PHOTOINTERPRETATION AND SMALL SCALE STEREOPLOTTING WITH DIGITALLY RECTIFIED PHOTOGRAPHS WITH GEOMETRICAL CONSTRAINTS $^{\rm 1}$

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KEY WORDS: Architectural photogrammetry, self-calibration, rectification, stereoscopy, vanishing points

ABSTRACT

It is well know that it is possible to use the vanishing point geometry to assess the orientation parameters of the photographic image. Here we propose a numerical or graphical procedure to estimate such parameters, assuming that in the imaged object are present planar surfaces, straight-line edges, and right angles. In addition, by means of the same estimated parameters, it is possible to project the same image onto a selected plane say to rectify the image. The advantages are that non-metric images, taken from archives or books also, provided a good geometry and quality, are suitable for the task. From one side the role of the classical line photogrammetry is taken more and more over by laser scanning, and on the other side, this simplified procedure enables the researcher to use photogrammetric techniques for stereoscopy and interpretation, thematic mapping, research. The convergent non-stereoscopic images rectified with digital photogrammetric techniques, are then made suitable for stereoscopy. In fact the rectification corrects for tilt displacement and not for relief displacement, but it is just relief displacement that enables stereoscopy. Some examples of stereoplotting and photo-interpretation with digitally rectified photographs are shown.

1 INTRODUCTION

The possibility to use non-metric images is normally linked to the availability of the control information in the imaged object, usually control points to be input in a bundle adjustment procedure or DLT. There is also the possibility to use a-priori knowledge of the geometry of the object, such as parallelism of lines and perpendicularity of planes (Williamston & Brill, 2, 1987,3, 1988, Ethrog, 8,1984, Krauss, 5, 1997, Van Heuvel, 7, 1999). In this way the study and the safeguard of the monuments can be helped by the biggest existing archive available to the researcher, the books. We propose an approach that has the advantage to be particularly simple and to be performed also in a graphical way by people not particularly expert in complicated computations. It uses the vanishing points geometry. We assume the geometry of the ideal pinhole camera. So far, the distortion is neglected. With the same estimated parameters it is possible to rectify the image and produce a stereo-couple from convergent non-stereoscopic photographs. Many computers have already main-board and graphical card suitable for stereoscopy (stereo-ready). In addition, the digital workstations do not have a device such as dove-prism to rotate the optical field and to improve stereoscopy; therefore a good stereo-capability is more important than before.

2 THE VANISHING POINT GEOMETRY

In an image the parallel object lines converge in points called Vanishing Points. There are many methods for the detection of the Vanishing Points in the images. A good review is given by (V.Heuvel, 6, 1998), together with a proposal for a new approach, based on strong statistical base. Here the detection of the vanishing points will not be discussed.

2.1 Perspective transformation with one vanishing point

The transformation matrix, composed by a perspective followed by a projection onto Z =0 plane, is written: $\begin{bmatrix} 1 & 0 & 0 & n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & n \end{bmatrix}$

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ l & m & 0 & 1 \end{bmatrix}$$
(1)

In the case that l = m = 0 (or we are not interested in the translation l, m), an arbitrary point P with homogeneous coordinates [x,y,z,1] is transformed or projected, in

$$[x,y,z,1]. \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [x,y,0,(x.p+1)] = \begin{bmatrix} x & y \\ x.p+1 & x.p+1 & 0 & 1 \end{bmatrix} = [X, Y,0,1]$$
(2)

When we have the image and we want the estimate the value of the parameter p, we can use the vanishing point in direction of X axis that is transformed:

¹ The present work has been financed by Cofin2001, Italian Ministry for Scientific Research

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \end{bmatrix} = \begin{bmatrix} 1/p & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_{PF1} & 0 & 0 & 1 \end{bmatrix}$$
(3)
$$p = 1/X_{PF1}$$
(4)

$$p = 1 / X_{PF1}$$

Once the image co-ordinate X_{PF1} of the vanishing point is determined, in a reference system where $Y_{PF1} = 0$, it is possible to rectify the image with eqns.(2), by means of the parameter p. In fact the inverse transformation of (2) is obtained from eqns. (2) and isolating the x and y

$$x = \frac{X}{1 - p.X}$$
$$y = \frac{Y}{1 - p.X}$$

(5)

This procedure is useful when we want to rectify an image with one vanishing point only. The case is very frequent with the images taken from books of architecture, where normally the lines, vertical in the reality, last vertical in the image.



Fig. 1 – One Vanishing Point Image



Figs. 2, 3 - Florence – S.Miniato al monte. The original one VP image and the rectified one

The ratio base/height depends on the translation m, that can not be determined. Therefore the rectified image has two unknown different scale factor.

2.2 Perspective transformation with two vanishing points

The transformation in this case is:

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(6)

An arbitrary point P[x,y,z,1] is then transformed in $\begin{bmatrix} 1 & 0 & 0 & p \end{bmatrix}$

$$[x,y,z,1] \cdot \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [x,y,0,(x,p+y,q+1)] = \begin{bmatrix} x & y \\ x,p+y,q+1 & x,p+y,q+1 & 0 & 1 \end{bmatrix} = [X,Y,0,1]$$
(7)

The two vanishing points of the X and Y axes, are projected:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \end{bmatrix} = \begin{bmatrix} 1/p & 0 & 0 & 1 \\ 0 & 1/q & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_{PF1} & Y_{PF1} & 0 & 1 \\ X_{PF2} & Y_{PF2} & 0 & 1 \end{bmatrix}$$
(8)

from where we get:

$$\begin{cases} p = 1/X_{PF1} & \{Y_{PF1} = 0 \\ q = 1/Y_{PF2} & \{X_{PF2} = 0 \end{cases}$$
(9)

Determinate the image co-ordinates image X_{PF1} and Y_{PF1} of the two vanishing point, *p* and *q* parameters are solved. The inverse transformation is obtained by developing (7) and ordering with respect to *x* and *y*

$$\begin{cases} (pX - 1).x + qX.y = -X \\ pY.x + (qY - 1).y = -Y \end{cases}$$
(10)



Fig. 4 – Two Vanishing points image

This procedure is useful when we want to rectify an image with two vanishing points only, using eqns (7 and (10), in a reference system where $Y_{PF1} = 0$ and $X_{PF2} = 0$.



Figs. 5, 6 - Assisi – Basilica of S. Francesco – Original -two VP- image of the façade - The rectified image, modified to fit the rose window in a circle

3 PERSPECTIVE TRANSFORMATION WITH THREE VANISHING POINTS

Let a reference system with horizontal X and Z-axes, and vertical Y-axis be fixed. The point of view M is placed along Z-axis at a distance *Zc* from the origin (fig. 1). The parallelogram ABCDEFGH is then rotated, translated and finally projected from M in a plane parallel to XY plane. The resulting image is shown in fig. 2); it is possible to detect three vanishing points, according to the three orthogonal directions. This is very often the case of images in architecture. Of course in a normal picture there are as many V.P. as many sets of parallel lines, but we restrict only to the case of the rectangular solid in fig. 1. A general conform transformation in 3d space can be performed by the concatenation of the three rotations tilt *k*, azimuth φ , and swing ϑ about the axes of the system and three translations *m*, *n*, *p* along three principal directions, (Roger, Adams, 1, 1980). [T₁] = [R_Z] [R_Y] [R_X][T_{XYZ}] =

$$= \begin{bmatrix} \cos \mathbf{k} & \sin \mathbf{k} & 0 & 0 \\ -\sin \mathbf{k} & \cos \mathbf{k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \mathbf{j} & 0 & -\sin \mathbf{j} & 0 \\ 0 & 1 & 0 & 0 \\ \sin \mathbf{j} & 0 & \cos \mathbf{j} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \mathbf{J} & \sin \mathbf{J} & 0 \\ 0 & -\sin \mathbf{J} & \cos \mathbf{J} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix}$$
(11)

When in the image beyond the solid ABCDEFGH other parallelograms are present also, whose main planes are not parallel to those of the first one, we select the most suitable, that is the one with the longest possible lines per VP detection. In this case there are more than three VP. Suppose that k = 0. To let K be =0, first the three vanishing points are found in the original image. Then the whole image is rotated to let the y of the two horizontal VP1 and VP3 be equal, so that k=0. The line connecting VP1 and VP3 is the so-called



True Horizontal Line, THL. Note that in the previous case,- two VP point, VP1 and VP2 - , such k rotation cannot be determined and the directions of x and y axis are arbitrary. This is the reason why there are two different unknown scale factor in the two directions. A perspective transformation [T] is given by the concatenation of the previous transformation [T₁] followed by a perspective projection $[P_{rz}]$ from the point of view M.

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$$[T] = [T_1][P_{r2}] = \begin{bmatrix} \cos j & \sin j \sin J & -\sin j \cos J & 0 \\ 0 & \cos J & \sin J & 0 \\ \sin j & -\cos j \sin J & \cos j \cos J & 0 \\ l & m & n & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/z_c \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos j & \sin J & 0 & \frac{\sin j \cos J}{z_c} \\ \sin j & -\cos j \sin J & 0 & -\frac{\cos j \cos J}{z_c} \\ l & m & 0 & \frac{z_c - n}{z_c} \end{bmatrix}$$
(12)

For the determination of the values of ϑ , φ , *Zc* the three vanishing points can be used. We assume that they are the vanishing points of the reference axes. The perspective transformation of the three V.P. is then

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} T_1 \end{bmatrix} = \begin{bmatrix} x_{VP1} & y_{VF1} & 0 & 1 \\ x_{VF2} & y_{VF2} & 0 & 1 \\ x_{VF3} & y_{VF3} & 0 & 1 \end{bmatrix}$$
(13)

The image co-ordinates $[x_{VP}y_{VP},0,1]$ can be found, manually or automatically, by intersection of the straight lines, or by many other methods (6, Van Heuvel, 1999). In this paper the image co-ordinates are reduced to the system with origin in the isocentre 0. The following relationships are then derived:

$$\frac{z_C \cdot \cos \mathbf{j}}{\sin \mathbf{j} \cdot \cos \mathbf{J}} = x_{PF1}$$

$$\frac{z_C \cdot \sin \mathbf{j} \cdot \sin \mathbf{J}}{\sin \mathbf{j} \cdot \cos \mathbf{J}} = y_{PF1}$$

$$-\frac{z_C \cdot \cos \mathbf{J}}{\sin \mathbf{J}} = y_{PF2} = \begin{cases} \frac{z_C \cdot \cot g \mathbf{j}}{\cos \mathbf{J}} = x_{PF1} \\ z_C \cdot tg \mathbf{J} = y_{PF1} \\ -z_C \cdot \cot g \mathbf{J} = y_{PF2} \\ -z_C \cdot \cot g \mathbf{J} = y_{PF2} \\ -z_C \cdot \cot g \mathbf{J} = y_{PF2} \\ -\frac{z_C \cdot tg \mathbf{j}}{\cos \mathbf{j}} = x_{PF3} \\ z_C \cdot tg \mathbf{J} = y_{PF3} \end{cases}$$
(14)

We get from there:

$$Z_C^2 = - y_{PF1} \cdot y_{PF2} \tag{15}$$

$$\boldsymbol{J} = a \tan \frac{y_{PF1}}{z_C} \tag{16}$$

$$\boldsymbol{j} = a \tan \frac{z_C}{x_{PF1} \cos \boldsymbol{J}}$$
(17)

$$\boldsymbol{j} = a \tan \frac{-x_{PF3} \cdot \cos \boldsymbol{J}}{z_C}$$
(18)

$$z_C^2 = -x_{PF1} \cdot x_{PF3} \cdot \cos^2 \boldsymbol{J}$$
(19)

For the computation of ϑ , φ , Z_C the V.P. image co-ordinates have been utilised; the parameters are invariant with respect to l, m, n. The Zc parameter can be regarded as the principal distance. The isocentre, i.e. the intersection on the three altitudes of the triangle formed by the three V.P., is then the principal point, say the origin of the axes. In fact the object vertical lines are not vertical any more in the projection plane, since they pass through the vertical VP2. The only object vertical line that remains vertical after the projection is the Y-axis, perpendicular to the THL horizontal line VP1-VP3. For symmetry the same holds for the other two altitudes in the triangle of the three vanishing points. Then the intersection of the three altitudes, the isocentre 0, is the intersection of the optical axis with the projection plane, that is, in the perfect pinhole camera model, the principal point. Note that we used only image co-ordinates and that the only control information from the object is the perpendicularity of the three main object plains and the horizontality and verticality of lines. The isocentre 0 and the principal distance Zc must be invariant with respect to the arbitrary selected solid ABCDEFGH. If we selected another solid, the vanishing points VP1 and VP2 would change, but THL, Z_C and isocentre 0, would remain the same. The angles ϑ , φ would be different. Note that a graphical-numerical solution is also possible:

- import the image in a CAD
- draw the intersecting lines to find the vanishing points, VP1, VP2, VP3,
- rotate the image to level the horizontal line VP1 VP3,
- draw the altitudes of the VP triangle,
- find the isocentre O, and reduce to it the image co-ordinates of the VP.,
- compute ϑ , φ , Z_C by (15), (16), (17), (18), (19).

A very common case is when the projection plane is parallel to the vertical line. In this case the rotation angle $\vartheta = 0$, the vertical VP2 goes to infinity, the isocentre 0 lies in the HL (horizontal line VP1 - VP3), the equations (14) reduce to one only with the remaining two unknowns Zc and φ . The procedure then fails. This is very often the case for images of buildings taken from books, since they have been produced with still life cameras that keep the negative plan vertical and shift the lens. So only eqns (4) and (5) can be used, relative to one V.P. case.

Finally for rectified images only one VP is finite, the remaining two are at infinity. The VP triangle reduces to one point only that coincides with the isocentre 0 and the finite VP.

3.1 The rectification with three VP image

Once ϑ , φ , $Zc_{,}$ are computed, it is not possible to apply the inverse transformation because in the projection on the plane z =0 there is a loss of information. Nevertheless the plane inverse transformation is possible, assuming as known one of the three object coordinates, or, that is the same, the equation of the new projection plane. The projection onto a co-ordinate plane is then a rectification. In fig.2, the rectified sides of the parallelogram, the rectangles ABCD, or BECF, or DCFG. Having set:

$$a_{1} = \cos\varphi; \qquad a_{2} = \sin\varphi \cdot \cos\vartheta; \qquad a_{3} = (\sin\varphi \cdot \cos\vartheta)/\mathbf{Z}c; a_{4} = 0; \qquad a_{5} = \cos\vartheta; \qquad (20)$$

$$a_{6} = -(\sin\vartheta)/\mathbf{Z}c; \qquad a_{7} = \sin\varphi; \qquad a_{8} = -\cos\varphi \cdot \sin\vartheta; \qquad a_{9} = -(\cos\varphi \cdot \cos\vartheta)/\mathbf{Z}c \qquad (20)$$
the projection of an arbitrary point from the 3D space onto Z=0 plane is:

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & 0 & a_3 \\ a_4 & a_5 & 0 & a_6 \\ a_7 & a_8 & 0 & a_9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} U & V & W & T \end{bmatrix} = \begin{bmatrix} (a_1x + a_4y + a_7z); & (a_2x + a_5y + a_8z); & 0; & (a_3x + a_6y + a_9z + 1); \end{bmatrix}$$

The normalisation is X = U/T; Y = V/T, that is to divide the first two co-ordinates by the third one:

$$\begin{cases} X = \frac{a_1 x + a_4 y + a_7 z}{a_3 x + a_6 y + a_9 z + 1} \\ Y = \frac{a_2 x + a_5 y + a_8 z}{a_3 x + a_6 y + a_9 z + 1} \end{cases}$$
(22)

(21)

Note that eqns. (22) are nothing but the homographic transformations. We reorder and isolate x and y as unknowns:

$$\begin{cases} (a_1 - a_3 X)x + (a_4 - a_6 X)y = X - (a_7 - a_9 X)z \\ (a_2 - a_3 Y)x + (a_5 - a_6 Y)y = Y - (a_8 - a_9 Y)z \end{cases}$$
(23)

Choosing for z an arbitrary value, x and y are then derived. The same holds when an equation of a plane is added. Note that the inverse transformation here is intended as rectification only, i.e. the projection into a plane Z=const. When Z varies, so does the

scale. The final scale of the rectified image is then unknown. In fig. 8 the three rectified rectangles have a side in common two by two, letting only a common scale factor to be established.

4. MUNICIPALITY HALL - PALAZZO DEL POPOLO - ANCONA

We show, as test, the survey of the Municipality Hall in Ancona, Italy (fig.3). First, we made a traditional photogrammetric survey. We used a calibrated camera Fuji GSW69 II, (plate format $6x9 \text{ cm}^2$) and a calibrated digital camera Fuji Finepix 4900 (resolution 1800x2400 pixel) (Fangi, 9, 1999). We determine 31 control points included the camera station points with centimetre accuracy. Then we apply the VP procedure to estimate the interior orientation parameters. In table 1 the comparison between the interior orientation parameters from the calibrations and the ones from the VP procedure are shown.

Table 1 - Calibration Vs Vanishing Points procedure – (pixel)						
			Calibrati	VP	Differences	
			on	Procedure		
1	Fuji GSW69	Х	956	1004	-48	-5.1%
	No dist. corr.	Y	659	657	+2	+0.3%
		С	1579	1569	+10	+0.6%
2	Fuji GSW69	Х	956	977	+21	+2%
	Dist. corr.	Y	659	680	-23	-3.3%
		С	1579	1599	-20	-1.3%
3	Rectified image	Х	-102	-104	+2	+1.9%
		Y	2359	2378	-19	-0.8%
		С	1137	1140	-3	-0.2%
4	Fuji Finepix4900	Х	1170	1177	-7	-0.6%
	No dist.corr.	Y	901	878	+29	+3.3%
		С	2484	2400	-84	-3.4%
5	Fuji Finepix4900	Х	1170	1200	-30	-2.6%
	No dist.corr.	Y	901	955	+54	+6.0%
		С	2484	2475	+9	+0.4%
6	Fuji Finepix4900	Х	1170	1194	-24	-1.4%
	Dist. corr.	Y	901	959	+58	+6.4%
		С	2484	2484	0	0%

In figs. 11 and 12 the rectified images for the main facade and the lateral façade are shown. These are derived from the same image. Although the lateral facade doesn't appear very attractive, the transformation is geometrically corrected. In the rectified images the principal point coincides with the only finite vanishing point: the self-calibration principal point has coordinates x = -102; y = 2359 while the transformed VP1 has co-ordinates x = -104; y =2378, and the difference is 2 pixel in x and 19 in y. From the rectified image the new principal distance cannot be derived from eqns. (5) or (9), since the image coordinates for VP1 are null. The differences of the orientation parameters range from -5,1% to +6.4%. The same test has been performed without lens distortion correction and after the correction. The distortion curve was obviously evaluated in advance (Fangi, 10, 1999). No significant improvement seems to derive from the correction of the lens distortion.



Fig. 9 - Calibration Vs Vanishing Points procedure. Differences in percentage for the orientation parameters.



Fig. 10 – Municipality Hall in Ancona – The original image ad the graphical solution.

Fig 11 – The rectified main facade



Fig. 12 – The rectified lateral facade



CONCLUSIONS

The present approach has some advantages.

First of all, it is simple, and a numerical-graphical solution is possible, allowing not very expert people to calibrate non-metric images. No measurement on the object is needed. Then last but not least, with the same parameters, it is possible to rectify the images, enabling the stereoscopy from non-stereoscopic images. The longer the lines of the three main directions, the better the determination of the VP. On the other side not high accuracy can be expected. Some assumptions on the objects are needed, such as perpendicularity of the main planes, and the presence of the horizontal and vertical lines, that often are not completely fulfilled, mainly for ancient buildings. Another possible shortcoming is the necessity of the presence in the image of three groups of lines, intersecting in three vanishing points, which is again not very often the case. In fact, unfortunately in the books of architecture almost any picture is made with still life cameras were the negative plane is kept vertical; the vertical vanishing point goes to infinity, making the procedure fail. Anyhow the described procedure permits to rectify the images, allowing stereoscopy from convergent non-stereoscopic imagery. While we are asking whether or not laser scanning will take the place of the traditional line photogrammetry, the simple and efficient tool of stereoscopy and photo-interpretation will last.

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