# QUICK SOLUTIONS PARTICULARLY IN CLOSE RANGE PHOTOGRAMMETRY

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**ABSTRACT:**

In this work we tried to solve some non-linear problems of analytic photogrammetry. These problems are also widely flowed into the digital photogrammetry. The model formation considering the solution of the Relative Orientation, and the object reconstruction considering the solution of the Absolute Orientation have non-linear functional models. These models need an exhaustive research of the preliminary values (of parameters) for the Relative Orientation, and a transformation of the parameters able to transform the problem of the Absolute Orientation in a simple linear problem. In model formation we used the coplanarity condition and even if we could not find the correct solution, we made (only for the Relative Symmetric Orientation) a research of all possible preliminary values and, among these values, we managed to select only 4 possible configurations. In the object reconstruction, using the Rodriguez matrix, we managed to solve a linear problem finding an exact solution. A wide experimentation, with many examples, gave us the expected results.

## 1. INTRODUCTION

We would like to suggest an easy way for close range photogrametric image orientations. This work tried to solve the non-linear problems typical in the analytic photogrammetry. It is well known that the Absolute Orientation and the Relative one have non-linear functional models. Therefore they need respectively a transformation of the parameters able to transform the problem of the object reconstruction in a simple linear problem and an exhaustive research of the preliminary values (of parameters) for the model formation. This way of working makes the orientation procedures much more flexibles, and permits wide applications also in close range photogrammetry.

Let us point out that close range photogrammetry is actually growing in importance. Indeed not only classical close range photogrammetry, e.g. for architectural and archaeological surveying as well as for industrial applications, but also data acquisition by equipped vehicles, and robots are nowadays typical data, largely diffused. Especially in the last cases, the need of real time (or quasi-real time) data processing and validation is very high. For these reasons, model formation and object reconstruction require the solutions of the problems of Relative Orientation and Absolute Orientation respectively, avoiding to waste time in an ‘a posteriori’ search of preliminary values.

## 2. FROM IMAGES TO OBJECT VIA MODEL

The main function of photogrammetry is the transformation of data from the image space to the object space. We can make this transformation in a direct way, with collinearity equations, or in two steps, with the formation of a model and, only in a second time, reconstructing the original object. First of all we have to take into consideration that:

- an image is not a map,
- at least two images are needed for reconstructing an object.

A relation of rototraslation with scale variation constitutes the links between the coordinates of the point Q (x, y, z), in an image, and the coordinates of the corresponding point P (X, Y, Z), in the object. Both reference systems are traditionally a Cartesian reference system, but the same is true, with minor changes, using different reference system, suitable linked to the previous ones. Let us show the above mentioned relation:

$$\begin{vmatrix} x^\circ \\ y^\circ \\ -c \end{vmatrix}_{ij} = \hat{\lambda}_{ij}\hat{R}_j \left( \begin{vmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{vmatrix}_i - \begin{vmatrix} \hat{X}_0 \\ \hat{Y}_0 \\ \hat{Z}_0 \end{vmatrix}_j \right) \quad (1)$$

where $x^\circ$, $y^\circ$, c = image coordinates and focal length
$\hat{X}_0, \hat{Y}_0, \hat{Z}_0$ = coordinates of projection center
$\hat{X}, \hat{Y}, \hat{Z}$ = object coordinates
$\lambda$ = scale factor, variable point by point

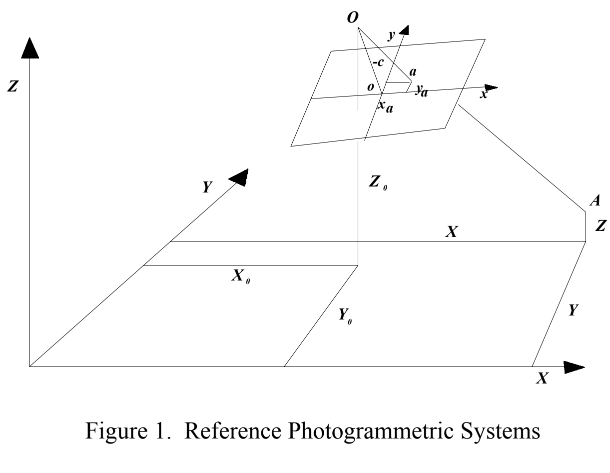



Figure 1. Reference Photogrammetric Systems

## 3. PROJECTION TRANSFORMATION

### 3.1 Parameters

The photogrametric technique is based on a transformation of a perspective (or a couple of perspectives) in a quoted orthogonal projection. In this transformation, we have non-linear parameters and, before starting the plotting, we need information about the preliminary value of those parameters. Our main aim is to find expressions working with parameters easy to be obtained. With two images, we can orient them; there are two different ways: a *'one step'* way, or a *'two steps'* one.

The first procedure is based on collinearity equations and needs *12 parameters*: $X_1$, $Y_1$, $Z_1$, $X_2$, $Y_2$, $Z_2$ (coordinates of the two projection centers), and $\omega_1$, $\varphi_1$, $\kappa_1$, $\omega_2$, $\varphi_2$, $\kappa_2$ (attituded angles of the two sensor). The second one separates the model formation (Relative Orientation) from the object reconstruction (Absolute Orientation). In this procedure, we define the problem of Absolute Orientation by means of 7 parameters: $t_x$, $t_y$, $t_z$ (shift vector), $\lambda$ (scale factor), $\Omega$, $\Phi$, K (Cardanic angles). On the contrary, to define the problem of Relative Orientation, we need 5 parameters: $\varphi_1$, $\kappa_1$, $\omega_2$, $\varphi_2$, $\kappa_2$ (Symmetric Relative Orientation), or $b_y$, $b_z$, $\omega_2$, $\varphi_2$, $\kappa_2$ (Asymmetric Relative Orientation).

**3.1.1 Absolute Orientation Parameters:** A rational alternative to classical Rotation Matrix is the *Rodriguez Matrix*. This matrix permits to find the exact solution, thanks to the solution of a linear system, after a suitable substitution of variables. We will briefly try to explain how to solve a linear problem. We start from rototraslation in the space (being *R* a rotation matrix and *t* a shift vector), with a global scale variation $\lambda$, we compute the expected value of all equations and then we subtract it from the previous equations:

$$\begin{array}{l} y_i = \lambda R x_i + t \\ \underline{\bar{y} = \lambda R \bar{x} + t} \\ y_i - \bar{y} = \lambda R(x_i - \bar{x}) \end{array} \tag{2}$$

We manage to eliminate the shifting contribute; indeed it is possible to calculate it lately using the following expression:

$$t = \bar{y} - \lambda R \bar{x} \tag{3}$$

If we make the square of the second equation in the formulas number (2), we also find an expression very easy to calculate the scale factor:

$$y^T y = \lambda^2 x^T R^T R x = \lambda^2 x^T x \Rightarrow \lambda = \sqrt{\frac{y^T y}{x^T x}} \tag{4}$$

Now we have to make a substitution of variables in a way to transform the non-linear problem in a linear one:

$$y_i = \lambda R x_i \tag{5}$$

With a further substitution, we obtain our linear system of equations:

$$y_i = R\sqrt{\frac{y^T y}{x^T x}} x_i = R x_i \tag{6}$$

In the previous expression, we use the Rodriguez Rational Matrix R:

$$R = (I - S)^{-1}(I + S) \tag{7}$$

where I = Identity Matrix
S = Emisymmetric Matrix

$$S = \begin{vmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{vmatrix} \tag{8}$$

After these simple substitutions, we obtain a linear solution, showing the direct proportion between the model coordinates $x = x(u°, v°, w°)$ and the object ones $y = y(X, Y, Z)$ :

$$y_i = R x_i = (I - S)^{-1}(I + S)x_i \;\Rightarrow\; (I - S)y_i = (I + S)x_i) \tag{9}$$

$$\begin{vmatrix} 1 & -c_j & b_j \\ c_j & 1 & -a_j \\ -b_j & a_j & 1 \end{vmatrix} \begin{vmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{vmatrix}_i = \begin{vmatrix} 1 & c_j & -b_j \\ -c_j & 1 & a_j \\ b_j & -a_j & 1 \end{vmatrix} \begin{vmatrix} u° \\ v° \\ w° \end{vmatrix}_{ij}$$

Reorganizing matrices and vectors, in a way which collects in a unique vector the three unknown parameters, coming from the above mentioned emismmetric matrix, we obtain the following final equation:

$$\begin{vmatrix} 0 & (\hat{Z}_i - w°_{ij}) & -(\hat{Y}_i - v°_{ij}) \\ -(\hat{Z}_i - w°_{ij}) & 0 & -(\hat{X}_i - u°_{ij}) \\ (\hat{Y}_i - v°_{ij}) & -(\hat{X}_i - u°_{ij}) & 0 \end{vmatrix} \begin{vmatrix} \hat{a}_j \\ \hat{b}_j \\ \hat{c}_j \end{vmatrix} + \begin{vmatrix} \hat{X}_i - u°_{ij} \\ \hat{Y}_i - v°_{ij} \\ \hat{Z}_i - w°_{ij} \end{vmatrix} = 0 \tag{10}$$

**3.1.2 Relative Orientation Parameters**: Regarding the Relative Orientation we make an exhaustive research of the preliminary values, solving a linearized problem in all its possible cases. Notice that an exact solution has been recently found, but it leads to an equation of order four, which supplies four plausible solutions, as we can easily achieve by repeating a linearized problem via an exhaustive research. The Relative Orientation is based on the *Coplanarity Condition.*

It shows that the point $P_1$, in the first image, and its homologue point $P_2$, in the second image, have a unique corresponding point Q on the object. In case of Asymmetric Relative Orientation, we have to define $b_y$, $b_z$, $\omega_2$, $\varphi_2$, $\kappa_2$, which are the parameters of position and attitude of the second image, compared to those of the first image. Notice that $b_x$ is already defined in the Absolute Orientation, as the scale factor $\lambda$. In case of Symmetric Relative Orientation, we have to define $\varphi_1$, $\kappa_1$, $\omega_2$, $\varphi_2$, $\kappa_2$, parameters which represent parameters of position and attitude of the two images. Notice that $\omega_1$ is missed because it is already defined in the Absolute Orientation, as the global attitude angle $\Omega$.

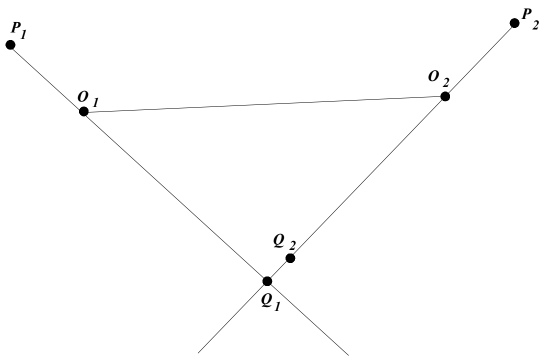



Figure 2. Coplanarity Condition

## 4. MODEL CONSTRUCTION

For the Relative Orientation, we should have previous information about the preliminary values of the parameters. It is not always possible to know them, before the plotting. Let us point out that non-conventional photogrammetry implies often camera acquisition without classical surveying measurement. If we consider the classical Symmetric procedure of Relative Orientation, we can make an exhaustive research of all possible preliminary parameters, because we work in a closed group (in the topological sense) of values compared to the rotations in the space.

The convergence of linearization of trigonometric functions is acceptable as far as values lower or near Π/4. Therefore we decided to explore all the admissible values for rotation angles with a step of Π/4, as shown below:

| | $\varphi_1$ | $k_1$ | $\omega_2$ | $\varphi_2$ | $k_2$ |
|---|---|---|---|---|---|
| Π/2 | ○ | | | ○ | |
| Π/4 | • | | | • | |
| 0 | • | • | • | • | • |
| Π/4 | • | • | • | • | • |
| Π/2 | ○ | • | • | ○ | • |
| 3Π/4 | | • | • | | • |
| Π | | • | • | | • |
| 5Π/4 | | • | • | | • |
| 3Π/2 | | • | • | | • |
| 7Π/4 | | • | • | | • |

Table 1. Exhaustive Research for Symmetric Relative Orientation parameters

where ○ $k_1 \equiv 0$ if $\varphi_1 \equiv \pm\Pi/2$ and/or $k_2 \equiv 0$ if $\varphi_2 \equiv \pm\Pi/2$

As known, if the φ angle is around ±Π/2, we can not individuate the *k* rotation, which is fixed equal to zero. Indeed in the polar zones (we assumed their range in a circle of one degree), the two angles are identical or quasi identical, and this fact produced singularity or ill-conditioning.

The exhaustive research explored $5 \times 8 \times 8 \times 5 \times 8 = 12800$ possible configurations. For each case, a linear system was solved, using the values of this configuration (case), as preliminary values of the parameters of the Symmetric Relative Orientation.
Examples were carried out in all the middle points of the possible configuration. Considering the 5 parameters of the Symmetric Relative Orientation, the angles $\kappa_1$, $\omega_2$, $\kappa_2$ are defined in a complete rotation (8 configurations), whilst $\varphi_1$, $\varphi_2$ are defined in a half rotation (5 configurations), which led to the above mentioned 12800 cases.

Each linear system solution gave us the estimate parameters for the Symmetric Relative Orientation. The convergence to admissible values is when $\sigma_0$ is small enough. Considering only the distinct solutions, we found four analytical acceptable configurations.

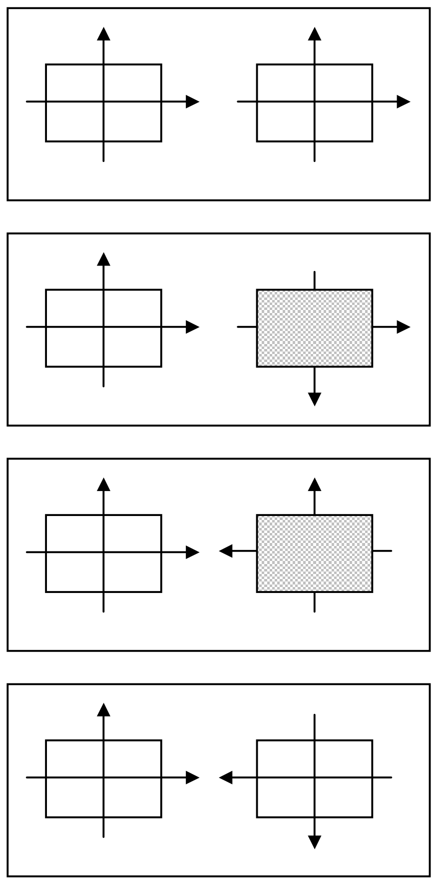


Figure 3. The 4 final possible configurations

These configurations are really different, so it is not so difficult to have information about the initial position of the images, in every specific case. Selecting the chosen case, it is possible to calculate the estimate parameters for the expected Symmetric Relative Orientation.

Notice that in the Asymmetric procedure of Relative Orientation, we have two shift parameters to be searched, but the group of shifting is not a closed one, so we had to use a different way to find the preliminary values. However with the following relations is possible to transform the Symmetric Relative Orientation parameters in the Asymmetric ones, and viceversa:

$$
\begin{aligned}
b_x &= \cos\varphi_1 \cos k_1 \\
b_y &= \cos\varphi_1 \sin k_1 \\
b_z &= \sin\varphi_1
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\varphi_1 &= \arcsin b_z \\
k_1 &= \arctan \frac{b_y}{b_x}
\end{aligned}
\tag{11}
$$

## 5. OBJECT RECONSTRUCTION

In our procedure for the Absolute Orientation, the object reconstruction does not need preliminary parameters, because we can reach the exact solution, by solving the linear system, mentioned in an above paragraph. We tested this procedure, considering 208 possible configurations. These cases come from an object rotation following the global attitude angles (Ω, Φ, K), with a step of Π/4. Exam was performed analyzing the rotation in the space of a cube with 27 control points, regularly distributed.

## 6. NUMERIC EXPERIMENTS

To verify precision, accuracy and reliability of these techniques, a program in Fortran 95 language (compiled and assembled with Lahey-Fujitsu Fortran 95 version 5.6) was written, implemented and tested. It runs on a Pentium 3 PC, with 933 MHz – 262 Mb / RAM – 30 Gb / Hard Disk. The exhaustive research for the Symmetric Relative Orientation works in 4 - 5 seconds, while all others procedures are immediate. In all the examples, we introduced random errors, with standard deviation of 20 μm, as usual in photogrammetry. Here we present an explanation of these programs:

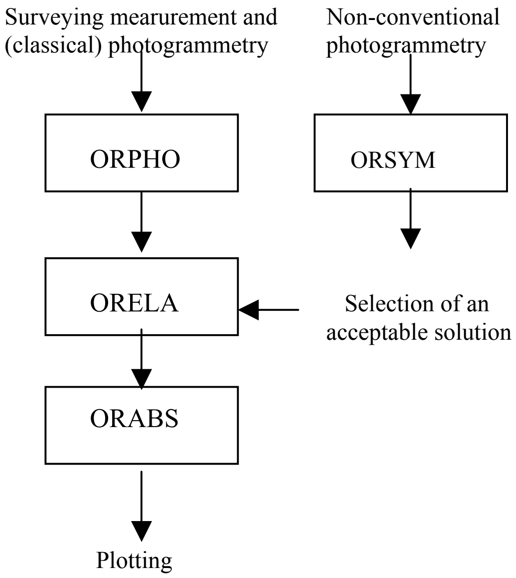
ORPHO_ it converts Cardanic angles in Eulerian angles and viceversa. This is a very large used transformation in close range photogrammetry, because it is essential for the image orientation, when the rotation angles are acquired by surveying measurements.

ORSYM_ it calculates the preliminary values for the Symmetric Relative Orientation. It solves 12800 linear problems, exploring all possible configurations in the space, with a step of Π/4. The same program, choosing one of the four distinct solutions, permits to calculate the preliminary parameters for the Asymmetric Relative Orientation.

ORELA_ it calculates the adjusted parameters of the Asymmetric Relative Orientation, starting from its preliminary ones. If these preliminary values are unknown at the data acquisition, it is possible to get them from the results of the previous program. On the contrary, if they are already known, it is possible to transform the Eulerian angles, more frequently and easily acquired, into the Cardanic ones, by means of ORPHO program.

ORABS_ it calculates the adjusted Absolute Orientation parameters. They are calculated with a simple substitution of variables, able to transform the non-linear problem of the Absolute Orientation in a linear one.

In the following flowchart, let us summarize the global procedure for the orientation of two images.




As evident, the analysis of the performance of the single programs and of the global procedure was quite heavy. Indeed it needed a long preparation of tools, which permitted to manage files of commands. Furthermore many different levels were prepared in order to collect, save and store the output files for the different steps.

Before to conclude we wish to presents some results of these experiments. We considered robust statistical index (mode, median, 1st and 3rd quantiles), able to analyze distribution free problems. On the following tables and figures, the difference among the nominal values and the preliminary ones are shown.

| | Ω | Ψ | K |
|---|---|---|---|
| **mode** | 17 | 1 | 0 |
| **percentile 0,25** | 10 | 5 | 10 |
| **median** | 21 | 13 | 22 |
| **percentile 0,75** | 38 | 25 | 39 |
| **max** | 93 | 56 | 86 |

Table 2. Absolute Orientation results

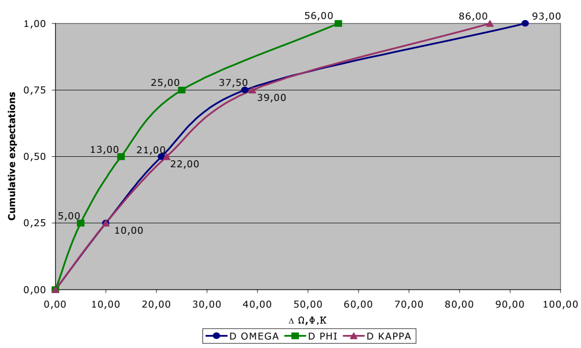



Figure 4. Absolute Orientation results

For the Absolute Orientation, we reached small values, less than 1/100 of grade.

| | $\Delta\varphi_1$ | $\Delta\kappa_1$ | $\Delta\omega_2$ | $\Delta\varphi_2$ | $\Delta\kappa_2$ |
|---|---|---|---|---|---|
| **mode** | 8 | 5 | 8 | 7 | 7 |
| **percentile 0,25** | 18 | 11 | 33 | 16 | 22 |
| **median** | 46 | 41 | 91 | 36 | 53 |
| **percentile 0,75** | 84 | 99 | 176 | 68 | 139 |
| **max** | 287 | 569 | 937 | 286 | 827 |

Table 3. Symmetric Relative Orientation results

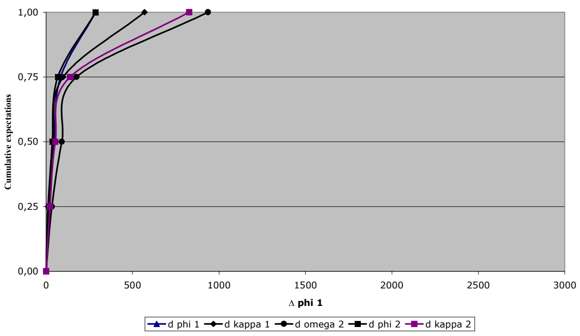



Figure 5. Symmetric Relative Orientation results

For the Relative Orientation (but for the Polar Regions), we reached again small values, less than 1/10 of grade. These values are bigger that the previous ones, but we have to underline that we worked only with preliminary values.

| | $\Delta\varphi_1$ | $\Delta\kappa_1$ | $\Delta\omega_2$ | $\Delta\varphi_2$ | $\Delta\kappa_2$ |
|---|---|---|---|---|---|
| **mode** | 96 | 491 | 208 | 14 | 275 |
| **percentile 0,25** | 36 | 224 | 362 | 19 | 250 |
| **median** | 68 | 432 | 760 | 47 | 514 |
| **percentile 0,75** | 102 | 682 | 1301 | 98 | 862 |
| **max** | 165 | 2205 | 2937 | 251 | 1895 |

Table 4. Symmetric Relative Orientation results (Polar regions)

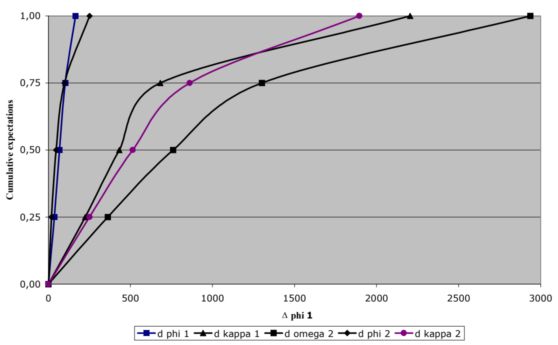



Figure 6. Symmetric Relative Orientation results (Polar regions)

For the Symmetric Relative Orientation, in the Polar Regions, we reached once more small values, less than 3/10 of grade. These values are bigger that the previous ones, but we have to underline that we worked with preliminary values and we explored the Polar Regions, i.e. a very critical zone.

## 7. CONCLUSION

New programs, which would permit an easy solution for the Relative and Absolute Orientations, gave very satisfied expected results. This procedure has great potenciality for non-conventional data acquisition ( eg. non-professional images, images coming from unknown and old sources, equipped vehicles, robots, and many other applications in close range photogrammetry). The advantage of a linear Absolute Orientation should also taken into account. Moreover even if the solution achieved in the Relative Orientation requires an exhaustive research, it is again quick and easy, and seems to solve positively the problem how to acquire the preliminary values of the Relative Orientation parameters.

## 8. REFERENCES

**References from Journals**:

Hattori, S. and Myint,Y., 1995. Automatic Estimation of Initial Approximations of Parameters for Bundle Adjustment. *Photogrammetric Engineering & Remote Sensin*, 61(7), pp. 909-915.

Longuet–Higgins, H. C, 1981. A computer algorithm for reconstructing a scene from two projections. *Nature*, 293(10), pp. 133-135

H.-P. Pan, 1996. A Direct Closed-Form Solution to General Relative Orientation. *Technical Report* on Photogrammetron, pp. 1-20

Abel-Aziz, Y. I., and Karara, H. M., 1971. Direct Linear Transformation into Objrct Space Coordinates in Close-Range Photogrammetry, *Proceding of Symposium on Close-Range Photogrammetry*, pp. 1-18

Thompson, E.H., 1959. A rational algebraic formulation of the problem of the relative orientation. *Photogrammetric Record*, 3(14), pp. 152-159.

Schut, G.H., 1961. On Exact Linear Equation for the Computation of Rotational Elements of Absolute Orientation. *Photogrammetria*. 17(1), pp. 34-37.

Stefanovic, P., 1973. Relative Orientation – a new approach. *ITC Journal*, pp. 417-448.

## APPENDIX A

In this appendix, as well as the following one, it is briefly reported very know formulas and relations, in order to help the reader to recognize what presented and explained in the previous paragraphs. Therefore taking into account the attitude angles, the following matrices represent the rotation matrices written respectively with the Cardanic angles and Eulerian ones:

$$\begin{vmatrix} \cos\varphi\cos k & \cos\omega \sin k + \sin\omega \sin\varphi\cos k & \sin\omega \sin k - \cos\omega \sin\varphi\cos k \\ -\cos\varphi \sin k & \cos\omega\cos k - \sin\omega \sin\varphi \sin k & \sin\omega\cos k + \cos\omega \sin\varphi \sin k \\ \sin\varphi & -\sin\omega\cos\varphi & \cos\omega\cos\varphi \end{vmatrix}$$

$$\begin{vmatrix} \cos\vartheta\cos\alpha - \sin\vartheta\cos\zeta \sin\alpha & \sin\vartheta\cos\alpha + \cos\vartheta\cos\zeta \sin\alpha & \sin\alpha \sin\zeta \\ -\cos\vartheta \sin\alpha - \sin\vartheta\cos\zeta\cos\alpha & -\sin\vartheta \sin\alpha + \cos\vartheta\cos\zeta\cos\alpha & \cos\alpha \sin\zeta \\ \sin\vartheta \sin\zeta & -\cos\vartheta \sin\zeta & \cos\zeta \end{vmatrix} \tag{12}$$

It is easy to derive the attitude angles from both matrices, noting that particular care is required in the Polar Regions due to the well know singularity of the rotation group in the space. Moreover the rotation matrix is obviously unique, for that reason this matrix allows for passing from the Cardanic angles to the Eulerian ones, and viceversa.

Furthermore as already explained, an advantageous alternative to the classical rotation matrix is given by the Rodriguez Rational Matrix:

$$R^T = \frac{1}{1+a^2+b^2+c^2}\begin{vmatrix} 1+a^2-b^2-c^2 & 2(ab+c) & 2(ac-b) \\ 2(ab-c) & 1-a^2+b^2-c^2 & 2(bc+a) \\ 2(ac+b) & 2(bc-a) & 1-a^2-b^2+c^2 \end{vmatrix} \tag{13}$$

where its parameters generally have not any physical sense.

## APPENDIX B

This appendix wants to present, very shortly, the analytical definition of the coplanarity condition, both in the Asymmetric configuration and in the Symmetric one.

The first case points out the coplanatity of four points, i.e. two projection centers and two image points, of a unique object point, in two different images:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & b_x & x_1 & b_x+\xi_2 \\ 0 & b_y & y_1 & b_y+\eta_2 \\ 0 & b_z & -c & b_z+\zeta_2 \end{vmatrix} = 0 \tag{14}$$

The same condition leads to an easier expression, which points out the coplanarity condition of three vectors: the baseline, the direction from the first image point to the observed object point and the direction from the second image point to the same observed object point:

$$\begin{vmatrix} b_x & x_1 & \xi_2 \\ b_y & y_1 & \eta_2 \\ b_z & -c & \zeta_2 \end{vmatrix} = 0 \tag{15}$$

Thus the calculation of this determinant leads to the following relation, know as coplanarity condition for the Asymmetric configuration:

$$\bar{b}_x(y_1\zeta_2 + c\eta_2) - b_y(x_1\zeta_2 + c\xi_2) + b_z(x_1\eta_2 - y_1\xi_2) = 0 \tag{16}$$

In the same way, assuming the same coplanarity condition of the same four points, in a different reference system, given by the Symmetric configuration:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & \bar{b}_x & \xi_1 & \bar{b}_x+\xi_2 \\ 0 & 0 & \eta_1 & \eta_2 \\ 0 & 0 & \zeta_1 & \zeta_2 \end{vmatrix} = 0 \tag{17}$$

and passing to the analogous coplanarity condition of the same three vectors, under the same (Symmetric) condition:

$$\begin{vmatrix} \bar{b}_x & \xi_1 & \xi_2 \\ 0 & \eta_1 & \eta_2 \\ 0 & \zeta_1 & \zeta_2 \end{vmatrix} = 0 \tag{18}$$

the calculation of this determinant leads to the following relation, know as coplanarity condition for the Symmetric configuration:

$$\eta_1\zeta_2 - \zeta_1\eta_2 = 0 \tag{19}$$

The general form of the two linearized coplanarity conditions are omitted, in sake of brevity, but it is not so difficult to derive them, taking into account the derivatives of the rotation matrix, respect to the attitude angles.