LEAST SQUARES 3D SURFACE MATCHING

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KEY WORDS: Least squares 3D surface matching, point clouds, registration, laser scanning.

ABSTRACT:

The automatic co-registration of point clouds, representing 3D surfaces, is a relevant problem in 3D modeling. This registration problem can be defined as a surface matching problem. We treat it as least squares matching of overlapping surfaces. The point cloud may have been digitized/sampled point by point using a laser scanner device, a photogrammetric method or other surface measurement techniques. In the past, several efforts have been made concerning the registration of 3D point clouds. One of the most popular methods is the Iterative Closest Point (ICP) algorithm. Several variations and improvements of the ICP method have been proposed. In photogrammetry there have been some studies on the absolute orientation of stereo models using DEMs (Digital Elevation Model) as control information. These works are known as DEM matching, which corresponds mathematically with least squares image matching. The DEM matching concept is only applied to 2.5D surfaces. 2.5D surfaces have limited value, especially in close range applications. Our proposed method estimates the 3D similarity transformation parameters between two or more fully 3D surface patches, minimizing the Euclidean distances between the surfaces by least squares. This formulation gives the opportunity of matching arbitrarily oriented 3D surface patches. An observation equation is written for each surface element on the template surface patch, i.e. for each sampled point. The geometric relationship between the conjugate surface patches is defined as a 7-parameter 3D similarity transformation. The constant term of the adjustment is given by the observation vector whose elements are the Euclidean distances between the template and search surface elements. Since the functional model is non-linear, the solution is iteratively approaching to a global minimum. The unknown transformation parameters are treated as stochastic quantities using proper weights. This extension of the mathematical model gives control over the estimation parameters. Furthermore, some experimental results based on registration of close-range laser scanner and photogrammetric point clouds will be presented. This new surface matching technique is a generalization of the least squares image matching concept and offers high flexibility for any kind of 3D surface correspondence problem, as well as statistical tools for the analysis of the quality of the final results.

1. INTRODUCTION

Laser scanners can measure directly 3D coordinates of huge amounts of points in a short time period. Since the laser scanner is a line-of-sight instrument, in many cases the object has to be scanned from different viewpoints in order to completely reconstruct it. Because each scan has its own local coordinate system, all the local point clouds must be transformed into a common coordinate system. This procedure is usually referred to as registration. Actually the registration is not a specific problem to the laser scanner domain. Since the problem is more general than the given definition, the emphasis of our work is to investigate the most general solution of the registration problem on a theoretical basis.

In the past, several efforts have been made concerning the registration of 3D point clouds, especially in the Computer Vision area. One of the most popular methods is the *Iterative Closest Point* (ICP) algorithm developed by Besl and McKay (1992), Chen and Medioni (1992), and Zhang (1994). The ICP is based on the search of pairs of nearest points in the two sets, and estimating the rigid transformation, which aligns them. Then, the rigid transformation is applied to the points of one set, and the procedure is iterated until convergence. The ICP assumes that one point set is a subset of the other. When this assumption is not valid, false matches are created, that negatively influence the convergence of the ICP to the correct solution (Fusiello et al., 2002). Several variations and improvements of the ICP method have been made (Masuda and Yokoya, 1995, Bergevin et al., 1996), but several problems still remain. From a computational expense point of view it is highly time consuming due to the exhaustive search for the nearest point (Sequeira, et al., 1999). Another problem is that it requires every point in one surface to have a corresponding point on the other surface. An alternative approach to this search problem was proposed by Chen and Medioni (1992). They used the distance between the surfaces in the direction normal to the first surface as a registration evaluation function instead of point–to–nearest point distance. This idea was originally proposed by Potmesil (1983). In (Dorai et al., 1997) the method of Chen and Medioni was extended to an optimal weighted least-squares framework. Zhang (1994) proposed a thresholding technique using robust statistics to limit the maximum distance between points. Masuda and Yokoya (1995) used the ICP with random sampling and least median square error measurement that is robust to a partially overlapping scene. Okatani and Deguchi (2002) propose the best transformation of two range images to align each other by taking into account the measurement error properties, which are mainly dependent on both the viewing direction and the distance to the object surface. The ICP algorithm always converges monotonically to a local minimum with respect to the mean-square distance objective function (Besl and McKay, 1992). Even if good initial approximations for the transformation parameters are provided, in some cases it might converge to a wrong solution due to its closest point (or tangent plane) search scheme. It does not use the local surface gradients in order to direct the solution to a global minimum. Another deficiency of the ICP method is to be not able to handle multi-scale range data.

In (Turk and Levoy, 1994) a method for combining a collection of range images into a single *polygonal mesh* that completely describes the object was proposed. This method first aligns the meshes with each other using a modified ICP, and zippers together adjacent meshes to form a continuous surface that correctly captures the topology of the object. Curless and Levoy (1996) proposed a *volumetric method* for integration of the range images. Two other volumetric approaches were given in (Pulli et al., 1997, Hilton and Illingworth, 1997).

A quite different registration approach has been proposed in (Johnson and Hebert, 1998, Johnson and Hebert, 1999). Pairwise registration is accomplished using *spin images*, an alternative representation finding point correspondence. The final transformation is refined and verified using a modified ICP algorithm. To generate the spin image of a point in a 3D point cloud, a local basis is computed at an oriented point (3D point with surface normal) on the surface of an object represented as a polygonal surface mesh. The positions with respect to the basis of other points on the surface of the object can then be described by two parameters. By accumulating these parameters in a 2D array, a descriptive image associated with the oriented point is created. Because the image encodes the coordinates of points on the surface of an object with respect to the local basis, it is a local description of the global shape of the object and is invariant to rigid transformations (Johnson and Hebert, 1998). In (Guarnieri et al., 2003) spin images were used for the automatic detection of common areas, and initial alignment between the range image pairs.

The *Iterative Closest Compatible Point* (ICCP) algorithm has been proposed in order to reduce the search space of the ICP algorithm (Godin et al., 1994, Godin and Boulanger, 1995, Godin et al., 2001). In the ICCP algorithm, the distance minimization is performed only between the pairs of points considered compatible on basis of their viewpoint invariant attributes (normalized color/intensity, curvature, and other attributes). In (Sharp et al., 2002) a conceptually similar method called *Iterative Closest Points using Invariant Features* (ICPIF) was introduced. This method chooses nearest-neighbor correspondences according to a distance metric, which is a scaled sum of the positional and feature distances. Roth (1999) proposed a method that exploits the intensity information supplied by the laser scanner device. It firstly finds the points of interest in the intensity data of each range image using and interest operator. Then, the 3D triangles, which are constructed by 2D interest points, are matched. In (Stamos and Leordeanu, 2003) another feature based registration approach, which searches the line and plane pairs in 3D point cloud space instead of 2D intensity image space, was adopted. The pairwise registrations generate a graph, in which the nodes are the individual scans and the edges are the transformations between the scans. Finally, the graph algorithm registers each individual scan with respect to a central pivot scan. There can be found many other *feature-based ICP* approaches in the literature (Chua and Jarvis, 1996, Feldmar and Ayache, 1996, Higuchi et al., 1995, Soucy and Ferrie, 1997, Thirion, 1996, Vanden Wyngaerd, et al., 1999, Yang and Allen, 1998).

In (Silva et al., 2003) *Genetic Algorithms* (GA) in combination with hill-climbing heuristics were applied to range image registration problem. In addition, some comparative studies of ICP variants have been made (Rusinkiewicz and Levoy, 2001, Dalley and Flynn, 2002). A highly detailed survey on the registration methods as well as recognition and 3D modeling techniques is given in (Campbell and Flynn, 2001).

Since most of the developed range image registration methods need an initial approximate alignment, there have been made some works on the pre-alignment. In (Murino et al., 2001) a method based on *3D skeletons* is introduced. 3D skeletons first extracted from both range images, and matched each other in order to find the pre-alignment. A *frequency domain* technique based on Fourier transformation is given in (Lucchese et al., 2002) as a pre-alignment method, also. An automatic pre-alignment method without any prior knowledge of the relative viewpoints of the sensor or the geometry of the imaging process is given in (Vanden Wyngaerd and Van Gool, 2002). It matches *bitangent* curve pairs, which are pairs of curves that share the same tangent plane, between two views. An interesting and problem specific pre-alignment method is given in (Sablatnig and Kampel, 2002). They present a method that pre-aligns the front- and backviews of rotationally symmetric objects, which are archeological ceramic fragments, using *3D Hough* transformation. An identical voting scheme (Habib and Schenk, 1999) based on Hough technique was used in order to find the initial approximations of the unknown 3D similarity transformation parameters between two overlapped airborne point clouds. This method can solve the transformation parameters in parameter space without point correspondence. The final registration is achieved using a similar method to Chen and Medioni's (1992) point-to-normal distance error minimization formula.

The well known approach for the multiple range image registration is to sequentially apply pairwise registration until all views are combined. Chen and Medioni (1992) proposed a method, which registers successive views incrementally with enough overlapping area. Each next view is registered and merged with the *metaview*, which is the topological union of the former pairwise registration. In (Blais and Levine, 1995) couples of images are incrementally registered together with a final registration between the first and last view. It is based on reversing the range finder calibration process, resulting in a set of equations which can be used to directly compute the location of a point in a range image corresponding to an arbitrary point in three dimensional space. Another multi image registration method based on inverse calibration, called *Iterative Parametric Point* (IPP), is given in (Jokinen and Haggren, 1995). In (Bergevin et al., 1996) an algorithm, which considers the network of views as a whole and minimizes the registration errors of all views simultaneously, is introduced. This leads to a *well-balanced network* of views in which the registration errors are equally distributed. Pulli (1999) proposed to use the pairwise alignments as constraints that the multiview step enforces while evenly diffusing the pairwise registration errors. In this work enforcing the pairwise alignments as constraint is called as *virtual mate*. In (Dorai et al., 1998) a seamless integration method based on weighted averaging technique for the registered multiview range data to form an unbroken surface is proposed. In (Eggert et al., 1998) a *force based optimization* technique for simultaneous registration of multiview range images is introduced. They announced that the final registration accuracy of their method typically approaches less than 1/4 of the interpoint sampling resolution of the range image.

Williams and Bennamoun (2001) proposed a new technique from the analytical calculation point view for the simultaneous registration of multiple point sets. The global point registration technique presented in this paper is a generalization of Arun et al.'s (1987) well known pairwise registration method, which uses the *Singular Value Decomposition* (SVD) to compute the optimal registration parameters in the presence of point correspondences. This method is a closed-form solution for 3D similarity transformation between two 3D point sets. It first reduces the unknown translation parameters shifting the all points to the center of gravity, and calculates the unknown rotation matrix using SVD of a 3x3 matrix, and finally calculates the translation parameters. During that time a similar method had been developed independently in MIT (Horn et al., 1988). But as pointed out by Horn et al. (1988) these methods were not entirely novel, since the same problem had already been treated in the *Psychometry* (Quantitative Psychology) literature (Schönemann, 1966, Schönemann and Carroll, 1970) in the name of *Procrustes Analysis*. An interesting note here is that the mathematical background of SVD, innovated by Eckart and Young (1936), comes from Psychometry area. It is also known as *Eckart-Young Decomposition*. From the mathematical point of view a similar method to Williams and Bennamoun's (2001) proposal is given in (Beinat and Crosilla, 2001). They propose the Generalized Procrustes Analysis as a solution for the multiple range image registration problem in the presence of point correspondences. More details for the Procrustes Analysis can be found in (Crosilla and Beinat, 2002). A further stochastic model taking into account for different a priori accuracy of the tie point coordinate components was proposed by Beinat and Crosilla (2002). In fact both of the presented methods (Williams and Bennamoun, 2001, Beinat and Crosilla, 2001) use *Gauß-Seidel* or *Jacobi* type iteration techniques in order to register multiple range images simultaneously. Photogrammetric *block adjustment by independent models* method has been proposed as another solution (Scaioni and Forlani, 2003) in the literature.

Masuda (2002) propose a method to register multiple range images using the *signed distance field* (SDF), which is a scalar field determined by the signed distance of an arbitrary 3D point from the object surface. In (Krsek et al., 2002) an automatic hybrid registration algorithm is presented. It works in a bottom-up *hierarchical* mode: points – differential structures – surface. The final refinement of the estimation is carried out using *Iterative Closest Reciprocal Point* (ICRP) algorithm (Pajdla and Van Gool, 1995). In (Castellani et al., 2002) a multiple range image registration method is given for 3D reconstruction of underwater environment from multiple acoustic range views acquired by a remotely operated vehicle. In addition, several reviews and comparison studies for the multiple range image registration are available in the literature (Jokinen and Haggren, 1998, Williams et al., 1999, Cunnington and Stoddart, 1999).

In (Dijkman and van den Heuvel, 2002) a semi automatic registration method based on least squares fitting the parameters of the models (cylinder and plane) is introduced. The registration is performed using the parameters of the models measured in different scans. The *Global Positioning System* (GPS) is also used to determine the 3D coordinates of the homologous points, which are used to merge the different scans (Balzani et al., 2002). Use of GPS allows combining all scans in a common system even if they do not have overlapping parts. To solve the point correspondence problem between two laser scanner point cloud before the 3D similarity transformation, an automatic method was proposed based on the assumption that the Z axis of two scans are vertical (Bornaz et al., 2002). In this work retro-reflective targets, which are attached on the object surface before the scanning process, are used as common points. The idea is to search the homologous points based on two spherical coordinates (range and elevation), whose the system are constructed in both sets of points choosing a point as origin. A similar automatic method has been given in (Akca, 2003) using the template shaped targets. In this work the space angles and the distances are used to solve the point correspondence problem, since they are translation and rotation invariant parameters among the different laser scanner viewpoints. The ambiguity problem, which is rare but theoretically and practically possible, is solved using *consistent labeling by discrete relaxation*.

This fairly exhaustive description of related research activities and achievements demonstrates the relevance of the problem. Also, we notice that still a satisfying solution has to be found, tested and implemented.

Since 3D point clouds derived by any method or device represent the object surface, the problem should be defined as a surface matching problem. In Photogrammetry, the problem statement of surface patch matching and its solution method was first addressed by Gruen (1985a) as a straight extension of *Least Squares Matching* (LSM).

There have been some studies on the absolute orientation of stereo models using DEMs as control information. This work is known as *DEM matching*. The absolute orientation of the models using DTMs as control information was first proposed by Ebner and Mueller (1986), and Ebner and Strunz (1988). Afterwards, the functional model of DEM matching has been formulated by Rosenholm and Torlegard (1988). This method basically estimates the 3D similarity transformation parameters between two DEM patches, minimizing the least square differences along the Z axes. Several applications of DEM matching have been reported (Karras and Petsa, 1993, Pilgrim, 1996, Mitchell and Chadwick, 1999, Xu and Li, 2000). Maas (2000) successfully applied a similar method to register airborne laser scanner strips, among which vertical and horizontal discrepancies generally show up due to GPS/INS accuracy problems. Another similar method has been presented for registering surfaces acquired using different methods, in particular, laser altimetry and photogrammetry (Postolov, Krupnik, and McIntosh, 1999). Furthermore, techniques for 2.5D DEM surface matching have been developed, which correspond mathematically with Least Squares Image Matching. The DEM matching concept can only be applied to 2.5D surfaces, whose analytic function is described in the *explicit* form, i.e. $z = f(x,y)$. Of course, this formulation has several problems in the matching of solid (3D) surfaces.

Although the registration of 3D point clouds is a very active research area in both Computer Vision and Photogrammetry, there is not such a method that has a complete capability to the following three properties: matching of multi-scale data sets, matching of real 3D surfaces without any limitation, fitting the physical reality of the problem statement as good as possible. The proposed work completely meets these requirements.

The Least Squares Matching concept had been applied to many different types of measurement and feature extraction problems due to its high level of flexibility and its powerful mathematical model: *Adaptive Least Squares Image Matching* (Gruen, 1984, Gruen, 1985a), *Geometrically Constrained Multiphoto Matching* (Gruen and Baltsavias, 1988), *Image Edge Matching* (Gruen and Stallman, 1991), *Multiple Patch Matching with 2D images* (Gruen, 1985b), *Multiple Cuboid (voxel) Matching with 3D images* (Maas, 1992, Maas and Gruen, 1995), *Globally Enforced Least Squares Template Matching* (Gruen and Agouris, 1994), *Least Squares B-spline Snakes* (Gruen and Li, 1996). For a detailed survey the authors refer to (Gruen, 1996). If 3D point clouds derived by any device or method represent an object surface, the problem should be defined as a surface matching problem instead of the 3D point cloud matching. In particular, we treat it as least squares matching of overlapping 3D surfaces, which are digitized/sampled point by point using a laser scanner device, the photogrammetric method or other surface measurement techniques. This definition allows us to find a more general solution for the problem as well as to establish a mathematical model in the context of LSM.

Our proposed method, *Least Squares 3D Surface Matching* (LS3D), estimates the 3D transformation parameters between two or more fully 3D surface patches, minimizing the Euclidean distances between the surfaces by least squares. This formulation gives the opportunity of matching arbitrarily oriented 3D surface patches. An observation equation is written for each surface element on the template surface patch, i.e. for each sampled point. The geometric relationship between the conjugate surface patches is defined as a 7-parameter 3D similarity transformation. This parameter space can be extended or reduced, as the situation demands it. The constant term of the adjustment is given by the observation vector whose elements are Euclidean distances between the template and search surface elements. Since the functional model is non-linear, the solution is iteratively approaching to a global minimum. The unknown transformation parameters are treated as stochastic quantities using proper weights. This extension of the mathematical model gives control over the estimation parameters. The basics of the mathematical modeling of the proposed method, the convergence behaviour, and the statistical analysis of the theoretical precision of the estimated parameters are explained in the following section. The experimental results based on registration of close-range laser scanner and photogrammetric point clouds are presented in the third section. The conclusions are given in the last section.

2. LEAST SQUARES 3D SURFACE MATCHING

2.1 The Estimation Model

Assume that two different surfaces of the same object are digitized/sampled point by point, at different times (temporally) or from different viewpoints (spatially). f(x,y,z) and g(x,y,z) are conjugate regions of the object in the *left* and *right* surfaces respectively. The problem statement is finding the correspondent part of the *template* surface patch f(x,y,z) on the *search* surface g(x,y,z).

$$f(x,y,z) = g(x,y,z) \tag{1}$$

According to Equation (1) each surface element on the template surface patch f(x,y,z) has an exact correspondent surface element on the search surface patch g(x,y,z), if both of the surface patches would be continuous surfaces. In order to model the random errors, which come from the sensor, environmental conditions or measurement method, a true error vector e(x,y,z) has to be added.

$$f(x,y,z) - e(x,y,z) = g(x,y,z) \tag{2}$$

The matching is achieved by minimizing a goal function, which measures the Euclidean distances between the template and the search surface elements. Equation (2) is considered observation equations, which functionally relate the observations f(x,y,z) to the parameters of g(x,y,z). The final location is estimated with respect to an initial position of g(x,y,z), the approximation of the conjugate search surface patch $g^0(x,y,z)$.

To express the geometric relationship between the conjugate surface patches, a 7-parameter 3D similarity transformation is used. Depending on the deformation between the template and the search surfaces, the geometric relationship could be defined using any other type of 3D transformation methods, e.g. a 12-parameter affine, 24-parameter tri-linear, or 30-parameter family of quadratic transformations.

The related linearized observation equations have been derived and are available to us. They result in the following linear system for Least Squares estimation:

$$-\mathbf{e} = \mathbf{A}\,\mathbf{x} - \ell \quad , \quad \mathbf{P} \tag{3}$$

where **A** is the design matrix, **x** is the parameter vector, and ℓ is the observation vector. With the statistical expectation operator E{} and the assumptions

$$e \sim N(0,\sigma_0^2\mathbf{Q}_{ll}) \ , \ \sigma_0^2\mathbf{Q}_{ll} = \sigma_0^2\mathbf{P}_{ll}^{-1} = \mathbf{K}_{ll} = E\{ee^T\} \tag{4}$$

the system (3) and (4) is a Gauβ-Markov estimation model. The unknown 3D similarity transformation parameters are treated as stochastic quantities using proper weights. This extension gives advantages of control over the estimating parameters (Gruen, 1986). In the case of poor initial approximations for unknowns or badly distributed 3D points along the principal component axes of the surface, some of the unknowns, especially the scale factor, may converge to a wrong solution, even if the scale factors between the surface patches are same.

$$-\mathbf{e}_b = \mathbf{I}\,\mathbf{x} - \ell_b \quad , \quad \mathbf{P}_b \tag{5}$$

The least squares solution of the joint system Equations (3) and (5) gives the unbiased minimum variance estimation for the parameters

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{P}_b)^{-1}(\mathbf{A}^T\mathbf{P}\ell + \mathbf{P}_b\ell_b) \qquad \text{solution vector} \tag{6}$$

$$\hat{\sigma}_0^2 = \frac{\mathbf{v}^T\mathbf{P}\mathbf{v} + \mathbf{v}_b^T\mathbf{P}_b\mathbf{v}_b}{r} \qquad \text{variance factor} \tag{7}$$

$$\mathbf{v} = \mathbf{A}\,\hat{\mathbf{x}} - \ell \qquad \text{residual vector for surface observations} \tag{8}$$

$$\mathbf{v}_b = \mathbf{I}\,\hat{\mathbf{x}} - \ell_b \qquad \text{residual vector for additional observations} \tag{9}$$

where ^ stands for the Least Squares (LS) estimator. The function values g(x,y,z) in Equation (2) are actually stochastic quantities. This fact is neglected here to allow the use of the Gauβ-Markov model and to avoid unnecessary complications, as typically done in LSM (Gruen, 1985a).

Since the functional model is non-linear, the solution iteratively approaches to a global minimum. In the first iteration the initial approximations for the parameters must be provided. The iteration stops if each element of the alteration vector $\hat{\mathbf{x}}$ in Equation (12) falls below a certain limit:

$$|dp_i| < c_i \quad , \quad i = \{1,2,...,7\} \tag{10}$$

The theoretical precision of the estimated parameters can be evaluated by means of the covariance matrix

$$\mathbf{K}_{xx} = \hat{\sigma}_0^2\mathbf{Q}_{xx} = \hat{\sigma}_0^2\mathbf{N}^{-1} = \hat{\sigma}_0^2(\mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{P}_b)^{-1} \tag{11}$$

The calculation of the numeric derivative terms depends on the analytical representation of the surface elements. We represent the search surface elements as planar surfaces and optionally, as parametric bi-linear surface patches, which are fitted to 3 and 4 neighbouring knot points, respectively.

2.2 Precision and Reliability Issues

The standard deviations of the estimated transformation parameters and the correlations between themselves may give useful information concerning the stability of the system and quality of the data content (Gruen, 1985a).

$$\hat{\sigma}_p = \hat{\sigma}_0\sqrt{q_{pp}} \quad , \quad q_{pp} \in \mathbf{Q}_{xx} \tag{12}$$

As pointed out in (Maas, 2000), the estimated standard deviations of the translation parameters are too optimistic due to stochastic properties of the search surface. Because of the high level redundancy of a typical data arrangement, a certain amount of occlusions and/or outliers do not have significant effect on the estimated parameters. Baarda's data-snooping method can be favourably used to localize the occluded or gross erroneous measurements.

2.3 Computational Aspects

The computational complexity is of order $O(N^2)$, where N is the number of employed points in the matching process. The actual problem is to search the correspondent element of the template surface on the search surface patch, whereas the adjustment part is a small system, and can quickly be solved using back-substitution followed by Cholesky decomposition. Searching the correspondence is an algorithmic problem, and needs professional software optimization techniques and programming skills, which are not within the scope of this paper.

Since the method needs initial approximations of the unknowns due to the non-linear functional model, one of the methods for pre-alignment in the literature (Habib and Schenk, 1999, Murino et al., 2001, Lucchese et al., 2002, Vanden Wyngaerd and Van Gool, 2002) should be utilized. In the case of multi-resolution data sets, in which point densities are significantly different on the template and search surface patches, higher degree C^1 continuous composite surface representations, e.g. bi-cubic Hermit surface (Peters, 1974), should give better results, of course increasing the computational expenses.

2.4 Convergence of Solution Vector

In a standard LS adjustment calculus in geodesy and photogrammetry, the function of the unknowns is unique, exactly known, and analytically continuous everywhere, e.g. the collinearity equations in the bundle adjustment. Here the function g(x,y,z) is discretized by using a definite sampling rate, which leads to slow convergence, oscillations, even divergence in some cases with respect to the standard adjustment. The convergence behaviour of the proposed method basically depends on the quality of the initial approximations and quality of the data content, and it usually achieves the solution after 4th or 5th iterations (Figure 1), as typically in LSM.

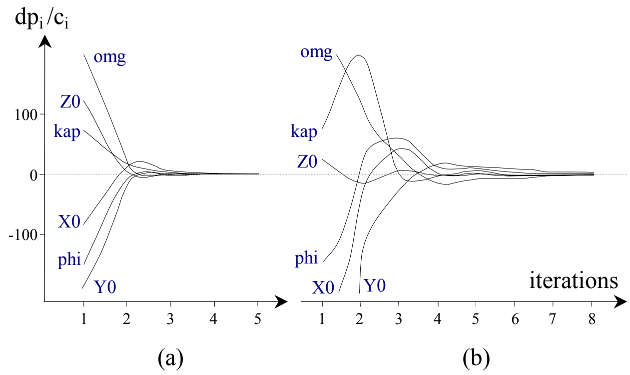


Figure 1: Typical examples for fast convergence (a) and slow convergence (b). Note that scale factor is fixed to unity.

3. THE EXPERIMENTAL RESULTS

Two practical examples are given to show the capabilities of the method. All experiments were carried out using own self-developed C/C++ software that runs on *Microsoft Windows®* OS. Processing times given in Table 1 were counted on such a PC, whose configuration is *Intel®* P4 2.53 GHz CPU, 1 GB RAM. The first example is the registration of three surface patches, which were photogrammetrically measured 3D point clouds of a human face from multi-images (Figure 2). For the mathematical and implementation details of this automatic surface measurement method the authors refer to (D'Apuzzo, 2002).

Left and right template surface patches (Figure 2-a and 2-c) were matched to the centre search surface patch (Figure 2-b) by use of LS3D. Since the data set already came in a common coordinate system, the rotation angles (ω,φ,κ) of the template surfaces were deteriorated by $\sim 10^g$ in the first iteration. Numerical results of the matching of the left surface and the right surface patches are given at parts I-L and I-R of Table 1 respectively. Relatively high standard deviations for the estimated t_x and φ (note that high physical correlation between x and φ due to a conventional axes configuration) exhibit the narrow overlapping area along the x-axis, nevertheless the matching result is successful. The estimated σ_0 values prove the accuracy potential of the surface measurement method, given as 0.2 mm by D'Apuzzo (2002).

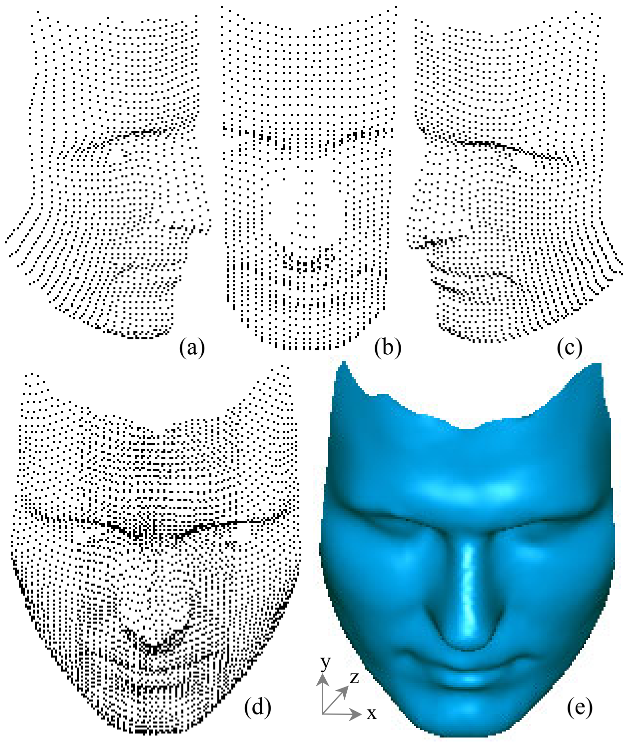
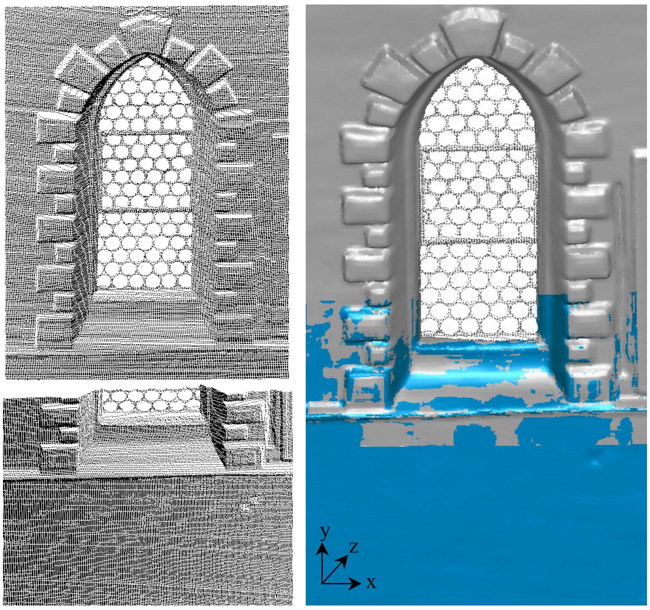


Figure 2: (a) left-template surface, (b) centre-search surface, (c) right-template surface, (d) obtained 3D point cloud after LS3D surface matching, (e) shaded view of the final composite surface.

The second experiment refers to the matching of two overlapping 3D point clouds (Figure 3), which are a part of a chapel in Wangen, Germany, and were scanned using *IMAGER 5003* terrestrial laser scanner (*Zoller+Fröhlich*). Initial approximations of the unknowns were provided by interactively selecting 3 common points on the both surfaces before the matching. Obtained results are given at part II of Table 1. The estimated σ_0 gives valuable information about the sensor noise level and the accuracy limit of the scanner as >1.7 mm.

The parametric bi-linear surface representation gives a slightly better convergence rate and a better a posteriori sigma value than the triangle plane representation, while increasing the computational expenses. The standard deviation of the z-component of the translation vector shows the excellent data content in the depth direction, but the relative precision is highly optimistic, which is ~1/1000 of the point spacing.

Since LS3D reveals the sensor noise level and accuracy potential of any kind of surface measurement method or device, it should be used for comparison and validation studies.



(a) (b)

Figure 3: (a) top - template surface patch, (a) bottom - search surface patch, (b) overlay of the shaded surfaces.

Table 1: Experimental results

	s	n	i	t	d	$\hat{\sigma}_0$	$\hat{\sigma}_{tx}/\hat{\sigma}_{ty}/\hat{\sigma}_{tz}$	$\hat{\sigma}_{\omega}/\hat{\sigma}_{\varphi}/\hat{\sigma}_{\kappa}$
				sec	mm	mm	mm	c
I-L	P	2497	7	0.6	1.5	0.19	0.15/0.07/0.05	0.96/2.44/1.90
	B		7	1.3		0.19	0.15/0.07/0.05	0.96/2.42/1.91
I-R	P	3285	6	0.5	1.5	0.21	0.13/0.03/0.05	0.68/2.25/1.73
	B		6	1.4		0.21	0.13/0.03/0.05	0.69/2.26/1.75
II	P	13461	5	3.8	10	1.74	0.23/0.62/0.01	0.69/0.17/0.46
	B		4	5.6		1.72	0.22/0.61/0.01	0.69/0.17/0.46

I-L: left face surface , I-R: right face surface, II: laser scanner data
s: surface representation, P: plane, B: bi-linear surface, n: number of employed points, i: iterations, t: process time, d: ~ point spacing

4. CONCLUSIONS

LSM is a fundamental measurement algorithm and has been applied to a great variety of data matching problems due to its strong mathematical model. Two well-known ones are LS image matching in 2D pixel space, and LS multiple cuboid matching in 3D voxel space. The LS3D is bridging the conceptual gap between the LS image matching and the LS cuboid matching.

This new 3D surface matching technique is a generalization of the least squares 2D image matching concept and offers high flexibility for any kind of 3D surface correspondence problem, as well as monitoring capabilities for the analysis of the quality of the final results by means of precision and reliability criterions. Another powerful aspect of this proposed method is its ability to handle multi-resolution, multi-temporal, multi-scale and multi-sensor data sets. The technique can be applied to a great variety of data co-registration problems. In addition time dependent (temporal) variations of the object surface can be inspected, tracked and localized using the statistical analysis tools of the method.

ACKNOWLEDGEMENT

The authors would like to thank Dr. Nicola D'Apuzzo for providing the face surface data sets, which were measured by use of his own software *Viewtriplet GTK v0.9©*. The laser scanner data set is courtesy of *Zoller+Fröhlich GmbH Elektrotechnik* (Wangen, Germany).

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