REAL TIME-BASED REPRESENTATION OF 3D OBJECTS

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ABSTRACT:

A realm is a planar graph over a finite resolution grid that has been proposed as a means of overcoming problems of numerical robustness and topological correctness in spatial databases. However, in spatial databases, current data structures based on realm only deal with 2D objects, and cannot be used to describe the complex phenomena in real 3D world. In order to represent 3D spatial objects and the topological relationships between them efficiently in spatial databases, we extend the original realm by following the next steps: (1) Extend the discrete space defined in 2D space to 3D discrete space. (2) Define 3D realm on 3D discrete space. (3) Define 3D realm-based data structures, such as R-block, R-cycle (planar/non-planar), R-face, R-hull and R-volume. (4) Define topological relationships between these 3D realm-based data structures.

1. INTRODUCTION

The application areas of GIS are very wide, such as, environment, network management (transportation, water, gas), agriculture, nation and region development, natural or technique exploration, geology, wireless telecommunication and so on. In all these applications, most of them are realized by using 2D data. Although some applications can use 2D data to get correct realization (such as network analyse, here topology is more important than geometry), the implementations of most applications are not reliable, because in databases, the expressions of geography objects is not corrected (Losa and Cervelle 1999).

Worboys (Worboys 1995) gave out the definition of 3D GIS: This type of system should be able to model, represent, manage, manipulate, analyze and support decisions based upon information associated with 3D phenomena. 3D GIS is not the simple extension of 2D GIS in altitude. In order to add the third dimension to 2D GIS, it is needed to do sufficient research on many aspects of GIS, such as various models and data organization.

Although the researches on 2D spatial analyse are very deep, the researches on 3D analyse are still in a very initial phase. Spatial relationships are the basis of lots of operations executed in GIS, such as including, adjacent, equal, direction, intersect, connecting and their suitable description and maintenance. Similar to 2D GIS, 3D GIS should be able to execute metric (distance, length, area, volume and so on), logic (intersection, union, difference), generalization, buffering, network (the minimal path) and fusing operations. Except metric operations, most of the operations need the knowledge of spatial relationships. Comparing to 2D GIS, the third dimension increases the total number and complexity of possible spatial relationships. Many researchers have adopted the formal method based on set topology to identify spatial relationships (Egenhofer 1990, Molenaar 1998), but the representations of spatial relationships have not been studied sufficiently. For example, in 9I model (Egenhofer and Franzosa 1991), the possible combinations are 512 kinds, but most of them are impossible, so it is very complex to determine the spatial relationships. (Zlatanova 1999) applies 9I model to distinguish the spatial relationships between 3D spatial objects. In order to eliminate the impossible conditions, she gives 13 rules. We provide a method “distribution of dimension” to describe the relationships between spatial objects. It improves 9I model from two aspects: (1) it decreases the possible combinations; (2) there are only two rules to eliminate the impossible conditions, which are very simple and straight.

In GIS, spatial objects are usually stored in relation tables according to spatial topology relationships. The numerical robust and topology correctness depend on the application programs that are not the parts of the database, which makes it complex for the development of GIS. Of course, some people stores the spatial objects to the file system according to his own format, which makes it difficult to be utilized efficiently and combined with database system, so it is difficult for these spatial objects to be integrated with other application systems.

A realm is a planar graph over a finite resolution grid that has been proposed as a means of overcoming problems of numerical robustness and topological correctness in spatial databases (Guting and Schneider 1993). However, in spatial databases, current data structures based on realm only deal with 2D objects, and cannot be used to describe the complex phenomena in real 3D world.

In order to describe 3D objects in spatial databases, we extend 2D realm to 3D realm. Based on 3D realm, we define the corresponding basic data structures, such as R-block, R-cycle (planar/non-planar), R-face, R-hull and R-volume, and give the topological relationships between these 3D realm-based structures.

This paper is organized as: Section 2 introduces the basic concepts of realm. Section 3 presents a method based on the distribution of the dimension. Section 4 extends the discrete space from 2D to 3D, and defines 3D realm. Section 5 discusses the spatial data structures on 3D realm and the topological relationships between these structures. Section 6 presents 3D spatial data types simply. The last section concludes the paper.

2. BASIC CONCEPTS OF REALM

2.1 2D Discrete Space and Realm (Guting and Schneider 1993)

In finite discrete space N×N (N={0, …, n-1}), **N-point** is a pair(x, y)∈N×N; **N-segment** is a pair of two different N points (p, q), and (p, q)=(q, p). P_N represents the set of all N-points; S_N represents the set of all N-segments.

Definition 1: A **Realm** R=P∪S, such that:

(1) $P \subseteq P_N$, $S \subseteq S_N$;
(2) ∀s∈S: s =(p, q) ⇒ p∈P ∧ q∈P
(3) ∀p∈P ∀s∈S: ¬(p in s)
(4) ∀s, t∈S, s≠t: ¬(s and t interest) ∧ ¬(s and t overlap)

2.2 Data Structures Defined on Realm

Definition 2: An **R-cycle** is defined as a set of R-segments $S(c)=\{s_0, \ldots, s_{m-1}\}$, such that:

(1) ∀i∈ {0, …, m-1}: s_i meets $s_{(i+1) \bmod m}$.
(2) There are no other touches of segments.

Definition 3: An **R-block** is defined as a set T of R-segments:

(1) ∀ r, t ∈ T $\exists s_1, \ldots, s_m \in$ T: $r=s_1$, $t= s_m$, and
(2) ∀ i∈{1, …, m-1}: s_i and s_{i+1} meet.

Definition 4: An **R-face** f is a pair (c, H), here c is an R-cycle, $H=\{h_1, \ldots, h_m\}$ is a set of R-cycles (may be empty), which satisfy next conditions (We use S(f) to represent the set of the segments in all the R-cycles of f):

(1) ∀ i∈ {1, …,m}: h_i edge-inside c;
(2) ∀ i, j∈ {1, …, m}, i≠ j : h_i and h_j are edge-disjoint;
(3) There do not exist R-cycles other than c and the R-cycles in H.

3. DISTRIBUTION OF DIMENSIONS - A METHOD TO IDENTIFY TOPOLOGICAL RELATIONSHIPS

3.1 Basic Concepts

A **topology space** is usually described as a set of any element, in this set the concept of continuity is defined (Clementini and Di Felice 1995). Let X and Y are two topological spaces, then:

Mapping f: x → f is continuous, if for every open subset V of Y, $f^{-1}(V)$ is a open subset of X.

If f is bijection, at the same time, f and f^{-1} are continuous, then f is called **topological isomorphism**. Topological isomorphism keeps the adjacent relationships between the mapping points, which includes transform, rotation and zoom. Topological relationships are those relationships that keep invariant under topological isomorphism.

3.2 The Definition of Spatial Objects

For a spatial object, we will define the following operations: lower dimensional parts(shortly as LDP, denoted as ∂), same dimensional parts(shortly as SDP, denoted as °), exterior(¯), set intersection(∩) and dimension(dim). dim is a function, which returns the dimension of a point set, for empty set, it returns –1. If a point set is composed of several parts, then it returns the highest dimension.

Here we do not consider complex geometry objects, because these objects can get by extending simple objects. In $\mathbb{R}^3$, the simple geometry objects are defined as follows:

1. A **simple volume** object is the closure of the connected 3D point set embedded in $\mathbb{R}^3$.
2. A **simple face** object is the connected 2D point set embedded in $\mathbb{R}^3$, no self-intersection, including one and only one cycle.
3. A **simple line** object is the connected 1D point set embedded in $\mathbb{R}^3$, no self-intersection, including two only two end points.
4. A **simple point** object is the 0D point set, and composed of only one point.

In topological space X, the neighbour of x is a subset of U, and U is an open set that includes x. If set A is a neighbour of its element x, then x is a interior point of A, the set of all the interior points is the interior of A, denoted as A° (Kelly 1955). The boundary of A is ∂A := A – A°, the exterior of A is A^{-} = X – A. In 4I/9I mode (Egenhofer and Herring 1990) A° is used to denote the interior of A, and ∂A is used to denote the boundary of A. If the dimension of A is the same with that of the embedding space, the explanation is right. However, if the dimension of A is less than that of the embedding space, the explanation is not right. Therefore, we use A° to denote the parts of A that have the points with the dimension equal to dim(A), and ∂A to denote the left parts of A:

$$A^\circ = \{x \in A \mid \dim(x) = \dim(A)\}$$
$$\partial A = \{x \in A \mid \dim(x) < \dim(A)\}$$

3.3 The Method Based on Distribution of Dimension (DD)

There are many methods about the definitions and verification of spatial topological relationships (Chen et al. 2001). Among these methods, 9I model provided by Egenhofer is the most popular. But there are so many conditions in 9IM that it needs too many restriction rules (Zlatanova 1999), so this method is not practical. Clementini (Clementini and Di Felice 1995) gives DEM method and CBM method. They also consider two exteriors of the two objects, which make it difficult for implementation, because it is difficult to implement the exterior of a spatial object.

The definitions of topology relationships before are usually defined using symmetrical structure, that is, they adopt the concepts of interior, boundary and exterior. The symmetrical structure is one of the main reasons to make these methods complex. However, these concepts are correct if the spatial objects and the embedding space have the same dimension, but not correct for spatial objects with different dimensions or the spatial objects with the embedding space in different dimensions. Hence, we provide an unsymmetrical representation method of spatial relationships. That is, we only consider the pair <A, B>, such that dim(A)≤dim(B).

Our method is based on dimension distribution. We do not use the intersection between every all the parts (interior, boundary, exterior or Voronoi exterior) of an object with all the parts of another object, but use an object's distribution in another object's lower dimensional part, same dimensional part and exterior.

Because of the non-splitting property of point, it cannot be divided into lower dimensional part, same dimensional part and exterior. Therefore: (1) There are only two possible conditions between two point objects: equal and unequal; (2) There are only three possible conditions between a point object and a non-point object: in lower dimensional part, in same dimensional part, in exterior. In the following, we mainly consider about non-point spatial object.

Suppose in ε^N, there are two simple spatial objects A and B, such that $0 \leq \dim(A) \leq \dim(B)$. Then the topological relationship R(A, B) between A and B can be represented by a triple **<En, Bn, In>**, such that:

- **En**:= $\dim(A \cap B^{-}) \in \{-1, \dim(A)\} \subset \mathbb{Z}$, that is, the dimension of A intersecting with the exterior of B.
- **Bn**:= $\dim(A \cap \partial B) \in [-1, \mathrm{MIN}(\dim(A), \dim(B)-1)] \subset \mathbb{Z}$, that is, the dimension of A intersecting with the lower dimensional part of B.
- **In**:= $\dim(A \cap B^{\circ})$, if dim(B)=N, then $\mathrm{In} \in \{-1, \dim(A)\} \subset \mathbb{Z}$, else if dim(B)<N, then $\mathrm{In} \in [-1, \dim(A)] \subset \mathbb{Z}$, that is, the dimension of A intersecting with the same dimensional part of B.

The method used triple <En, Bn, In> to identify the topological relationships is called "**Distribution of Dimensions**" (shortly as **DD**).

Note:

(1) Let dim(A)=α, MIN(dim(A), dim(B)-1)=β;
(2) $\mathrm{MAX}(x_1, \ldots, x_n)$ denotes the maximal value among $x_1, \ldots, x_n$; $\mathrm{MIN}(x_1, \ldots, x_n)$ denotes the minimal value among $x_1, \ldots, x_n$.

In the set of all of <En, Bn, In>, not all the combinations are possible. Here are two restriction rules:

(1) ¬(MAX(En, Bn, In)<α), that is, in continuous space, spatial object O is divided into finite parts $o_1, o_2, \ldots, o_n$, then $\dim(O)=\mathrm{MAX}(\dim(o_1), \dim(o_2), \ldots, \dim(o_n))$. This rule restriction rule is called "dimensional invariant".

(2) If dim(B)=N, (a)if α=N, then $\mathrm{En}=\alpha \wedge \mathrm{In}=\alpha \rightarrow \mathrm{Bn}=(\alpha-1)$, and β = α-1; (b)if α<N, then $\mathrm{En}=\alpha \wedge \mathrm{In}=\alpha \rightarrow \mathrm{Bn} \in \{\alpha, \alpha-1\}$, and β=α. This restriction rule shows the separating effect of lower dimensional part, which is called "low dimensional part separating"

Comparing with the restriction rule of 9I model (Zlatanova 1999), we can see these two rules are very simple.

According to the definition of DD and two restriction rules, we give the possible conditions of R(A, B) under dim(B)=N and dim(B)<N separately:

(1) If dim(B)=N, then $\mathrm{En} \in \{-1, \alpha\}$, $\mathrm{Bn} \in [-1, \beta]$, $\mathrm{In} \in \{-1, \alpha\}$. Hence, all of the possibilities are:

2×(β+2)×2 All the combinations
- 1×(MIN(α-1, β)+2)×1 (Remove the conditions violating "dimension invariant")
- IFF(α=N, β+1, β) (Remove the conditions violating "lower dimensional part separating")
= 4β + 6 - MIN(α-1, β) - IFF(α=N, β+1, β)

(2) If dim(B)<N, then $\mathrm{En} \in \{-1, \alpha\}$, $\mathrm{Bn} \in [-1, \beta]$, In=-1. Hence, all of the possibilities are:

2×(β+2) (All of the combinations)
- (MIN(α-1, β)+2) (Remove the conditions violating "dimensional invariant")
= 4β + 4 - (MIN(α-1, β)+2)

Next we illustrate the topological relationships involving at least one simple volume in 3D space, for the restriction of pages, we omitted other conditions.

3.3.1 A simple line – a simple volume: $\mathrm{En} \in \{-1, 1\}$, $\mathrm{Bn} \in \{-1, 0, 1\}$, $\mathrm{In} \in \{-1, 1\}$，α=1, β=1, so the possible relationships are:
4×1+6-MIN(1-1, 1) – IFF(1=3, 1+1, 1)=4+6-0-1=9。

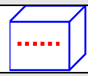
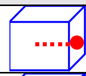
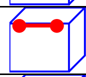
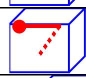
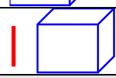
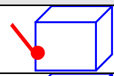
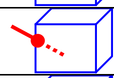
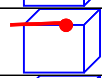
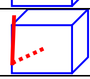
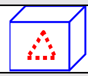
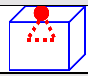
SN	En	Bn	In	relationship
	-1	-1	-1	Violate rule (1)
(1)	-1	-1	1	
	-1	0	-1	Violate rule (1)
(2)	-1	0	1	
(3)	-1	1	-1	
(4)	-1	1	1	
(5)	1	-1	-1	
	1	-1	1	Violate rule (2)
(6)	1	0	-1	
(7)	1	0	1	
(8)	1	1	-1	
(9)	1	1	1	

Table 1. Topological Relationships between a simple line and a simple volume

3.3.2 A simple face - a simple volume: $\mathrm{En} \in \{-1, 2\}$, $\mathrm{Bn} \in \{-1, 0, 1, 2\}$, $\mathrm{In} \in \{-1, 2\}$，α=2, β=2, so the possible relationships are:
4×2+6-MIN(2-1, 1) – IFF(2=3, 2+1, 2)=8+6-1-2=11。

SN	En	Bn	In	relationship
	-1	-1	-1	Violate rule (1)
(1)	-1	-1	2	
	-1	0	-1	Violate rule (1)
(2)	-1	0	2	
	-1	1	-1	Violate rule (1)

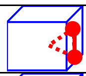
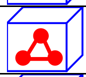
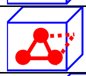
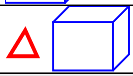
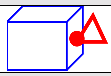
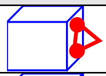
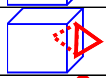
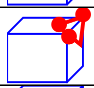
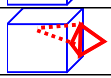
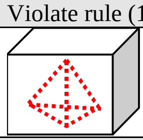
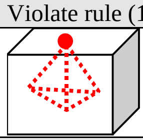
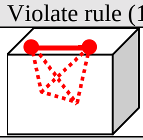
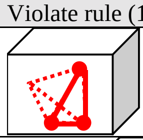
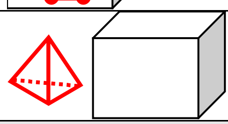
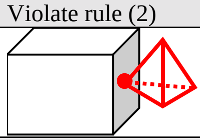
(3)	-1	1	2	
(4)	-1	2	-1	
(5)	-1	2	2	
(6)	2	-1	-1	
	2	-1	2	Violate rule (2)
(7)	2	0	-1	
	2	0	2	Violate rule (2)
(8)	2	1	-1	
(9)	2	1	2	
(10)	2	2	-1	
(11)	2	2	2	

Table 2. Topological Relationships between a simple face and a simple volume

3.3.3 A simple volume – a simple volume: En∈{-1, 3}, Bn∈{-1, 0, 1, 2}, In∈{-1, 3}, α=3, β=2, so the possible relationships are:

$4\times2 + 6 - MIN(3-1, 2) - IFF(3=3, 2+1, 2) = 8+6-2-3 = 9$

SN	En	Bn	In	relationship
	-1	-1	-1	Violate rule (1)
(1)	-1	-1	3	
	-1	0	-1	Violate rule (1)
(2)	-1	0	3	
	-1	1	-1	Violate rule (1)
(3)	-1	1	3	
	-1	2	-1	Violate rule (1)
(4)	-1	2	3	
(5)	3	-1	-1	
	3	-1	3	Violate rule (2)
(6)	3	0	-1	
	3	0	3	Violate rule (2)

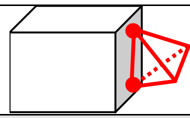
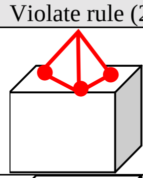
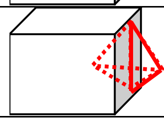
(7)	3	1	-1	
	3	1	3	Violate rule (2)
(8)	3	2	-1	
(9)	3	2	3	

Table 3. Topological Relationships between two simple volumes

4. EXTENSION OF BASIC SPACE AND 3D REALM

Definition 5: In finite discrete space N×N×N (N={0,…,n-1}), an **N-point** is a pair (x,y,z)∈N×N×N; an **N-segment** is different N-point pair (p, q); we define (p, q)=(q, p). $\mathbf{P_N}$ represents the set of all N-points; $\mathbf{S_N}$ represents the set of all N-segments.

In the follow table, we give out the topological relationships and operations between N-points and N-segments.

N-point×N-point	→ BOOL	=
N-point×N-point	→ N-segment	connection
N-point×N-segment	→ BOOL	on, in, disjoint
N-segment×N-segment	→ BOOL	intersected(=, overlap, meet, aligned), disjoint (parallel, aligned)
N-segment×N-segment	→ N-point	intersection

Table 4. Primitives defined on 3D grid

All these relationships and operations can be gotten by basic math operations, for details see the appendix of (Guting and Schneider 1993)

Definition 6: 3D Realm R=P∪S, such that:

(1) $P\subseteq P_N$, $S\subseteq S_N$;
(2) $\forall s\in S$: s =(p,q) ⇒ p∈P ∧ q∈P
(3) $\forall p\in P$ $\forall s\in S$: ¬(p in s)
(4) $\forall$s, t∈S, s≠t: ¬(s and t interest) ∧ ¬(s and t overlap)

5. DATA STRUCTURES AND TOPOLOGICAL RELATIONSHIPS BASED ON 3D REALM

5.1 Data Structures in 3D Realm

Definition 7: An R-block b is a set T of R-segments, such that:

(1) $\forall$ r, t ∈ T $\exists s_1, \ldots, s_m \in$ T: $r=s_1$, $t=s_m$, and
(2) $\forall$ i∈{1, …, m-1}: s_i and s_{i+1} meet.

Definition 8: An R-cycle c is a set of R-segments $S(c)=\{s_0, \ldots, s_{m-1}\}$, such that:

(1) $\forall i \in \{0, \ldots, m-1\}$: s_i meets $s_{(i+1) \bmod m}$
(2) There are no other touches between segments.

If all the segments of an R-cycle are all in the same plane, it is called a R-planar cycle, shortly as RP-cycle, otherwise it is called a non-R-planar cycle, shortly as RNP-cycle (it can be represented by the combination of RP-cycles).

In fact, if m=1, then R-cycle is degenerated to a R-segment, furthermore, a R-segment can be degenerated to a R-point.

In 2D space, it is enough to know all the intersection points of segments, but in 3D space, we have to know all the intersection between all faces. The intersections may be points and segments.

Definition 9: An R-face f is a quaternion (c, H, EP, ES), here c a RP-cycle, $H=\{h_1, \ldots, h_m\}$ is the set of RP-cycle (may be empty), $EP=\{ep_1, \ldots, ep_n\}$ is the set of R-points, $ER=\{es_1, \ldots, es_o\}$ is the set of all R-segments in c, c, h1, …, hm are all on a same plane, and they satisfy the next conditions (Here we use S(f) to denote the set of all the segments in all the RP-R-cycles):

(1) $\forall i \in \{1, \ldots, m\}$: h_i edge-inside c.
(2) $\forall i, j \in \{1, \ldots, m\}, i \neq j$: h_i and h_j are edge-outside.
(3) The RP-R-cycles of c and those in H are the only RP-R-cycles composed by the segments in S(f). (That is, an R-face cannot be divided into two non-intersecting R-faces.)
(4) $\forall i \in \{1, \ldots, n\}$: ep_i ***in*** c $\wedge$ $ep_i \notin$ h1 $\wedge \ldots \wedge$ $ep_i \notin$ hm, it ensures that the separated points in face, that is, the embedding points.
(5) $\forall i \in \{1, \ldots, o\}$: ($es_i$ ***in*** c $\vee$ es_i ***touch*** c) $\wedge$ (es_i ***disjoint*** h_1 $\vee$ es_i ***touch*** h_1) $\wedge \ldots \wedge$ (es_i ***disjoint*** h_m $\vee$ es_i ***touch*** h_m), it ensures that the segments in the face, that is, the embedding segments.

The relationships between face and other spatial objects can be represented through the relationships between the RP-R-cycles composing the face and other spatial objects.

If an R-face f is (c, $\varnothing$, EP, ES), that is, the embedding points and embedding segments have no effect on f, they are only useful in distinguishing spatial relationships.

Definition 10: An R-hull h is a set of R-faces $F(h)=\{f_0,\ldots,f_{m-1}\}$, S(h) denotes the set of all the segments in h, such that:

(1) $\forall i \in \{0, \ldots, m-1\}$: $f_i=(c_i, \varnothing, \varnothing, \varnothing)$.
(2) $\forall i \in \{0, \ldots, m-1\}$: f_i is area_outside to $f'_1, \ldots, f'_n \in F(h)$, such that n is the number of segments in f_i.
(3) $\forall r \in S(h)$, r only belongs two faces.
(4) $\forall f_i, f_j \in F(h), f_i \neq f_j, \exists f'_1, \ldots f'_n \in F(h)$, f_i is area-outside to $f'_1, \ldots, f'_n$ is area-outside to f_j. (That is, a face can connect to any other face through none face or several faces).

R-hull is the simplest volume, convex or concave, and it cannot be divided into two R-hulls through points or segments.

Definition 11: An R-Volume v is a pair (h, H), here h is an R-hull, $H=\{h_1, \ldots, h_m\}$ is a set of R-hulls (may be empty), they satisfy the next conditions:

(1) $\forall i \in \{1, \ldots, m\}$: h_i area-inside h;
(2) $\forall i, j \in \{1, \ldots, m\}, i \neq j$: h_i is vertex-outside, edge-outside or area-outside hj;
(3) There is no any other R-hull except for h and the R-hulls in H.

This kind of volume has holes, and those holes can be point, line, face and volume.

5.2 Topological Relationships

For simplicity, we only discuss the topological relationships between a N-point *p* and an R-hull *h*, a N-segment *s* and an R-hull *h*, an R-segment *s* and an R-hull *h*, an RP-cycle *c* and an R-hull *h*, and two R-hulls.

5.2.1 The relationships between an N-point *p* and an R-hull *h:* Similarly to the way to define the relationships between R-point and R-cycle in (Guting 1993), we can define on(p, *h*), in(p, *h*) and out(p, *h*).

5.2.2 The relationships between an N-segment *s* and an R-hull *h*: There are nine kinds of relationships between a line and a volume, which are all possible here:

- vertex_inside: All points of *s* are in *h*.
- edge_inside: *s* has end points on *h*, and other parts in *h*.
- on: *s* is in the ∂*h*.
- edge_boundary_inside: *s* has partial segments on *h*, and other parts in *h*.
- vertex_outside: All points of *s* are out of *h*.
- edge_outside: *s* has end points on *h*, and other parts out of *h*.
- point_intersect: *s* has only intersection points with ∂*h*, and partial segments both in and out of *h*.
- edge_boundary_outside: *s* has partial segments on *h*, and other parts out of *h*.
- edge_intersection: *s* has intersection segments with ∂*h*, and partial segments in and out of *h*.

5.2.3 The relationships between an R-segment *s* and an R-hull *h*: The relationships between R-segment *s* and R-hull *h* are the special cases of that between N-segment *s* and R-hull *h*, the next four conditions will not appear:

edge_boundary_inside, point_intersect, edge_boundary_outside, edge_intersect.

Only the following five kinds of conditions are possible:

vertex_inside, edge_inside, on, vertex_outside, edge_outside.

5.2.4 The relationships between an RP-cycle *c* and an R-hull *h*: There are eleven kinds between face and volume, which are all possible here, so the relationships between RP-cycle *c* and R-hull *h* are:

- vertex_inside: All points of *c* are in *h*.
- edge_inside: *c* has end points on *h*, and other parts in *h*.
- area_inside: *c* has segments on *h*, and other parts in *h*.
- on: *c* is in the ∂*h*.
- area_boundary_interior: *c* has partial face on *h*, other parts in *h*.
- vertex_outside: All points of *c* are out of *h*.
- edge_outside: *c* has end points on *h*, and other parts out of *h*.
- area_outside: *c* has segments on *h*, and other parts out of *h*.

- edge_intersect: *c* has segments (but no faces) on *h*, and partial faces both in and out of *h*.
- area_boundary_outside: *c* has partial face on *h*, and other parts out of *h*.
- area_intersect: *c* has partial face on *h*, and partial faces both in and out of *h*.

5.2.5 The relationships between R-hull *h1* and R-hull *h2*: There are nine kinds of relationships between two simple volume objects, which are all possible here, so the relationships between two R-hulls are:

- vertex_inside: All points of h1are in *h2*.
- edge_inside: *h1* has end points on *h2*, and other parts in *h2*.
- area_inside: *h1* has segments on *h2*, and other parts in *h2*.
- volume_inside: *h1* has faces on *h2*, and other parts in *h2*.
- vertex_outside: All points of *h1* are out of *h2*.
- edge_outside: *h1* has end points on *h2*, and other parts out of *h2*.
- area_outside: *h1* has segments on *h2*, and other parts out of *h2*.
- volume_outside: *h1* has faces on *h2*, and other parts out of *h2*.
- area-intersect: *h1* has partial face on *h2*, and partial volumes both in and out of *h2*.

6. 3D SPATIAL DATA TYPES

Here we give out the concepts of spatial data types based on 3D realm, we will give the details in another paper. The basic types imported are points, lines, faces and volumes.

For a given 3D realm R:

- The value of type **points** is a set of R-points.
- The value of type **lines** is a set of blocks that both of them are not vertex-intersected;
- The value of type **faces** is a set of faces that both of them are not edge-intersected;
- The value of type **volumes** is a set of volumes that both of them are not area-intersected.

7. CONCLUSIONS

This paper discusses the disadvantages in the method of realm. We extend 2D realm to 3D realm and define the spatial data structures on it. In order to give a better description of the topological relationships between spatial objects, we provide a method of Distribution of Dimension, which is not sensitive to the increase of dimensions, and the restriction rules are simpler than other methods. (The following table gives the number of distinguished topological relationships using different methods in 2D space.)

Method	A/A	L/A	P/A	L/L	P/L	P/P	Total
4IM	6	11	3	12	3	2	37
9IM	6	19	3	23	3	2	56
DEM	9	17	3	18	3	2	52
DE+9I	9	31	3	33	3	2	81
V9I	13	13	5	8	4	3	46
DD	7	9	3	8	3	2	32

Table 5. Compares of distinguished topological relationships using different methods (modified from (Clementini and Di Felice 1995)

REFERENCES

Chen, J., Li, C.M., Li, Z.L., Gold, C., 2001. A Voronoi-based 9-intersection model for spatial relations. Int. J. Geographical Information Science, 15(3), pp.201-220.

Clementini, E., Di Felice, P., 1995. A Comparison of Methods for Representing Topological Relationships. Information Sciences 3, pp.149-178.

Egenhofer, M.J., Herring, J.R., 1990. A mathematical framework for the definition of topological relationships. In Brassel, K. and Kishimoto, H. (Eds) Proceedings of the 4th International Symposium on Spatial Data Handling, Zurich, Columous, OH: International Geographical Union, pp.803-813.

Egenhofer, M.J.,and Franzosa, R.D.,1991. Point-set topological spatial relations. International Journal of Geographical Information Systems, 5 ,pp.161-176.

Guting, R.H., Schneider, M., 1993. Realms: A Foundation for Spatial Data Type in Database Systems. In D. Abel and B.C. Ooi, editors, Proc. 3rd Int. Conf. on Large Spatial Databases (SSD), Lecture Notes in Computer Science, pages 14-35. Springer Verlag.

Kelly, J.L., 1955. *General Topology*. VAN NOSTRAND REINHOLD COMPOANY, 1955.

Losa, A. de la, Cervelle, B., 1999. 3D Topological modeling and visualization for 3D GIS. Computers & Graphics 23, pp.469-478.

Molenaar, M., 1998. An Introduction to the theory of spatial object modeling for GIS. Taylor & Francis.

Rahman, A., Pilouk, M., Zlatanova, S., 2001. The 3D GIS software development: global efforts from researchers and vendors. Geoinformation Science Journal, 1(2),

Worboys, M., 1995. *GIS: a computing perspective*. Taylor & Francis publication.

Zlatanova, S., 1999. On 3D togical relationships. In A. Camelli, A. M. Tjoa, and R. R. Wagner, editors, Tenth International Workshop on Database and Expert Systems Applications. DEXA99, pp.913-919.

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