in Section 3, curves were fitted on the experimental validation curves, as shown in Figure 6. The fitting of a hyperbolic model to the means of the mismatches was not as good as with synthetic data: Figure 6(b) shows that the fitting residuals were approximately one order of magnitude larger than with synthetic data.

Finally, predictions were made using the hyperbolic curves to assess the mean of the residuals of matching and the mean of the mismatches for given densities (number of points) exceeding the 7000-point surface used for the validation. The predictions were then checked by matching the surface S2 to reference surfaces of the same densities. The comparisons are tabulated in Table 3 and Table 4.
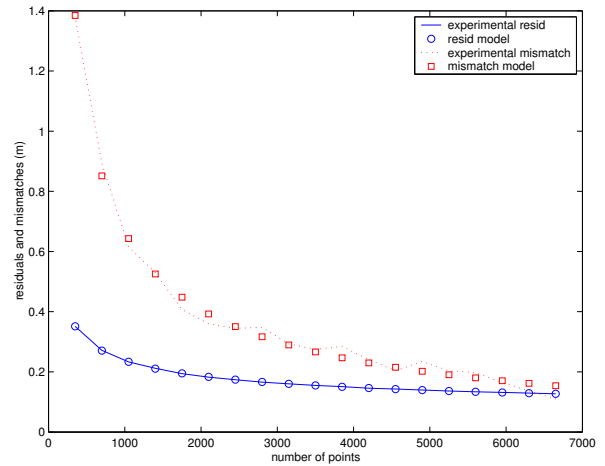
(a) experimental and fitted curves
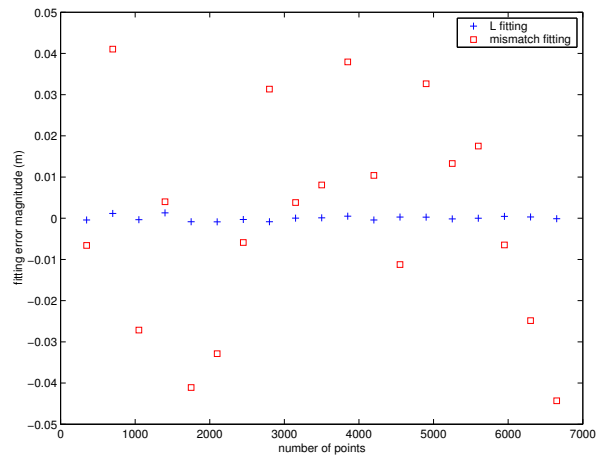
(b) residuals of fittings

Figure 6: Experimental and fitted curves for ALS data

The predictions of the mean of the matching residuals were accurate to the sub-millimetre in all the cases but two (9000 and 10000 points), where the differences between predicted and matched were approximately 4mm. By contrast, the predictions for the mismatch do not reflect the results obtained by the matching program in an accurate manner, with differences between predicted and obtained ranging from 9mm to 134mm. The mismatch results obtained for a density of 11000 points highlight the fact that the closest fit for the least squares adjustment does not always correspond to the best registration, due to the fact that the surface fitting uses a triangulated model of the true surface, and not the true surface. The correspondence improves

	Mean matching residuals (m)		
No of points	predicted	obtained	Δ
8000	0.1204	0.1213	-0.0009
9000	0.1164	0.1208	-0.0044
10000	0.1130	0.1166	-0.0036
11000	0.1100	0.1097	+0.0003
12000	0.1074	0.1080	-0.0006
13860	0.1034	0.1025	+0.0009

Table 3: Predicted and experimental means of residuals obtained with ALS data

	Mean mismatches (m)		
No of points	predicted	obtained	Δ
8000	0.1288	0.0306	+0.0982
9000	0.1142	0.1575	-0.0433
10000	0.1019	0.1154	-0.0135
11000	0.0914	0.2260	-0.1346
12000	0.0824	0.1264	-0.0440
13860	0.0683	0.0593	+0.0090

Table 4: Predicted and experimental mismatches obtained with ALS data

with the density of the reference surface, as the model becomes a better representation of the true surface. The most accurate prediction is obtained with the denser data set (13860 points).

Many parameters can influence matching results and matching predictions. Two possible explanations that come to mind to justify the difference between predicted and obtained mismatch are that the reference data used for the validation is not dense enough, nor is the data used for the testing. This resulted in poor confidence in the means of the mismatches. This would explain why the best prediction is obtained with the denser data set. Another explanation is the lack of texture of the data. Software which minimises the heights along the z axis is more appropriate to match surfaces in flat terrain, while better matching is obtained in hilly terrain with algorithms that minimise normal distances (Schenk et al., 2000).

5 CONCLUSION

Experimental curves which relate accuracy of matching to the density of the reference data were generated, using a validation method. Matching a cloud of points to a reference surface whose density is changed repeatedly by eliminating randomly chosen data allows plotting of mean results and confidence error bars as a function of the density of the reference data. Two curves were generated with the same data set. One was the mean of the means of the surface matching residuals versus the density. The second one relates the mean of the mismatches to the density. The term mismatch denotes the difference between the spatial position of a matched point and its true position in the surface.

The experimental curves were modelled as curves of the hyperbolic family. The mathematical models obtained were used to predict accuracy for denser reference data than that used for the validation.

Two experiments were undertaken. The first one used artificial data. The second one was undertaken with an ALS data patch. The curves obtained with the validation of the artificial data were successfully fitted with hyperbolic curves, with small fitting residuals for both curves. The differences between predictions and matching results were under 2mm for the mean of the means of the residuals and under 1mm for the mean of the mismatches.

The curves generated in the second experiment were also modelled as hyperbolic functions. The hyperbolic model

proved to be an efficient prediction tool for the mean of the surface matching residuals: 4 out of 6 predictions were within one millimetre of the matching results. The accuracy of the predictions of the mismatch were ranging from 9mm to 134mm, and were therefore not as accurate as the other predictions. The predictions improve with increasing density to the lowest difference (9mm) at the highest density (13860 points). The difference between the registration accuracy of both experiments can be attributed to the difference of roughness of the terrains: the topography of the ALS data is rather flat and the lack of texture results in the automatic registration needing a denser reference set.

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