IMAGE FUSION USING WAVELET TRANSFORM

Zhu Shu-long [a, b]

[a] Department of Remote Sensing Information Engineering, Zhengzhou Institute of Surveying & Mapping, Zhengzhou, China, Postcode: 450052 -zsl1964@public2.zz.ha.cn

[b] LIESMARS of Wuhan University, Wuhan, China, Postcode: 430079

Commission IV, Working Group IV/7

KEY WORDS: Multi-source images, Fusion, Wavelet transform, Edge processing, Wavelet based fusion, Fused Images

ABSTRACT:

Extracting more information from multi-source images is an attractive thing in remotely sensed image processing, which is recently called the image fusion. In the paper, the image fusion algorithm based on wavelet transform is proposed to improve the geometric resolution of the images, in which two images to be processed are firstly decomposed into sub-images with different frequency, and then the information fusion is performed using these images under the certain criterion, and finally these sub-images are reconstructed into the result image with plentiful information.

1. THE INTRODUCTION

For the remotely-sensed images, some have good spectral information, and the others have high geometric resolution, how to integrate the information of these two kinds of images into one kinds of images is a very attractive thing in image processing, which is also called the image fusion. For the purpose of realization of this task, we often need some algorithms to fuse the information of these two kinds of images, and however we find few such algorithms. In recent ten years, a new mathematical theory called "Wavelet theory" (Charles K. Chui, 1992) has gradually been used in the fields of graphics and imagery, and been proved to be an effective tool to process the signals in multi-scale spaces (Thierry Ranchin et al.). In this paper , the image fusion algorithm based on wavelet transform is proposed to improve the geometric resolution of the images, in which two images to be processed are firstly decomposed into sub-images with the same resolution at the same levels and fifferent resolution among different levels, and then the information fusion is performed using high-frequency sub-images under the "gradient" criterion, and finally these sub-images are reconstructed into the result image with plentiful information. Since the geometric resolution of the image depends on the high-frequency information in it, therefore this image fusion algorithm can acquire good results.

2. THE ALGORITHM OF WAVELET TRANSFORM

2.1 Mallat Algorithm of Wavelet Transform

Suppose that $\{V_j, j \in Z\}$ is a multi-resolution Analysis in $L^2(R)$, $\varphi(x)$ is the scaling function of subspace V_0 , W_j is the orthogonal complement of V_j with respect to V_{j+1} , i.e. $V_{j+1} = V_j + W_j$, $\psi(x)$ is wavelet function of subspace W_0. If a signal $f(x) \in V_{j+1}$, then it can be expressed as :

$$f(x) = \sum_n c_n^{j+1} \varphi_{j+1,n} \tag{1}$$

since $V_{j+1} = V_j + W_j$, then $f(x)$ can also be expressed as:

$$f(x) = \sum_n c_n^j \varphi_{j,n} + \sum d_n^j \psi_{j,n} \qquad (2)$$

Combining formula (1) with (2), we get :

$$\sum_n c_n^{j+1} \varphi_{j+1,n} = \sum_n c_n^j \varphi_{j,n} + \sum d_n^j \psi_{j,n} \qquad (3)$$

since $\varphi_{j,k}$ is orthogonal with respect to different j and k , if two sides of formula (3) are multiplied by $\varphi_{j,k}$ and then integrated with respect to x , we can obtained :

$$c_k^j = \sum_n c_n^{j+1} \left\langle \varphi_{j+1,n}, \varphi_{j,k} \right\rangle = \frac{1}{\sqrt{2}} \sum_n h_{n-2k} c_n^{j+1} \qquad (4)$$

Using the same method, we also have:

$$d_k^j = \frac{1}{\sqrt{2}} \sum_n g_{n-2k} c_n^{j+1} \qquad (5)$$

The formulas (4) and (5) are the decomposition formula of signal, where c^j is an approximation of c^{j+1}, and d^j is the detailed part of c^{j+1}.

If having the decomposed signals $\{c_n^j,\ n \in Z\}$ and $\{d_n^j,\ n \in Z\}$, then two sides of formula (3) are multiplied by $\varphi_{j+1,k}$, and then integrated with respect to x, we can obtained :

$$\begin{aligned} c_n^{j+1} &= \sum_k c_n^j \left\langle \varphi_{j,k}, \varphi_{j+1,n} \right\rangle + \sum_k d_k^j \left\langle \psi_{j,k}, \varphi_{j+1,n} \right\rangle \\ &= \frac{1}{\sqrt{2}} \sum_n c_k^j h_{n-2k} + \frac{1}{\sqrt{2}} d_k^j g_{n-2k} \\ &= \frac{1}{\sqrt{2}} \left(\sum_k c_k^j \widetilde{h}_{2k-n} + \sum d_k^j \widetilde{g}_{2k-n} \right) \end{aligned}$$

(6)

Where $\widetilde{h}_n = h_{-n}$, $\widetilde{g}_n = g_{-n}$, when the signal is finite, $\widetilde{h}_n = h_{1-n}, \widetilde{g}_n = g_{1-n}$. The formula (6) are the reconstruction formula of signal. The formulas (4)、(5) and (6) are called Mallat algorithm (S. Mallat, 1989) .

For two-dimensional signal c^{j+1}, its decomposition formulas are :

$$\begin{aligned} c_{m,n}^j &= \frac{1}{2} \sum_{k,l \in Z} c_{k,l}^{j+1} h_{k-2m} h_{l-2n} \\ d_{m,n}^{j1} &= \frac{1}{2} \sum_{k,l \in Z} c_{k,l}^{j+1} h_{k-2m} g_{l-2n} \\ d_{m,n}^{j2} &= \frac{1}{2} \sum_{k,l \in Z} c_{k,l}^{j+1} g_{k-2m} h_{l-2n} \\ d_{m,n}^{j3} &= \frac{1}{2} \sum_{k,l \in Z} c_{k,l}^{j+1} g_{k-2m} g_{l-2n} \end{aligned} \qquad (7)$$

and its reconstruction formula is :

$$c_{m,n}^{j+1} = \frac{1}{2} \left(\sum_{k,l \in Z} c_{k,l}^j \widetilde{h}_{2k-m} \widetilde{h}_{2l-n} + \sum_{k,l \in Z} d_{k,l}^{j1} \widetilde{h}_{2k-m} \widetilde{g}_{2l-n} + \sum_{k,l \in Z} d_{k,l}^{j2} \widetilde{g}_{2k-m} \widetilde{h}_{2l-n} + \sum_{K,l \in z} d_{k,l}^{j3} \widetilde{g}_{2k-m} \widetilde{g}_{2l-n} \right)$$

(8)

where c^j is an approximation of c^{j+1}, d^{j1} 、 d^{j2} and d^{j3} are the detailed parts of c^{j+1}. Fig. 1 illustrates the Mallat algorithm of 2D images intuitively.

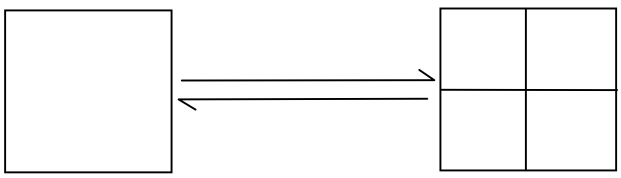

Figure 1. The Mallat algorithm of 2D image

2.2 Edge Processing for Wavelet Transform

Since our image fusion algorithm need to use the forward and the inverse algorithm of wavelet transform, in order to reconstruct the signals precisely, the image edges must be processed under the process of wavelet transform, otherwise some information will be lost in this operation.

Assuming that the low-pass filter and the corresponding high-pass filter are four coefficient Daubechies' (I. Daubechies, 1988), i.e. $H=(h_0,h_1,h_2,h_3)$ and $G=(g_{-2},g_{-1},g_0,g_1)$ respectively, the samples of the original discrete signal are $C^0=(c_0^0,c_1^0,\ldots,c_{M-1}^0)$, then the image edges are processed according the following precise formulas (Zhu Shu-long and Geng zhe-Xun, 1999) in our experiments:

For left image edges, we have：

$$\begin{cases} c_{-1}^0 = c_{-2}^0 = 0 \\ c_{-1}^1 = \dfrac{h_2}{h_1} d_0^1 \end{cases}$$

(9)

For right image edges, we have：

$$\begin{cases} c_M^0 = 0, \quad d_{M/2}^1 = 0 \\ c_{M+1}^0 = -\dfrac{h_2}{h_0} c_{M-1}^0 + \dfrac{h_3}{h_0} c_{M-2}^2 \end{cases}$$

(10)

If the image edges are processed according (9) and (10), then the original image can be restored after wavelet forward transform and then wavelet backward transform.

3. IMAGE FUSION ALGORITHM BASED WAVELET TRANSFORM

The main idea of our algorithm is that: (1) the two images to be processed are resampled to the one with the same size; and (2) they are respectively decomposed into the subimages using forward wavelet transform, which have the same resolution at the same levels and fifferent resolution among different levels; and (3) information fusion is performed based on the high-frequency subimages of decomposed images; and finally the result image is obtained using inverse wavelet transform.

Let $A(x,y)$ and $B(x,y)$ be the images to be fused，the decomposed low-frequency sub-images of $A(x,y)$ and $B(x,y)$ be respectively $lA_J(x,y)$ and $lB_J(x,y)$ (J is the parameter of resolution) ,the decomposed high-frequency sub-images of $A(x,y)$ and $B(x,y)$ be respectively $hA_j^k(x,y)$ and $hB_j^k(x,y)$ (j is the parameter of resolution and $j=1,2,\cdots,J$. for every j, $k=1,2,3$) , the gradient image generated from $hA_j^k(x,y)$ and $hB_j^k(x,y)$ be respectively $GA_j^k(x,y)$ and $GB_j^k(x,y)$, then the fused high-frequency sub-images $F_j^k(x,y)$ are :

if $GA_j^k(x,y) > GB_j^k(x,y)$

$$F_j^k(x,y) = hA_j^k(x,y),$$

if $GA_j^k(x,y) < GB_j^k(x,y)$

$$F_j^k(x,y) = hB_j^k(x,y),$$

and the fused low-frequency sub-image $F_J(x,y)$ are :

$$F_J(x,y) = k1 \cdot lA_J(x,y) + k2 \cdot lB_J(x,y)$$

where $k1$ and $k2$ are given parameters, if image B is fused into image A, then $k1 > k2$. If the decomposed high-frequency sub-images $hA_j^k(x,y)$ of $A(x,y)$ is replaced by be respectively $F_j^k(x,y)$, the decomposed low-frequency sub-image $lA_J(x,y)$ of $A(x,y)$ is replaced by be respectively $F_J(x,y)$, and $F_J(x,y)$ and $F_j^k(x,y)$ are used to reconstruct the image $A'(x,y)$ using the inverse wavelet transform, then $A'(x,y)$ not only has the low-frequency information of $A(x,y)$ and $B(x,y)$, but also has the high-frequency information of $A(x,y)$ and $B(x,y)$ at the same time. This algorithm shows that the high-frequency information fusion between two images is completed under the "gradient" criterion.

For multi-spectral images, we can use the same method proposed above to process every band images respectively. For color images, we can convert the color images from RGB (i.e.

Red, Green, Blue) space to HIS (i.e. Hue, Intensity, Saturation) space, and then only process the intensity component of HIS space using the same method proposed above, and finally acquired the fused the images.

4. THE ASSESSMENT FOR THE FUSED IMAGES

The index "Average gradient" is applied to measure the detailed information in the images. The formula of average gradient "g" can be denoted as follows:

$$g=\frac{1}{(M-1)\cdot(N-1)}\sum_{x=1}^{M-1}\sum_{y=1}^{N-1}\sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2+\left(\frac{\partial f(x,y)}{\partial y}\right)^2} \quad (11)$$

Where $f(x,y)$ is image function, M and N are the length and width of the image respectively. Generally, the larger g is, the clearer the image is. For color images, average gradient "g" can be calculated from the "intensity" images.

5. THE EXPERIMENTS

In our experiments, two images are used to verify our algorithm, the first is the image composed of SPOT multi-spectral images with the resolution of 20 m (as shown in Fig.2), the second is SPOT panchromatic image with the resolution 10 m (as shown in Fig.3). In order to fully utilize the spectral information of the former and geometric information of the latter, the image fusion algorithm proposed above is used to introduce the details of the latter into the former so that the result image (as shown in Fig.4) has not only good spectral information but also good geometric information. The table 1 illustrates the average gradient "g" in these three images quantitatively, in which the average gradient of the result image is obviously greater than that of the original image 1.

Now this algorithm has been a practicable function in our Image Processing System (IPS). The further research is to adapt this algorithm to the large-data images.

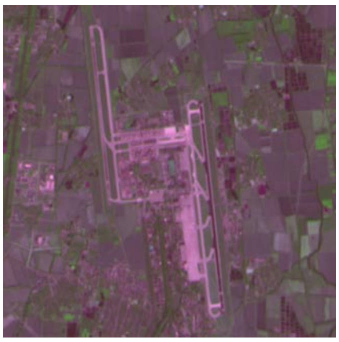

Fig. 2. The original image 1 with the resolution 20 m

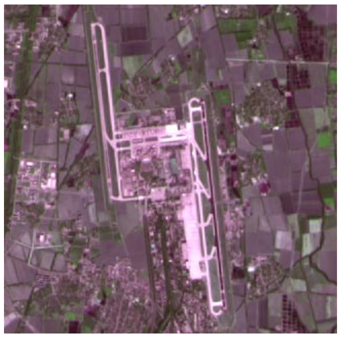

Table 1 the image and its average gradient

Image Name	The average gradient
the original image 1 with the resolution 20 m	4.78
the original image 2 with the resolution 10 m	12.33
the result image generated by our algorithm	11.16

REFERENCES:

Thierry Ranchin et al., Efficient data fusion using wavelet transform: the case of SPOT satellite images, in Mathematical imaging: Wavelet Application in Signal and Image Processing, Andrew F. Laine, Editor, Proceeding of SPIE 2034, pp171-179.

Charles K. Chui, 1992, An Introduction to Wavelets, Academic Press, Inc., America.

S. Mallat, 1989, A theory for multiresolution signal decomposition: The wavelet representation, IEEE Transaction on PAMI, Vol.11, No.7, pp.674-693.

I. Daubechies, 1988, Orthonormal bases of compactly supported wavelets, Communication on Pure and Applied Mathematics, Vol. XLI, pp.909-996.

Zhu Shu-long and Geng zhe-Xun, 1999, The Wavelets and Its Applications to Graphics and Imagery, The PLA Press, P.R.C.