A Voronoi K-order Approach for Digital Elevation Model (DEM) Interpolation

Renliang ZHAO [1,3] Jun CHEN[2] Zhilin LI[3] C.M. Gold[3]

[1] The Central South University Changsha, China, 410083

[2] National Geometrics Center of China, Baishengcun, Zizhuyuan, Beijing, China, 100044

zhaorl@263.net, chenjun@nsdi.gov.cn

[3] Department of Land Surveying and Geo-Informatics

The Hong Kong Polytechnic University, Kowloon, Hong Kong

lsllz@polyu.edu.hk

Abstract

Digital Elevation Model （DEM） interpolation is one basic functions for spatial description and spatial analysis in GIS and related spatial information fields. DEM interpolation can be viewed as a function for determining the heights of unknown points using a set of proper known data. So, selecting a set of appropriate neighboring reference data points is one of the key steps for DEM interpolation. The selected reference points are used for estimating the value of elevation at any location in the given area. The commonly used search radius and quadrant search methods take reference points according to metric distance and are considered as metric methods. The reference points selected by a metric method might not be well distributed in space. This leads to the discontinuity problem of interpolated DEM surface and some artifacts might be generated.

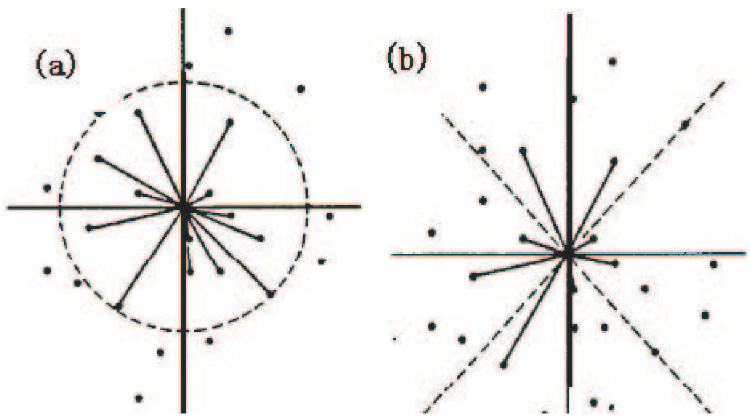
The use of Voronoi diagrams and 'area-stealing' (or natural neighbor interpolation) has been shown to adapt well to poor data distributions because insertion into the mesh of a sampling point generates a well–defined set of neighbors. The 'stolen area' of the immediate neighbors (also called first order neighbors) are used for determining the weighting function. However, other non-first order neighbors may also have influence or contribution to the value of estimated elevation because the topographic surface is continuous. In other word, it is rational to take K-order neighbors into account in DEM interpolation.

A Voronoi K-order based approach for DEM interpolation is discussed in this paper.
The limitations of traditional metric methods and the improvement achieved by Voronoi 'area-stealing' are discussed in the first part of the paper. The concept and computation of Voronoi k order neighbor are introduced in the second part. The use of Voronoi k order neighbors for selecting reference points and determining weighting functions are discussed in the third part. The k order-based interpolation approach is compared with metric and 1st order approaches in the last section of the paper.

1. Introduction

Digital Elevation Model （DEM） interpolation is one of basic functions for spatial description and spatial analysis in GIS and related spatial information fields [Carrara, 1997, Ferenc Sárközy,1998, Okabe, et al., 2001]. Interpolation can be viewed as a function for estimating the heights of unknown points using a set of proper known data. Generally, interpolation comprises two types, one type of global methods in which all data are used, and one type of local methods in which only partial data are used. Global methods are usually used for only trend surface analysis because of the limitation of the huge costs of computation. In contrast, local methods are more popular and practical for DEM since they involve only partial data and represent real surface more precisely. DEM interpolation described here is restricted to local methods.

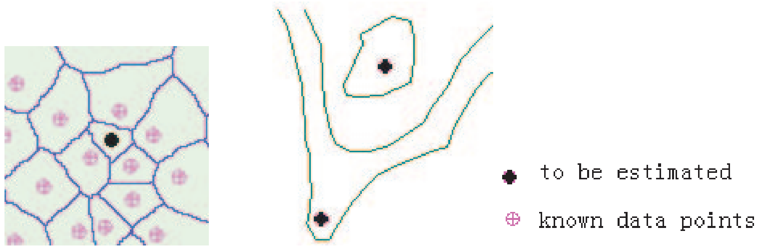
So far a large variety of local interpolation methods has been proposed or developed such as weighted moving averages, bicubic splines, kriging, finite elements, etc [Carrara,1997]. Different from global interpolation methods, local interpolation methods depend on the selection of these appropriate neighbors [Gold, 1992a]. They must select a set of nearby data points. Therefore, one of the key steps for all these local methods, is how to select a set of appropriate neighboring reference data points to the sample location being estimated. Traditionally, most methods adopt "distance" measure to determine necessary neighboring known height data points to the location to be estimated [Gold 1992a, Du 1996]. Usually, this procedure is implemented using the commonly "radius search" and "sector search" methods seen in Figure 1. Radius search method takes those points fallen into a circle of predefined radius, and sector search method requires respectively no less than one data points located in each sector such as quadrant or octant. These methods are considered as pure metric methods. However, the reference points selected by a metric method might not be well distributed in space. They could not accurately reflect spatial correlation between unknown location and known data points because spatial correlation depend other factors other than distance. This leads to the discontinuity problem of interpolated DEM surface and the generation of some artifacts [Gold, 1992a].



(a) circle search method (b) sector search method

Figure 1 Radius search and sector search method for selecting reference points

In order to improve the situation of above traditional methods, a Voronoi diagram based method is introduced into DEM interpolation [Sukumar , 1997, Sibson, 1981, Gold, 1989, Gold, 1992a, Du, 1996]. The use of Voronoi diagrams and 'area-stealing' has been proved to adapt well to poor data distributions because the insertion of unknown into the mesh of a sampling point generates a well–defined set of neighbors. The 'stolen area' of the immediate neighbors (also called first order neighbors, natural neighbors) is used for determining the weighting function. Up to now, only first order neighbors are taken into account in this method. However, closer points might be excluded if only first order neighbors are considered because closer points could belong to non-first order neighbors in term of the definition of Voronoi neighbors and regions. Therefore, other non-first order neighbors may also have influence or contribution to the value of estimated elevation since the surface is often supposed to be continuous. Furthermore, the number of first order neighbors will not meet the needs of DEM interpolation when the number is too little, for example, in the case of Figure 2(a). At the same time, this might cause the generation of "flat terrain" in some case shown as Figure 2(b) and (c), as in the methods based on Triangular Irregular Networks (TIN).



(a) (b)

(a) the number of the selected points is less than 6

(b) flat terrain caused by the use of only 1 order neighbors

Figure 2 First order neighbors can not meet the needs of interpolation

It indicates that it is rational to take non-first order neighbors on Voronoi diagrams into account. However, so far there is a lack of investigation into the role and contribution of non-first order neighbors in local DEM interpolation. In this study, it is attempted that Voronoi based k order neighbors are introduced into DEM interpolation. A Voronoi K-order based approach for DEM interpolation is proposed in this paper.

The remaining of this paper is structured as follows. Section 2 gives the definition of k order neighbor on raster Voronoi diagrams. Section 3 describes how to employ k order neighbors for selecting the set of appropriate neighboring data points to the unknown location being estimated, and then how to use these selected data for DEM interpolation. Some experiments and discuss are given in Section 4. Section 5 concludes this paper and discusses further related researches.

2. Voronoi K Order Neighbors

A Voronoi diagram is one of fundamental geometric structure, which was first propsed by Dirichlet in 1850 and Voronoi in 1908 (Aurenhammer, 1991 for a survey). A Voronoi diagram is composed of Voronoi regions (or called influent regions), which is associated with a spatial object (generator). Each point in the Voronoi region is closer to the spatial object than the others. Ordinarily, Voronoi diagrams are referred as Voronoi diagrams for points, i.e., the generators are composed of discrete spatial points. In fact, Voronoi diagrams has been extended to spatial lines, areas, even higher dimensional objects, generally called generalized Voronoi diagrams (Okabe, et al, 2001). Figure 3 shows such a generalized Voronoi diagram.

Figure 3 A Voronoi diagram

Generally, a Voronoi diagram can be obtained with vector based or raster based methods in terms of the nearest rule that all the nearest “empty space” to a spatial object is associated with this object. Most current methods are vector-based, for example, the classic divide and conquer method, incremental method, sweepline method (Aurenhammer, 1991, Okabe et al, 2001). Raster based methods are implemented more simply and easily. In this study, a raster method using distance transform based on mathematical morphology is adopted to generate the required Voronoi diagrams. The Voronoi diagram in Figure 3 is constructed with this raster method.

A Voronoi diagram actually describes the spatial influent region for each a generator and well defines neighboring relations among generators in a space. The neighboring relation between two objects can be defined clearly through topological solution, i.e., two objects are neighboring if they share common Voronoi boundaries [Gold, 1992b]. This definition is just that of common Voronoi immediate neighbor. Immediate neighboring relation is only a basic neighboring relation on a Voronoi diagram. In many cases, immediate neighbors to immediate neighbors to an object, i.e., an ojbect's *immediate neighbors' immediate neighbors* are also of interests very much because such neighbor's neighbors imply important neighboring information like immediate neighbors. For the sake of clearness, one can say immediate neighbors as first order neighbors, immediate neighbor's immediate neighbors as second order neighbors, seen in Figure 4. In a similar way, one can define k-order neighbors to an object as follows:

(1) One object *A* are first order neighbour to another object *B*, if they share Voronoi boundaries;

(2) An object *A* is a k order neighbour to another object *B*, if *A* is not *B*'s a k-2 order neighbour and shares Voronoi boundaries with *B*'s a (k-1) order neighbour.

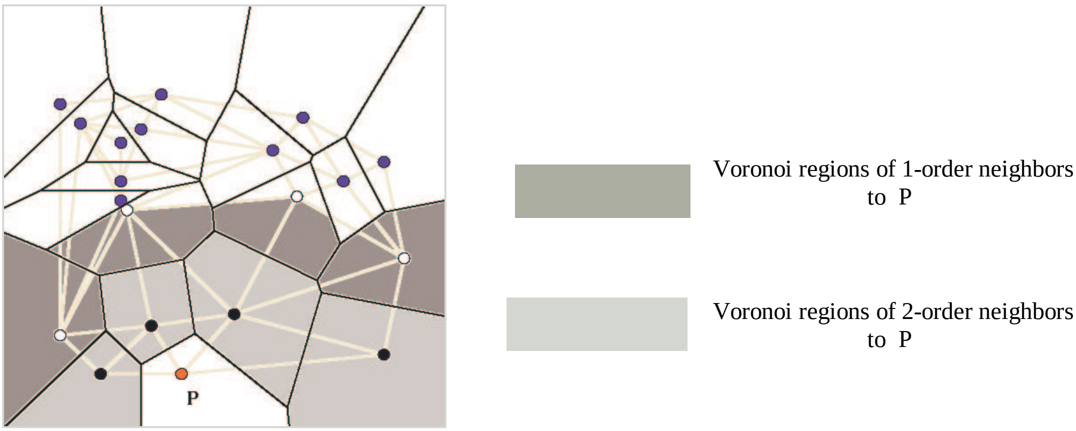


Figure 4 1 order and 2 order neighbors to point P

K order neighboring relation actually describe the "close" or "neighboring" degree between two objects more detailed than immediate neighbor, it can be viewed as a metric of spatial neighborhood. One can prove the number "K" of k order neighbors meets the properties of a metric. Let *vd* (*X*, *Y*) denote the number "K" of k order neighbors between object *X* and *Y*, then

1) *vd* (*B*, *A*) = *0* if two objects are equal;

2) *vd* (*B*, *A*) = *vd* (*A*, *B*);

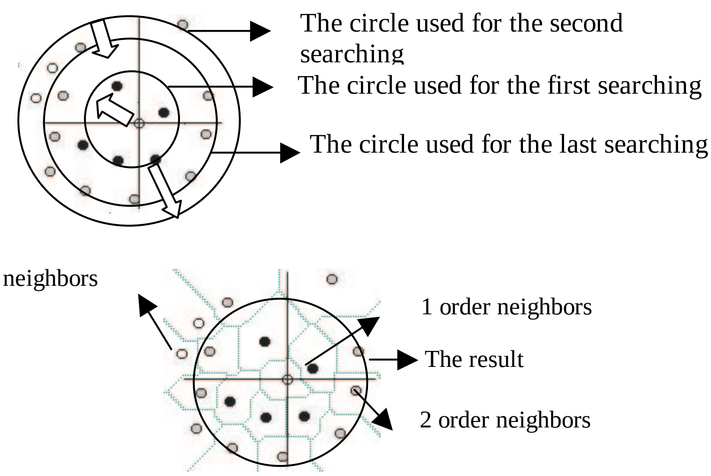
3) If *vd* (*A*, *P*) > *vd* (*B*, *P*), *vd* (*B*, *P*) > *vd* (*C*, *P*) then *vd* (*A*, *P*) > *vd* (*C*, *P*).

3. Selection of data and interpolation based on k order neighbors

3.1 Strategy for Selection of the data points for interpolation

As described above, the selection of data points is one of key steps and determines the quality and efficiency of DEM interpolation. Traditional radius search method or sector search method takes those points fallen into a circle of predefined radius. These methods are simple, however, the reference points selected by a metric method might not be well distributed in space. This shortcoming will cause the problem of DEM quality. At the same time, there is some blindness in searching data points, due to the difficulty in determining the reasonable and appropriate radius of search circle. The points needed by an interpolating point are often found after repeating to search for several times with deferent radiuses, even this procedure will not stop for ever if the radius is not appropriate, illustrated in Figure 5 (a). Therefore, this is another sever deficiency and reduces the efficiency of DEM interpolation.

In order to overcome the shortcoming of traditional methods at the same time, here K order spatial neighboring relation is adopted into spatial interpolation and a new strategy for searching neighboring points is formed: search for neighboring points in turn of the order from the near to the far instead of the radius of search circle, and cease until the number of selected points meets the requirements of interpolation methods, shown in Figure 5(b). The selected points can be used for many DEM interpolation methods such as weighted moving averages, local polynomial patches, kriging, least square configuration, etc. This strategy can avoid the random of searching data points caused by the circle search method by reducing the searching range to a local range and the order of k order neighbors. Also, it can prevent the occurrence of ill-distribution of selected data points in the search circle method. Moreover, it is more feasible and improves the problems in Figure 2 caused by the use of only immediate neighbors described in Section 1.



(a) Selecting data points by searching circle randomly and blindly
(b) Selecting data points by k-order neighbors in turn from near to far

3.2 Interpolation based k order neighbors

After acquiring the appropriate data points, the following work is just to weight the importance of these data points to the location to be estimated and compute the heights. There are several methods for computing the weights. One common and simple method is to use inverse distance as weights. When this method is used, height can be fast computed by the following equation.

$$z_p = \sum_{i=1}^{n} w_i z_i / \sum_{i=1}^{n} w_i \,,\; w_i = [(x_p - x_i)^2 + (y_p - y_i)^2]^{-1}$$

where, z_p denotes the height of point P (x_p, y_p) to be estimated and w_i denotes the weight of point (x_i, y_i).

In this equation, although the selected points are k order neighbors to point P, the weight function could not reflect the difference between different k order neighbors. In terms of the definition of k order neighbors, for instance, first order neighbors is the immediate neighbors to P, second order neighbors is the immediate neighbors' immediate neighbors, they are so different, thus they should play different role in the estimation of heights. For this purpose, the following equation might be adopted,

$$z_p = \sum_{i=1}^{n} w_i z_i / \sum_{i=1}^{n} w_i \text{ , } w_i = [(x_p - x_i)^2 + (y_p - y_i)^2]^{-1}(1 + v_i)$$

where, *vi* denotes the influence of the order of neighbors, $v_i = 1/k_i$ and k_i is the number of neighboring order between point P (x_p, y_p) and point (x_i, y_i).

4. Experiments

In order to compare and analyze the interpolation method based on k order neighbors, two types of experiments are implemented. These experiments involve four methods for DEM interpolation, the selected data points in four methods are respectively the nearest neighbor, circle search, 1 order neighbors, 2 order neighbors. These experiments are performed in the same environments such as the same machine and software.

In first experiments, a comparison in time consuming, shown in Figure 6. It is shown that the consuming time of circle search method depends on the number of data points and increases with the increase of the number of data points faster, but for the other three methods, the time varies very little with the increase of points. Especially, when the distribution of data points is not good, time cost will become larger.

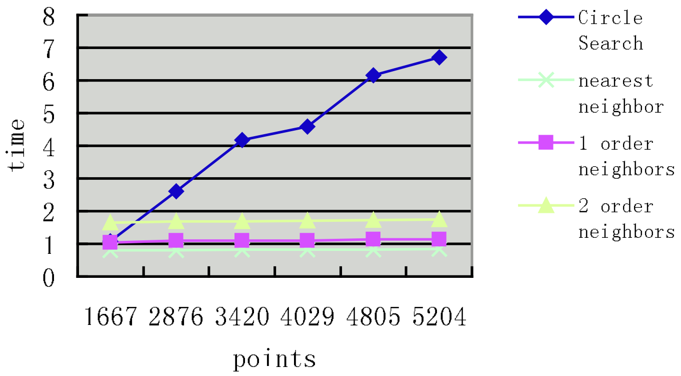
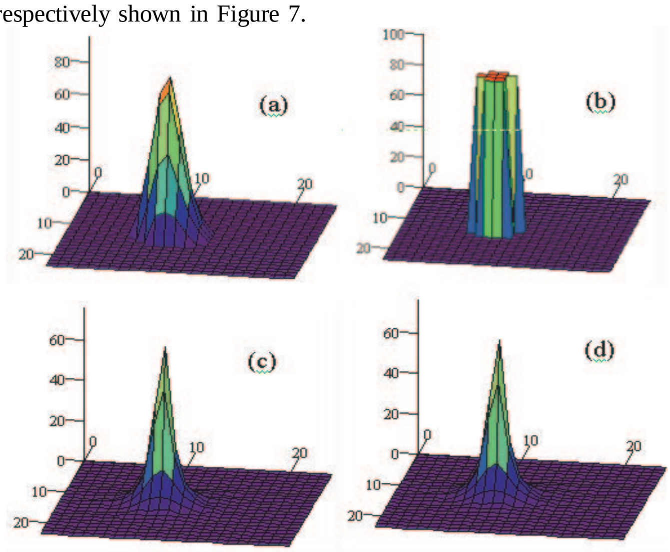


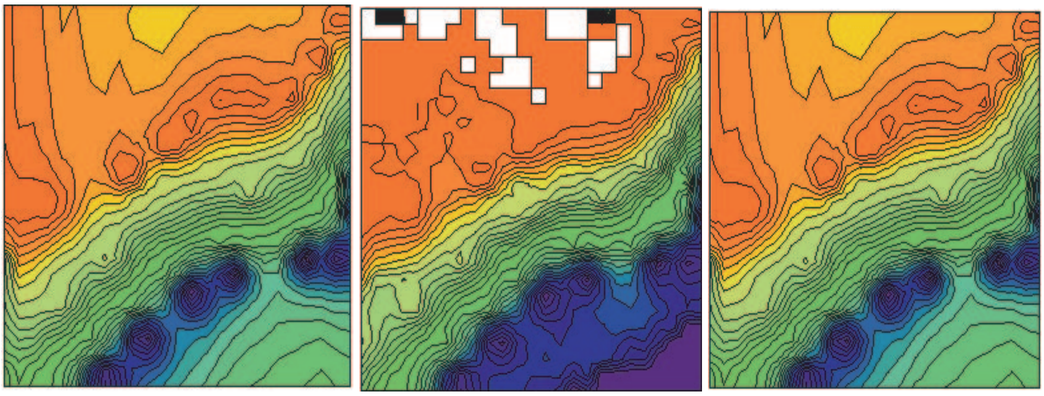
Figure 6 A comparison of four methods in time efficiency

In Second experiments, the original data consist of a set of sparse points of which the height of one point is very high and the others are the same. The results of the four methods are respectively shown in Figure 7. They indicate that the use of the order of neighbors makes the surface become more smoothing and reasonable to some degree.



(a) circle search method (b) the nearest neighbor
(b) (c) 1 order neighbors (d) 2 order neighbors
Figure 6 A comparison of the result of four methods

In third sample, a set of contours is transformed into gird DEM using the method of k order neighbor, illustrated in Figure 8.



(a) circle search method (b) 1 order neighbors (c) 2 order neighbors
Figure 8 A comparison of the generation of contours

5. Conclusions

In this paper, an alternative method is described for the data selection and interpolation with the combination of distance and topological methods based on k order neighbors using Voronoi diagrams. Several comparative experiments and analysis show the data points for interpolation can be selected in a more reasonable through Voronoi K order neighbors. The use of k order neighbors can avoid the searching blindness and ill-distribution of searching data points in the circle search method by the order of k order neighbors. Moreover, it is more feasible and improves the problems caused by the use of only immediate neighbors.

Acknowledgements:

The work described in this paper was supported by a grant from Research Grants Council of the Hong Kong Special Administrative Region (Project PolyU 5048/98E) and by the National Natural Science Foundation of China (No. 69833010).

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