A Description Method of Spatial Complexity in Terms of Visibility

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Abstract

The final goal of this research is grasping the relation between visual quality and shape of a real space. The visual quality of a real space means effects on human psychology or difficulty of evacuation. As the first step, this paper argues a description method of spatial complexity in terms of visibility. A simple concave figure can be used as an examine space to calculate distribution of visible quantity. Firstly, we define formulas to obtain exact value of visible quantity in the space. Then, visible quantity distribution of the space is visualized based on the formulas. Finally, we try to describe the complexity of the space. Average and standard deviation of the visible quantity distribution are used to describe the complexity and transition of them is observed with transforming the space. The relationship between the intuitive impression of space and visible quantity was suggested as a result of examination. We present the possibility of evaluating visual quality of a real space by visible quantity of the space.
Keywords: visibility, visible quantity, spatial complexity, concave space

1 Introduction

Various attempts have been made to link physical quantity based on geometric shape of real spaces and psychological responses of humans. Visible quantity is one example and is being studied from various perspectives. Visible quantity is a fixed quantity in respect to the position of the visual point, not affected by visual direction. It can therefore be discussed in relation to psychological effects imposed by the geometrical shape at that position, without being confined to the scope of restricted visual direction.

However, most studies on visible quantity until now have focused on discretely investigating and describing visible and invisible relations in actual space, and do not clarify the intrinsic values and distribution of visible quantity. The author is studying the possibility of the relation between physical quantity and psychological response of numerous space shapes amounting to relation between visible quantity and psychological response in all directions. As phase 1 of this study, this paper proposes a description method of spatial complexity in terms of visibility.

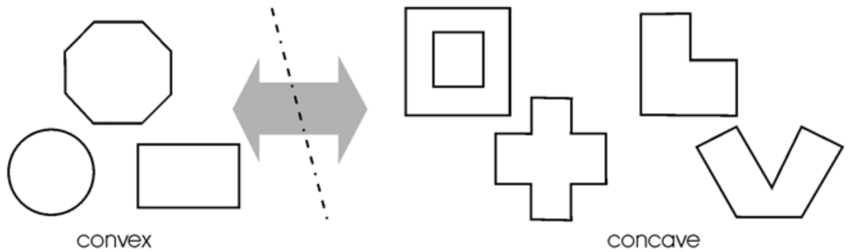

Fig. 1. The evaluation of concave complexity in terms of visibility

A wide range of research results in the computational geometry field can be given for visible quantity. In relation to methods for dividing visible domains based on spaces from given drawings, museum problems, fortification problems, lighting problems, etc. have been pinpointed(for example Akiyama et al(1996)). High speed algorithms for obtaining visibility relation has been studied by many researchers(for example Asano(1990) or B. Chazelle et al (1989)). In the field of architectural engineering and urban planning, studies such as the following have also been conducted: study on arrangement characteristics of Mosque focusing on visibility of the mosque from inside Islamic cities by Oikawa(2000), analysis of urban factors which have a large effect on visibility by Kametani et al(1997) and Lee et al(1997), study on relation between urban structure and shape restrictions and visibility by Hsiao et al(1997). etc.

This study aimed at grasping the relation between visual quality and shape of a real space using visible quantity defined in continuous space. The visual quality of a real space means effects on human psychology or difficulty of evacuation. This report does not go into the relation between visible quantity and psychological level. It will however study the feasibility of applications to the realistic space by clarifying that visible quantity occurs due to the shape of space.

The approach employed here is corresponding to evaluate degree of concave instead of classify plane figures into only two categories, convex figure and concave figure as illustrated in Figure 1.

2 Definition of Visible Quantity and Calculation Method

2.1 Definition of Visible Quantity

Taking into consideration the closed domain Ω shown in Figure 2, when the line connecting the two points in the given Ω does not intersect the boundary of domain Ω (when Ω is a polygon, the sides), these two points are visible. Here, the visibility $\Phi_{vis}(x,y)$ is defined as the area of the domain made up by the aggregate of points that are visible from $P(x,y)$ inside Ω (see figure 2). $\Phi_{inv}(x,y)$ is defined as the invisible area of the domain Ω from $P(x,y)$. If Ω is a convex drawing, since any combination of 2 points inside Ω is visible, visible quantity is equivalent to area S_Ω inside Ω at any point. In this paper, as the first step in studying visible quantity in space, concave figures made by cutting out part of a square space with primitive drawings (squares, rectangles, circles) are used to study visible quantity. And we observe changes in visible quantity due to the cutting out method and differences in the shape of drawings cut out. Regarding the effects of the shape of space on psychological aspects, since the size of space may also play a part, calculation using visible quantity directly is also possible. However in this report, based on the aim of investigating changes in visible quantity due to differences in shape, visible rate $\phi_{vis}(x,y)$ was calculated by dividing visible quantity $\Phi_{vis}(x,y)$ by area $S_{\Omega,}$ and this value was compared. In addition, the average μ_Ω of visibility rates over the whole domain Ω and standard deviation σ_Ω were calculated. μ_Ω is considered to be corresponding to the degree of ease in grasping the whole space, while σ_Ω is considered to be equivalent to the intensity of changes in visibility quantity in space.

Table 1. Symbols and notation

Ω	closed domain
$P(x,y)$.	point inside the domain Ω
S_Ω	the area of the domain Ω
$\Phi_{vis}(x,y)$	the visible area of the domain Ω
$\Phi_{inv}(x,y)$	the invisible area of the domain Ω
$\phi_{vis}(x,y)$	the visible ratio of the domain Ω (=$\Phi_{vis}(x,y)$ / $S_{\Omega)}$
$\mu_\Omega(x,y)$	the average of visibility rates over the whole domain Ω
$\sigma_\Omega(x,y)$	the standard deviation of visibility rates over the whole domain

Here, the relation with physical quantity used in past studies is described. In the study in Oikawa(2000), the number of minarets which are visible from the measuring points at the center of streets in actual space are taken as number of visibilities. This is equivalent to the case of limiting visible subjects and discretely calculating in the visibility rates of this paper. By distributing measuring points at regular intervals in a domain and taking the subjects of visibility evaluation as other measuring points, the value obtained by dividing the number of visibilities by the number of measurements approaches the visibility rate in this paper when the distribution density of measuring points is increased. In the studies in Kametani *et al*(1997) and Lee *et al*(1997), they arranged two types of visible subjects, whole domain and inside streets, and also arranged two physical quantities, relative visibility rate and visibility rate in the visible domain. The visible domain is also calculated discretely and the domain is considered as the three-dimensional space. in these studies. Despite differences, the idea is same as the visibility rate used in this paper.

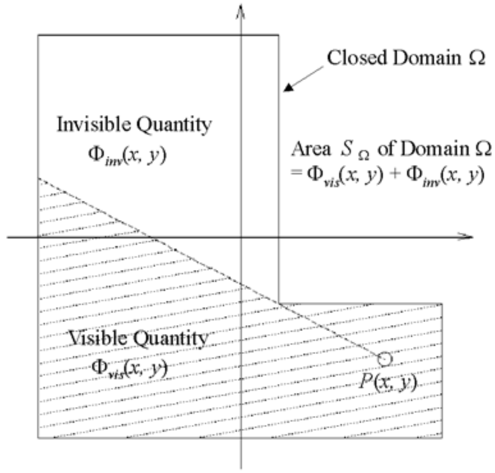

Fig. 2. Definition of visibility quantity

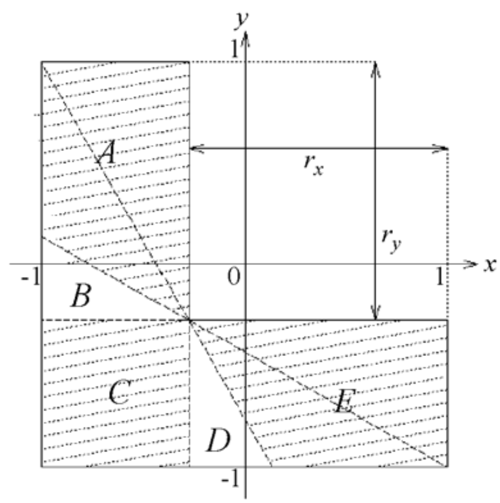

Fig. 3. Parameters and partial domain for calculation of visible quantity for L-shaped space

The following shows how visibility rate is calculated for a L-shaped space, square cut out space, and circular cut out space

2.2 Calculation for L-shaped space

As shown in Figure 3, the domain made by cutting out a rectangular shape measuring r_x x r_y from a square space(2 x 2) is called L-shaped space. In order to seek the visible quantity of such spaces, it is necessary to divide the Ω into five sub domains from *A* to *E* based on the extended lines of the sides of the domain boundary, and the lines joining vertices of the polygon. The following shows the visible quantity for segment domains *A* to *E* shown in Figure 3. Here x, y, r_x and r_y follow the symbols in Figure 2 and Figure 3. Taking the visible quantity as $\Phi_{vis}(x,y)$ and invisible quantity as $\Phi_{inv}(x,y)$:

$$\Phi_{vis} + \Phi_{inv} = S_\Omega = - r_x r_y + 4 \tag{1}$$

Φ_{inv} is expressed in the following for convenience.

$$\text{domain } A\text{: } \Phi_{inv} = \frac{-r_y + 2}{2(y + r_y - 1)}\{(2 - r_y)x + 2r_x y + r_x r_y + r_y - 2\} \tag{2}$$

$$B\text{: } \Phi_{inv} = \frac{r_x^2}{2(-x - r_x + 1)}(y + r_y - 1) \tag{3}$$

$$C\text{: } \Phi_{inv} = 0 \tag{4}$$

$$D\text{: } \Phi_{inv} = \frac{r_y^2}{2(-y - r_y + 1)}(x + r_x - 1) \tag{5}$$

$$E\text{: } \Phi_{inv} = \frac{-r_x + 2}{2(x + r_x - 1)}\{2r_y x + (2 - r_x)y + r_x r_y + r_x - 2\} \tag{6}$$

$\Phi_{inv}(x,y)$ in equations (2) to (6) are continuous in domain Ω. Thus both $\Phi_{vis}(x,y)$ and $\phi_{vis}(x,y)$ are also continuous in Ω according to equation (1).

2.3 Calculation for square cut out space

As shown in Figure 4, the square spaces(2 x 2) are cut out using the square with side $2r$ arranged parallel to the square making up the external circumference of space, taking the origin as the center. In this case, it is necessary to study only the $y \le x$ of the first quadrant from the symmetry. Still as shown in Figure 4, there is a need to divide it into the five sub domains from A to E using the extension lines of the sides of the cut out square and straight lines linking the vertices of two squares. Because this calculation becomes slightly complicated than the L-shaped space, a separate calculation method is applied. As shown in Figure 5, taking P_1 to be the first point intersecting the square shape cut out by rotating the visual line direction starting from the positive direction of the x axis with *P* as the visual point, P_2 to be the final point of intersection, S_1 to be the area of the domain enclosed by straight line PP_1, straight line *PO* and the sides of the outer square, and S_2 to be the area of the domain enclosed by straight line PP_2, straight line *PO*, and the sides of the outer square. Here, when the x coordinate of the point of intersection between straight line PP_1 and straight line $y = 1$ is taken as X_1 and that of the point of intersection between straight line PP_2 and straight line $y = 1$ is X_2, then

$$X_1 = \frac{x - rx + ry - r}{y - r}, X_2 = \frac{rx - x + ry + r}{y + r}$$

Using these, S_1 will be as follows for domains A and B or when $y \geq r$,

$$A,B: \; S_1 = \frac{x+1}{2x(x+r)}(rx^2 + rxy + rx + ry) \tag{7}$$

For domains C and D, or when $y \leq r$ and $X_1 \leq -1$, then

$$C,D: \; S_1 = \frac{r(x+1)^2(x-y)}{2x(x-r)} \tag{8}$$

For domain E, or when $X_1 \geq -1$, then

$$E: \; S_1 = \frac{(x+y)^2 + r((y-2)x^2 - (y^2 - 2y + 3)x - y)}{2x(y-r)} \tag{9}$$

Likewise S_2 will be as follows for domains B and C, $X_2 \leq -1$,

$$B,C: \; S_2 = \frac{r(x+1)^2(x+y)}{2x(x-r)} \tag{10}$$

For domains A, F, and E, or $X_2 \geq -1$,

$$A,D,E: \; S_2 = \frac{-(x-y)^2 + r((y+2)x^2 + (y^2 + 2y + 3)x - y)}{2x(y+r)} \tag{11}$$

As S_1 and S_2 include the visible domains as shown in Figure 5, it is impossible to simply total these results. Taking the areas of the visible domains included in S_1 and S_2 respectively as S_1' and S_2', then:

$$A,B: \; S_1' = \frac{r}{2x}(x^2 - 3rx + xy + ry) \tag{12}$$

$$C,D,E: \; S_1' = \frac{r}{2x}(x-r)(x-y) \tag{13}$$

$$\text{Whole domain: } S_2' = \frac{r}{2x}(x+y)(x-r) \tag{14}$$

Taking equations (7) to (14) into account, the invisible quantity is then

$$\Phi_{inv}(x,y) \quad = S_1 + S_2 - S_1^{'} - S_2^{'} - 4r^2 \tag{15}$$

This is a continuous function in $\Omega^{(1)}$. Both visible quantity and visibility rate are also continuous in domain Ω.

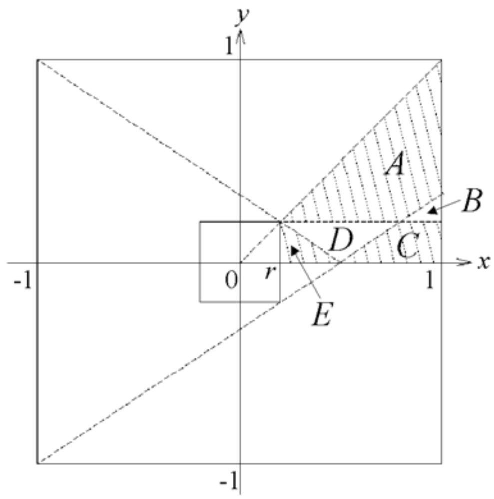

Fig. 4. Parameters and partial domain for calculation of visible quantity for square cut out space

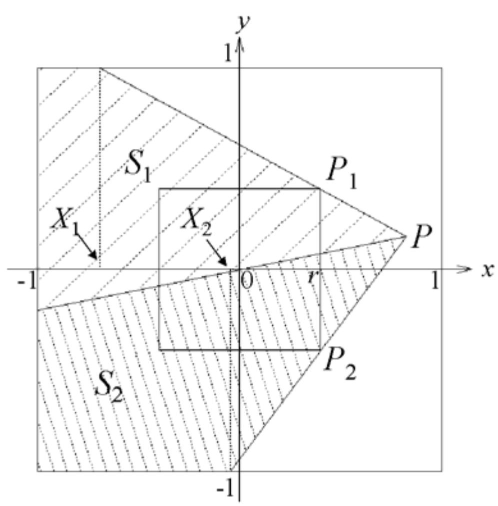

Fig. 5. Calculation of square cut out of space

2.4 Calculation for circular cut out space

Figure 6 shows the cutting off of a square space(2 x 2) using a circle with a radius r around the origin. In the same way as cutting out using a square space, only the range $y \le x$ of the first quadrant needs to be taken into consideration. There is a need to split the area into the four sub domains A to D shown in Figure 6 according to the tangent of the circle passing through the vertex of the outer square. As shown in Figure 7, like 2.3, calculation is done using S_1 and S_2. P_1 and P_2 are tangent points lying on the tangent of the circle passing through P. Taking P as the visual point, the first tangent point appearing when the visual line is rotated starting from the normal direction of the x axis is taken as P_1 and the other point as P_2. Here, when the y coordinate of the point of intersection between straight line PP_1 and straight line x = -1 is taken as Y_1 and the x coordinate of the point of intersection between straight line PP_2 and straight line y = -1 is taken as X_2, then

$$Y_1 = \frac{r\left(x^2+y^2\right)+rx-y\sqrt{x^2+y^2-r^2}}{x\sqrt{x^2+y^2-r^2}+ry} \qquad X_2 = \frac{r\left(x^2+y^2\right)+ry-x\sqrt{x^2+y^2-r^2}}{rx+y\sqrt{x^2+y^2-r^2}}$$

Using these equations, S_1 is as follows for domains A and B, or when $Y_1 \le 1$

$$A,B\colon\ S_1 = \frac{r(x+1)^2\left(x^2+y^2\right)}{2x(x\sqrt{x^2+y^2-r^2}+ry)} \tag{16}$$

It is as follows for domains C and D, or when $Y_1 \ge 1$

$$C,D\colon\ S_1 = \frac{(x+y)^2\sqrt{x^2+y^2-r^2}+rx\left((y-2)\left(x^2+y^2\right)-2x\right)}{2x(y\sqrt{x^2+y^2-r^2}-rx)} \tag{17}$$

Likewise, S_2 is as follows for domain B, or when $X_2 \le -1$

$$B\colon\ S_2 = \frac{r(x+1)^2\left(x^2+y^2\right)}{2x(x\sqrt{x^2+y^2-r^2}-ry)} \tag{18}$$

It is as follows for domains A and C, or when $-1 \le X_2 \le 1$

$$A,C\colon\ S_2 = \frac{-(x-y)^2\sqrt{x^2+y^2-r^2}+rx\left((y+2)\left(x^2+y^2\right)+2x\right)}{2x(rx+y\sqrt{x^2+y^2-r^2})} \tag{19}$$

It is as follows for domain D, or when $X_2 \ge 1$

$$D\colon\ S_2 = \frac{\left(4x^2\sqrt{x^2+y^2-r^2}+r\left(x^4-2x^3+x^2+y^2x^2-2y^2x-4yx+y^2\right)\right)}{2x(x\sqrt{x^2+y^2-r^2}-ry)} \tag{20}$$

Like 2.3, S_1 and S_2 include the visible domain S_3 as shown in Figure 8. As shown in Figure 8, S_3 is the quantity depending only on the distance from the origin, and can be expressed by the following equation regardless of the domain.

$$\text{All domains: } S_3 = r\sqrt{x^2+y^2-r^2} - r^2 \cos^{-1}\frac{r}{\sqrt{x^2+y^2}} \tag{21}$$

The invisible quantity is as follows considering equations (16) to (21)

$$\Phi_{inv}(x,y) = S_1 + S_2 - S_3 - \pi r^2 \tag{22}$$

This is a continuous function inside $\Omega^{(1)}$. Both the visible quantity and visible rate are continuous in domain Ω as well.

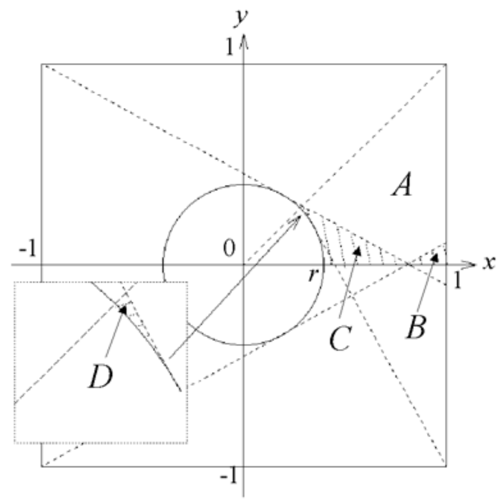

Fig. 6. Parameters and partial domain for calcuation of visisble quantity for circular cut out space

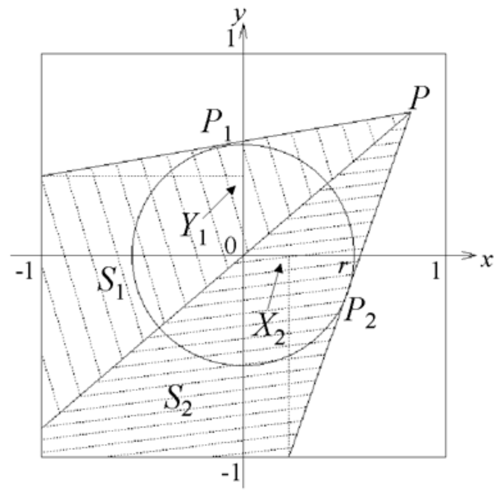

Fig. 7. Calculation of ciruclar cut out space

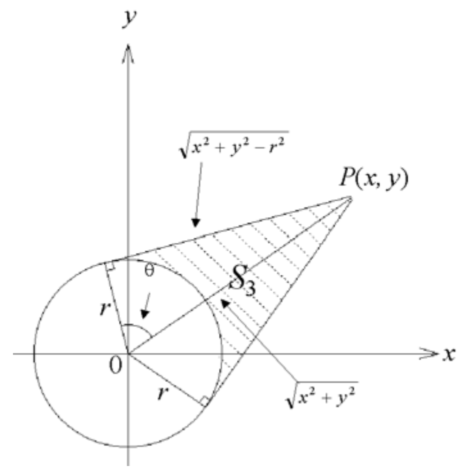

Fig. 8. Visible domain S_3 included in S_1 and S_2

3 Changes in Visible Quantity According to Difference in Shape of Given Space

The previous chapter discusses the methods of calculating the visible quantity when the corners of a square space are cut by a rectangle and when the center is cut out by a square and by a circle. The following describes the actual calculations performed and evaluation of changes in the visible quantity while changing parameters. Also studied were the differences in the changes in the visible quantity according to the method of arranging the cut drawing and the shape of the drawing with cut out center. As shown in Figure 9 to 12, the distribution drawings on the visibility rate in the following investigations plot $\phi_{vis}(x,y)$ to obtain an isometric projected drawing from the normal directions of x, y, and z. Contour lines are drawn every 0.1[2], and a domain composing Ω is drawn on the xy plane. Other than the distribution of the visibility rate, the average visibility rate and the standard deviation of visibility rate in domain Ω were also calculated. These were obtained numerically by dividing the inside of Ω into a mesh of 0.01 x 0.01, not obtained analytically.

3.1 When the size of the square shape is changed for a L-shaped space with $r_x = r_y$

Figure 9 shows the distribution of the visibility rate when r_x (= r_y)= 1.5, 1.0, and 0.5, and the average visibility rate and standard deviation inside Ω when r_x (= r_y) is changed continuously from 2 to 0. The maximum visibility rate is 1.0. As the figure shows, the visibility rate is constant (=1.0) in the segment domain where whole domain is visible. However for the domains at the two ends of L-shaped space, the visibility rate decreases towards the ends. At the same time, the further away is Ω from a square, in other words, the greater r_x is, the more intense are the fluctuations in the visibility rate near straight line $x = 1 - r_x$, and straight line $y = 1 - r_y$. As the graph on the average visibility rate shows, the greater r_x is, the more the average visibility rate approaches 0.5, and the more the two spaces become independent visually. Regardless of whether r_x is small or large, the standard deviation becomes small, and is a consistent space visually.

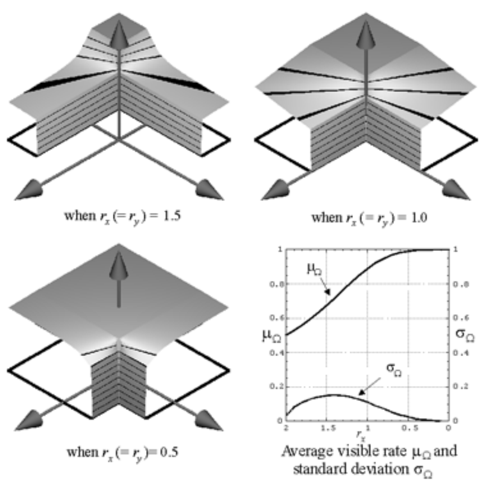

Fig. 9. Distribution of visible rates $\phi_{vis}(x,y)$ when size of cut out square shape is changed, average visible rate and standard deviation

3.2 When the aspect ratio of a cut rectangular shape is changed by changing only the size of r_x, taking $r_x = 1.5$ (constant) in a L-shaped space

Figure 10 shows the distribution of the visibility rate when r_x is varied to 1.5, 1.0, and 0.5, and the average visibility rate and standard deviation inside Ω when r_x is changed continuously from 2 to 0. As the graph on the average visibility rate shows, domain Ω forms a rectangular shape when r_x is small and large, and the average visibility rate is maximum at the two ends of r_x and is minimum at between them. Looking at the space distribution, even when $r_x = 0.5$ at which the average visibility rate is large, local visibility rate changes intensely, indicating that it is difficult to pinpoint distribution only from the average value. The standard deviation of the visibility rate when r_x is large is large compared to other cases.

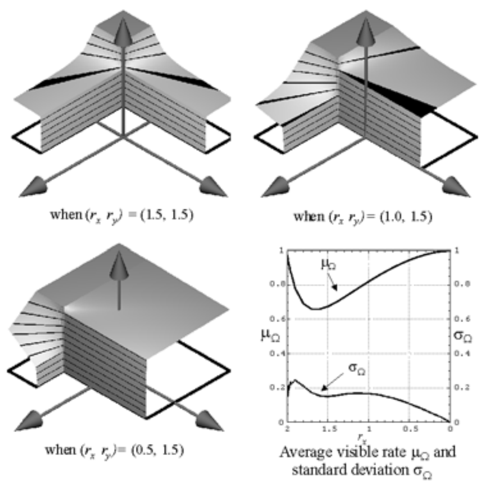

Fig. 10. Distribution of visible rates $\phi_{vis}(x,y)$ when aspect of cut out rectangular shape is changed, average visible rate and standard deviation

3.3 When r_x is changes in the square cut out space

Figure 11 shows the distribution of the visibility rate when r is 0.2, 0.5, and 0.8, and the average visibility rate and standard deviation inside Ω when r is changed continuously from 1 to 0. When r is near 1, the space is divided into four convex domains, thus the average visibility rate approaches 0.25. Though the visibility rate is large at the corners of a square space than the center in most cases, if r is considerably small, or the B domain in Figure 4 exists, the B and C domains become opposite, where the center has a large visibility rate (see distribution diagram of $r = 0.2$ in Figure 11). The smaller r gets, the high the visibility rate becomes. The smaller and the larger r gets, the more consistent the space becomes.

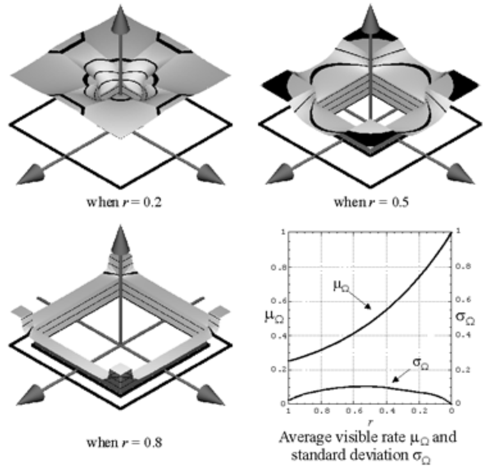

Fig. 11. Distribution of visible rates $\phi_{vis}(x,y)$ when r of cut out square shape is changed, average visible rate and standard deviation

3.4 When *r* is changed in the circular cut out space

Figure 12 shows the distribution of the visibility rate when r is 0.2, 0.5, and 0.8, and the average visibility rate and standard deviation inside Ω when r is changed continuously from 1 to 0. When r is near 1, the space is divided into four domains. However because each of the domains are not perfectly convex, the average visibility rate drops below 0.25. As apparent from the distribution diagram of visibility rates, there are no intense changes in the visibility rate. Corresponding to this, the average deviation of the visibility rate changes at a small value compared to other cases. Like the square cut out area, if r is considerably small, or if the *B* domain in Figure 6 exists, the *B* domain and *A* domain become opposite, and the visibility rate (see distribution diagram of r = 0.2 in Figure 12) becomes large at the center.

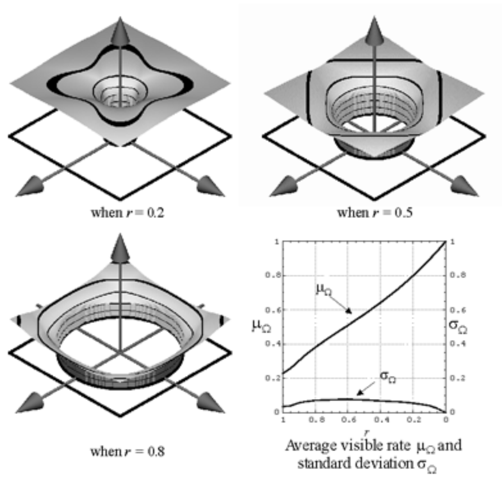

Fig. 12. Distribution of visible rates $\phi_{vis}(x,y)$ when r of cut out circular shape is changed, average visible rate and standard deviation

3.5 Comparison of L-shaped square cut out space and square cut out space

In 3.1 and 3.3, an obstacle area is set up in part of the square space using squares in both. Focusing on the area of the square cut out for both, we studied the average visibility rate for both. Figure 13 shows the average visibility rate and standard deviation of both. As evident from this diagram, when a square with the same area is used, cutting at corners have an overwhelmingly larger visibility rate than cutting out at the middle. This matches empirical findings that in a real space, placing an object at the corners than the middle provides better view on the whole. Judging from the physical quantity that is visibility rate, placing at corners always provide larger view than placing at the center. When the area is above 0.36, placing at corners is a value more than 50% of the visibility value when placed at the center. From when area exceeds 0.9, the standard deviation starts to reverse, and when the object becomes large, findings suggest that asymmetrical placement at corners result in inconsistency of space visually compared to the center.

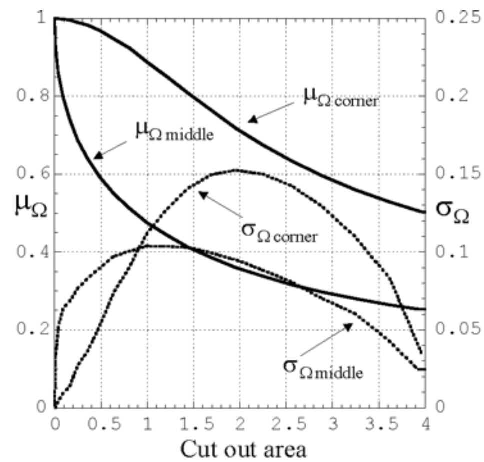

Fig. 13. Changes in visible quantity according to area of cut out square shape

3.6 Comparison of square cut out space and circular cut out space

With both 3.3 and 3.4, obstacle area is placed at the center. The relation between the area of the layout drawings and visibility rate was investigated and the differences in the visibility rate according to the shape of the drawings were studied. Figure 14 shows the average visibility rate and standard deviation in each case. Though the average visibility rate was more or less the same, in the range of spatial shapes which may exist, in other words, range where parameters are not extreme values, the square cut out area has lower average visibility rate. However, the standard deviation is always small with the circular cut out space, indicating that an even space with no sudden changes in the visible quantity is realized by circular shapes without corners.

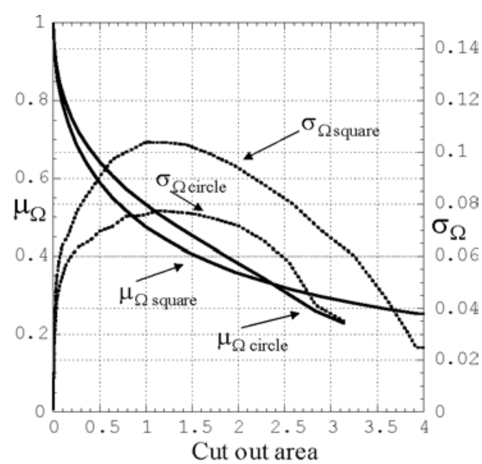

Fig. 14. Changes in visible quantity according to cut out area of cut out space

4. Summary and Future Tasks

Focusing on visible quantity in space, the author clarified that the distribution of visible quantity changes diversely according to the shape of space using primitive drawings. The effects of differences in shape on visible quantity were also studied, and results suggested the feasibility of using visible quantity in actual space. Though the stage of linking visible quantity with specific psychological quantity has not yet been reached, prospects of relation between the impression of space obtained intuitively and shape of distribution of visible quantity have been achieved, and efforts will be started from the job of expressing intuitive impressions quantitatively. Also planned are investigations on relations with other physical quantities obtained by graphic sciences approach such as (peripheral length) / (area), (peripheral length)^2/(area), visibility rate of outer circumference sides, taking into account mathematical background.

The visible quantity can also be applied for complicated cases by a discrete approach using values of sampling points. Particularly from a viewpoint of visual environment field, it would be interesting to study how physical quantities related to visible quantity change when part of the boundary has specular reflecting characteristic. By extending visible graphs investigated in the field of calculation geometry to the matrix composed by the value at all points in space, applications to disaster prevention is also possible by investigating ease of visual recognition of the space and studying the visibility of evacuation lamps.

This report discussed two-dimensional investigations, however the definition of visible quantity itself can easily be extended to three dimensions as seen in previous studies. It should be possible to link the attraction of three-dimensional complex urban space and visible quantity, and evaluate the attraction of given space quantitatively. In the future, we hope to continue studies on the nature of visible quantity and feasibility of applications to establish means to clarify the impressions of spatial shapes on people.

Acknowledgements

Finally, the author would like to express thanks to all the prominent referees who provided valuable comments and views.

Notes

(1) Punched square domains and circular domains are not counted as visible domains nor non-visible domains.
(2) As the portion obtained by the intersection of the curved surface drawing of the three-dimensional graph expressing distribution of visibility rate and domain enclosed by the two sides and is black, the width of the black area differs according to the cosine of the angle between the normal vector of the curved surface and z axis.

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