FUNDAMENTAL PRINCIPLES OF IMAGE ORIENTATION USING ORTHOGONAL PROJECTION MODEL

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ABSTRACT

The central perspective model is generally used in photogrammetry because of the high reliability, but it has a weak point that the initial values of orientation parameters are necessary due to the non-linearity. On the other hand, the model based on projective geometry, which is widely used in computer vision, can be treated as linear forms, but many indefinitenesses in the model leads to the lower precision and the lower stability. The authors developed an alternative model called the orthogonal projection model, which is as reliable as the central perspective model and does not need initial values of orientation parameters. The orthogonal projection model is derived from affine projection model with a constraint of orthogonality. The model is therefore appropriate to long distance observation like affine projection model, and it is also applicable to close range photogrammetry with high accuracy. This paper describes the derivation of the orthogonal projection model and the geometric characteristics, and also verifies the effectiveness by various simulations and a field test.

1 INTRODUCTION

Time series monitoring of displacement of cliff faces is very important for disaster prevention at mountainous districts. Unstable rock displacement has to be measured within a few millimeters from long distance over 100m in some cases. Long distance photogrammetry with a telescopic lens camera is suitable for these applications. However, the conventional central perspective orientation model is hardly applicable for these cases, because it has a weak geometric aspect in very narrow view angle. This paper presents an alternative orientation model called the orthogonal projection model, which is stable for almost occasion including long distance observation. Firstly, in order to emphasize the advantages of the proposed model, the disadvantages of the existing orientation models are focused on. Secondly, equations of the orthogonal projection model are derived and its geometrical characteristics are described. Finally, various simulations and a field test investigate the effectiveness of the proposed model.

2 THE EXISTING ORIENTATION MODELS

2.1 Central perspective model

The central perspective model is highly reliable for close range applications as well as aerial photogrammetry. This model directly treats Euclidean geometric parameters such as rotations and translations as orientation parameters. It can achieve rigorous solutions, whereas initial values of unknowns are necessary for adjustment due to its complicated non-linear form. GPS and INS will support stable measurement by obtaining approximate values of orientation parameters, but these equipments cannot be used on every occasion. On field observations, almost initial values of orientation parameters and object coordinates may be possibly unavailable. Furthermore, object points accuracy becomes worse under bad conditions such as observations with super telescopic lens due to deterioration of the linear independence among orientation parameters, even if highly approximated initial values are available.

2.2 DLT

If some control points are available, DLT (Abdel-Aziz & Karara, 1971) may be applicable. DLT is based on projective geometry and can be treated as a linear model. However, DLT needs many control points (at least 6 points), because the form consists of 11 parameters with respect to each photo. In fact precise solutions cannot be obtained unless using a lot of control points. The model space formed by coplanarity condition of overlapped photos can be transformed into object space by the three-dimensional projective transformation. The projective transformation has 15 parameters, whereas the similarity transformation has only 7 parameters. This means that the free network solutions by DLT have 15-7=8 indefinitenesses against similarity to object space. The many indefinitenesses lead to low precision and low reliability. Anyway, DLT can be a good approach for acquiring initial values.

2.3 Affine projection model

Okamoto (1992) proposed the affine projection based orientation model which can overcome the problems in long distance observation. Considering the relationship between an affine image and an object space, the basic equations relating an object point P (X, Y, Z) and the corresponding image point p (x_a, y_a) are described with generalized orientation parameters A_i $(i = 1, ..., 8)$ as

$$\begin{aligned} x_a &= A_1X + A_2Y + A_3Z + A_4 \\ y_a &= A_5X + A_6Y + A_7Z + A_8 \end{aligned} \quad (1)$$

These equations are quite simple and linear. Therefore, the linear independence among orientation parameters is high. If affine image coordinates (x_a, y_a) are observed and more than 4 control points are given, the orientation parameters A_i can be obtained on any occasion. However, ordinarily affine image coordinates are not directly observed and they have to be transformed from central perspective image coordinates (x, y) for rigorous solutions. The transformation into affine images requires approximate values of internal and external orientation parameters (Ono et al, 1996, Okamoto et al, 1998).

The relationship between the model space constructed by affine projection model and the object space is described as three-dimensional affine transformation, which has 12 degrees of freedom. Therefore, free network solutions by affine projection model has 12-7=5 indefinitenesses against similarity to object space.

3 ORTHOGONAL PROJECTION MODEL

3.1 Basic concept

The central perspective model directly treats observed images, whereas the orthogonal projection model treats parallelly projected images which is transformed from observed ones. The conceptual diagram of orthogonal projection model is illustrated in Figure 1. The orthogonal projection model is a kind of affine projection model with constraints. Affine projection allows oblique projection to an image plane, whereas orthogonal projection allows only the projection perpendicular to an image plane.

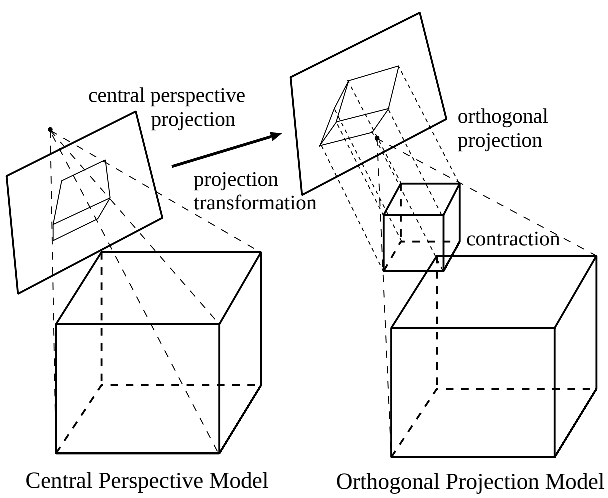

Figure 1: Conceptual diagram of orthogonal projection model

3.2 Derivation of model equations

If the lens distortions and the shift of the principal points are negligible, the central perspective model is expressed as:

$$\begin{pmatrix} x \\ y \\ -c \end{pmatrix} = \lambda \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{pmatrix} \quad (2)$$

where λ is a scale factor, c is a principal distance, a_{ij} is components of rotation matrix and (X_0, Y_0, Z_0) is perspective center.

The value of λ changes in proportion to distance to object points. By substituting λ by constant scale parameter m, Equation 2 is described as :

$$\begin{aligned} \begin{pmatrix} x_a \\ y_a \\ -m/\lambda c \end{pmatrix} &= m/\lambda \begin{pmatrix} x \\ y \\ -c \end{pmatrix} \\ &= m \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{pmatrix} \end{aligned} \quad (3)$$

By transposing (X_o, Y_o, Z_o) to left site, equation (3) is described as following.

$$\begin{pmatrix} x_a - x_o \\ y_a - y_o \\ -m/\lambda c - z_o \end{pmatrix} = m \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (4)$$

where

$$\begin{pmatrix} x_o \\ y_o \\ z_o \end{pmatrix} = m \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X_o \\ Y_o \\ Z_o \end{pmatrix}$$

The orthogonal projection with contraction is expressed by the first and second equations of (4).

$$\begin{aligned} x_a &= m\{a_{11}X + a_{12}Y + a_{13}Z\} + x_o \\ y_a &= m\{a_{21}X + a_{22}Y + a_{23}Z\} + y_o \end{aligned} \quad (5)$$

The number of independent parameters is six. They consist of x_o, y_o, m and three rotation angles.

Mathematically m is an arbitrary constant, which is involved with image coordinates x_a, y_a. From a practical standpoint, m is adjusted so as to scale down the average photographing distance to be same length as principal distance c (Figure 2).

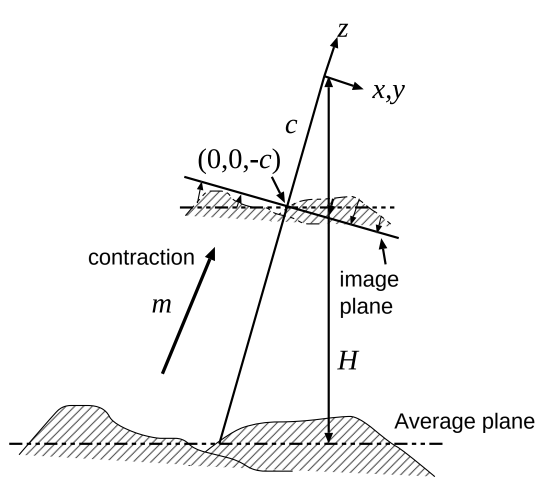

Figure 2: Constant scale parameter m

Let H be the average photographing distance to Z direction ($H = \bar{Z} - Z_o$), m is described as following.

$$m = -\frac{a_{33}c}{\bar{Z} - Z_o} = -\frac{a_{33}c}{H} \quad (6)$$

Equation (2) is reversely transformed as following.

$$\begin{pmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ -c \end{pmatrix} \quad (7)$$

Taking notice of the third equation of (7), λ is expressed by:

$$\lambda = \frac{a_{13}x + a_{23}y - a_{33}c}{Z - Z_o} \quad (8)$$

By substituting (6) and (8) into (3), the equations of transformation from central perspective image coordinates to orthogonal projection ones are derived.

$$\begin{aligned} x_a &= \frac{Z - Z_o}{H} \frac{a_{33}c}{a_{33}c - a_{13}x - a_{23}y} x \\ y_a &= \frac{Z - Z_o}{H} \frac{a_{33}c}{a_{33}c - a_{13}x - a_{23}y} y \end{aligned} \quad (9)$$

Both of equations (5) and (9) are derived by the central perspective model without approximation. In this sense, the model consisting of (5) and (9) is as rigorous as the central perspective model.

3.3 Generalization of Orthogonal Projection Model

By simply generalizing equations (5), collinearity equations (1) of affine projection model are derived. Addition of constraints for orthogonal projection to (1) leads to the generalized orthogonal projection model. Because the generalized coefficients $A_i (i = 1, 2, 3, 5, 6, 7)$ of equations (1) are derived from components of rotation matrix a_{ij} and scale parameter m, they have following features:

Constraint 1: vector (A_1, A_2, A_3) and (A_5, A_6, A_7) are perpendicular to each other.

Constraint 2: norm of (A_1, A_2, A_3) is equivalent to that of (A_5, A_6, A_7).

Constraints 1 and 2 are described as:

$$A_1 A_5 + A_2 A_6 + A_3 A_7 = 0 \quad (10)$$

$$A_1^2 + A_2^2 + A_3^2 = A_5^2 + A_6^2 + A_7^2 \quad (11)$$

respectively. Constraint 1 means that an image plane and incident rays from objects are orthogonalized to each other. Constraint 2 means that scale of x_a direction is equivalent to that of y_a direction.

Affine projection model (1) with constraints (10) and (11) is defined as generalized orthogonal projection model. As mentioned above, orthogonal projection model has six independent orientation parameters. Two constraints reduce the degrees of freedom of equation (1) from 8 to 6 in generalized orthogonal projection model.

By generalizing the model, some advantages arise. Geometric orientation parameters of equation (5) are not linear to each other. This means that the initial values of unknowns are necessary just like the central perspective model. On the contrary, the orientation parameters of the generalized model are linear in equation (1). Equations of constraints are not linear, but there is no problem because equation (1) can give the approximation values. Furthermore, the generalized model has higher linear independence than the geometric model. Thus, the generalization of orthogonal projection model presumably conduces to robust adjustment.

From here, the generalized orthogonal projection model is treated as orthogonal projection model.

3.4 Estimation of Geometric Orientation Parameters

As mentioned above, orthogonal projection image coordinates (x_a, y_a) have to be transformed from observed image coordinates (x, y). The transformation equation (9) requires values of components of rotation matrix a_{13}, a_{23}, a_{33}, Z, Z_o and c. These parameters can be estimated with the generalized parameters A_i.

Components of rotation matrix are estimated by following approaches. By definition of A_i,

$$\begin{pmatrix} A_1 & A_2 & A_3 \\ A_5 & A_6 & A_7 \end{pmatrix} = m \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad (12)$$

Because norm of each line of rotation matrix is 1,

$$m^2 = A_1^2 + A_2^2 + A_3^2 \quad (13)$$

With equations (11) and (12), $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$, are easily determined. The other components a_{31}, a_{32}, a_{33} can be estimated by considering geometric feature of rotation matrix.

$$a_{11}^2 + a_{21}^2 + a_{31}^2 = 1$$

Thus

$$a_{31} = \pm\sqrt{1 - a_{11}^2 - a_{21}^2}$$

In the same way,

$$a_{32} = \pm\sqrt{1 - a_{12}^2 - a_{22}^2}$$

$$a_{33} = \pm\sqrt{1 - a_{13}^2 - a_{23}^2}$$

Furthermore,

$$\begin{aligned} a_{11}a_{31} + a_{12}a_{32} + a_{13}a_{33} &= 0 \\ a_{21}a_{31} + a_{22}a_{32} + a_{23}a_{33} &= 0 \end{aligned}$$

There are two sets of a_{31}, a_{32}, a_{33} which satisfy these all equations. A set closer to initial value is selected.

If c is given, Z_o is calculated with equation (6).

$$Z_o = \frac{a_{33}c}{m} + \bar{Z} \quad (14)$$

If precise value of c is unknown, compensation value Δc has to be calculated in collinearity equations. Even if X, Y, Z

and A_i are correct, transformation errors by Δc cause large residuals of image coordinates. Conversely, Δc can be estimated from residuals of image coordinates. By equation (9) partial differential coefficients of x_a and y_a with respect to c are described as following.

$$\frac{\partial x_a}{\partial c} = \frac{Z - Z_o}{H} \frac{-a_{33}x(a_{13}x + a_{23}y)}{(a_{13}x + a_{23}y - a_{33}c)^2} \quad (15)$$

$$\frac{\partial y_a}{\partial c} = \frac{Z - Z_o}{H} \frac{-a_{33}y(a_{13}x + a_{23}y)}{(a_{13}x + a_{23}y - a_{33}c)^2} \quad (16)$$

By adding $\Delta x_a = \partial x_a/\partial c \Delta c$, $\Delta y_a = \partial y_a/\partial c \Delta c$ to equation (9), Δc can be adjusted as well as other unknowns.

3.5 Similarity to Object Space

Orthogonal projection image has a smaller number of indefinitenesses than affine projection one. Therefore it is conceivable that the 3-D model image constructed with overlapped orthogonal projection images has also a smaller number of indefinitenesses than 3-D affine model one.

For simplifying the problem, it is assumed that internal orientation parameters are known. The number of independent orientation parameters on stereo pair images is 6×2=12. By expressing with suffixes l and r to parameters of left and right images respectively, coplanarity condition of corresponding rays is described as following.

$$\begin{vmatrix} A_{1l} & A_{2l} & A_{3l} & A_{4l} - x_{al} \\ A_{5l} & A_{6l} & A_{7l} & A_{8l} - y_{al} \\ A_{1r} & A_{2r} & A_{3r} & A_{4r} - x_{ar} \\ A_{5r} & A_{6r} & A_{7r} & A_{8r} - y_{ar} \end{vmatrix} = 0 \quad (17)$$

By rearranging this equation, the following linear equation is derived.

$$x_{al} = B_1 y_{al} + B_2 x_{ar} + B_3 y_{ar} + B_4 \quad (18)$$

This shows that the coplanarity condition can mathematically provide 4 orientation parameters among the 12 ones of the stereo pair of orthogonal projection images. Hence, the number of the parameters determined by absolute orientation is 12 - 4 = 8. This means that free network solutions by orthogonal projection model have 8 - 7 = 1 indefiniteness against similarity to object space. In concrete terms, an angle between the corresponding rays of stereo pair images becomes indefinite, and the constructed space deforms to the depth direction.

In the next place, considering the orientation problem on the triplet orthogonal projection images, the number of orientation parameters is 6×3 = 18. On the other hand, the number of parameters determined by coplanarity condition is $4times3$ = 12, but all of 12 parameters are not completely independent to each other. Because the following coplanarity condition with regard to all of three images is formed, one degree of freedom decreases.

$$\begin{vmatrix} A_{1l} & A_{2l} & A_{3l} & A_{4l} - x_{al} \\ A_{5l} & A_{6l} & A_{7l} & A_{8l} - y_{al} \\ A_{1c} & A_{2l} & A_{3c} & A_{4c} - x_{ac} \\ A_{5c} & A_{6l} & A_{7c} & A_{8c} - y_{ac} \\ A_{1r} & A_{2r} & A_{3r} & A_{4r} - x_{ar} \\ A_{5r} & A_{6r} & A_{7r} & A_{8r} - y_{ar} \end{vmatrix} = 0 \quad (19)$$

Therefore, the coplanarity conditions can mathematically provide 12 - 1 = 11 orientation parameters. Thus, the degrees of freedom of the free network solutions are 18 - 11 = 7. This means the free network 3D model space constructed with triplet orthogonal projection images has high similarity to objects.

Finally, the case where principal distance c is unknown is discussed. If c is unknown and fixed in triplet images, the number of unknown parameters increases to 19. As mentioned above, Δc can be estimated from residuals of image coordinates. In the other words, Δc can be determined by the coplanarity conditions. The number of parameters determined by coplanarity condition comes to be 12. Therefore, the degrees of freedom of the free network solutions come to be 19 - 12 = 7. High similarity to the object space is retained in case where c is unknown and fixed.

4 SIMULATIONS

In order to investigate the geometrical characteristics of the orthogonal projection model, simple simulations were performed on the following cases.

1. stereo pair images are used and c is given
2. triplet images are used and c is given
3. triplet images are used and c is unknown

Configuration of camera and object points is illustrated in Figure 3 and Table 1.

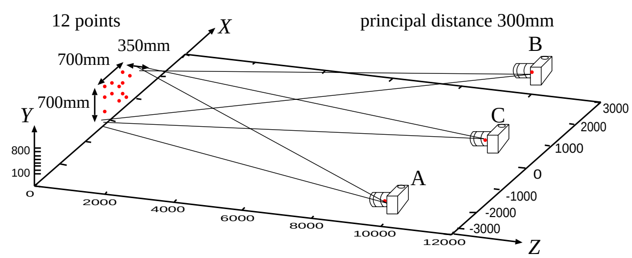

Figure 3: Configuration of camera and object points

Table 1: Coordinates of object points and camera position (mm)

No.	X	Y	Z
1	-200	800	0
2	-200	500	0
3	-300	100	0
4	-100	700	200
5	-150	350	300
6	100	700	0
7	50	450	0
8	250	800	350
9	300	550	0
10	250	250	250
11	400	800	0
12	400	150	0
A	-3000	500	10000
B	3000	400	10000
C	0	400	11000

Observed values and initial values were given by perturbing true values by normal random with the following standard deviations: 0.001mm in image coordinates, 10mm in object coordinates, 10mm in camera position, 2 degrees in inclination of camera.

4.1 Case 1

Only two images taken at point A and B were used. Principal distance c was fixed to true value 300mm.

The following indexes are shown in tables.

1. RMSE of image coordinates σ_0
2. Internal errors
3. RMSE between true values of object points and free network solutions transformed to center of objects by similar transformation
4. RMSE between true values of object points and free network solutions transformed to center of objects by 3-D affine transformation

3. appreciates all deformations of obtained 3-D space, whereas 4. does not appreciate the overall deformation. By comparing values of 3. and 4., similarity to object space can be estimated.

Table 2: Results with stereo pair images (mm)

1	σ_0=	0.00169		
	X	Y	Z	XYZ
2	0.0854	0.0836	0.1299	0.1019
3	0.9334	0.5566	2.2348	1.4347
4	0.0572	0.0849	0.0922	0.0796

3. has more than 10 times larger errors than 4. This result confirms that the orthogonal projection model cannot construct 3-D model with high similarity to the object space in case where only two overlapped images are used. Further, it is shown that indefiniteness appears in depth direction Z.

4.2 Case 2

Triplet images taken at point A, B and C were used. Principal distance c was fixed to the true value 300mm.

Table 3: Results with triplet images (mm)

1	σ_0=	0.00087		
	X	Y	Z	XYZ
2	0.0417	0.0411	0.0647	0.0504
3	0.0390	0.0499	0.0773	0.0577
4	0.0258	0.0328	0.0599	0.0422

Differences between 3. and 4. are small and the accuracies of both are high. It was confirmed that the proposed model is effective in the case where triplet images are used.

4.3 Case 3

Triplet images taken at point A, B and C were used. Principal distance c was fixed to the false value 290mm.

Table 4: The case where false value 290mm is given to c

1	σ_0=	0.00229		
	X	Y	Z	XYZ
2	0.3014	0.2985	0.3892	0.3324
3	0.1077	0.0884	0.2655	0.1731
4	0.0350	0.0536	0.1170	0.0770

The value of σ_0 was more than twice larger than that in the case 2. And the object coordinates were much worse than those in the case 2. This indicates that Δc affects residuals of image coordinates.

In the next test principal distance c was treated as unknown and the initial values 290mm was given to c.

Table 5: The case where c is treated as unknown

1	σ_0=	0.00083		
	X	Y	Z	XYZ
2	0.0407	0.0401	0.0636	0.0494
3	0.0682	0.0454	0.1072	0.0779
4	0.0269	0.0216	0.0767	0.0485

The obtained value of c was 299.3mm. Compared to the case where false value is given to c, the accuracy was obviously improved.

5 FIELD TEST

A field test was carried out around Kamo-river at Kyoto Japan. Configuration of camera and object points is illustrated in Figure 4.

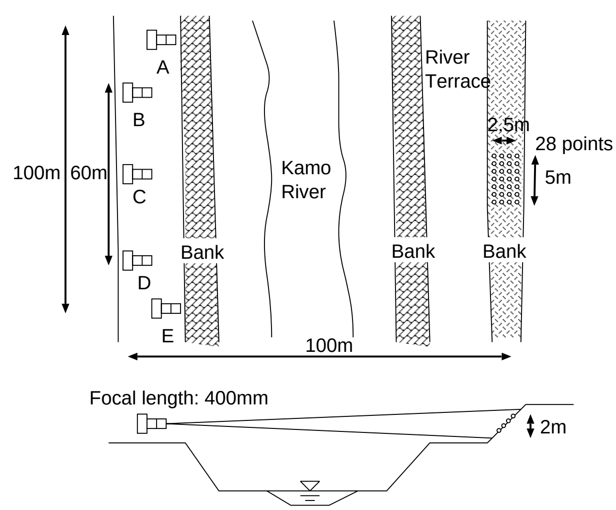

Figure 4: Configuration of camera and object points

Conditions on the test are described below.

Camera: Canon D30 (Digital Camera equipped with CMOS sensor)

Image size: 2160 x 1440 pixels
Resolution: 10.5 μm / pixel
Lens: EF100-400mm F4.5-5.6L IS USM (Zoom lens)
Focal length: 400mm (fixed with tape)
Distance to objects: 100 – 110m
Target size: 2cm in diameter
The number of object points: 4 x 7 = 28
The number of images: 5

Figure 5: One of the photo images taken with D30

5.1 Bundle adjustment

The object coordinates was observed by the ground triangulation with a total-station as check data at the following accuracy.

Estimated standard errors: X:0.4767 Y:0.5031 Z:0.5173 (mm)

Initial values of object coordinates were calculated by resampling the check data by 50cm. Initial values of orientation parameters were estimated with the initial values of object coordinates by using DLT.

For comparison purpose, free network bundle adjustments were performed with both of the central perspective model and the orthogonal projection model.
c was treated as unknown. Zeros were given to the other internal orientation parameters.

5.2 Results

Two tests with different number of images were carried out. The indexes mentioned at previous section were calculated with both of the two models in each test.

Table 6: Results with 5 images (mm)

central perspective model				
1	σ_0=	0.002611		
	X	Y	Z	XYZ
3	7.9642	2.6021	6.2955	6.0507
4	2.1044	0.7834	3.4743	2.3884
orthogonal projection model				
1	σ_0=	0.002658		
	X	Y	Z	XYZ
3	3.4556	1.0566	3.4874	2.8994
4	2.1074	0.7836	3.4505	2.3777

Table 7: Results with 3 images (B,C,D) (mm)

central perspective model				
1	σ_0=	0.002633		
	X	Y	Z	XYZ
3	99.5854	43.8100	96.2873	83.8805
4	3.8363	1.3900	6.7535	4.5555
orthogonal projection model				
1	σ_0=	0.002755		
	X	Y	Z	XYZ
3	9.8375	3.9887	9.4912	8.2213
4	2.1514	0.9260	4.8342	3.1014

These results show that the orthogonal projection model is more effective than the central perspective model for long distance observation. Especially, the proposed model is robust in bad condition.
These results were achieved by using the initial values of orientation parameters derived by DLT, but it was also confirmed that initial values derived by affine projection model are available for the proposed model in the same condition.

6 CONCLUSIONS

This paper described the principles of the orthogonal projection model, which is appropriate for long distance observation. In addition, the following several characteristics on the proposed model were confirmed by some simulations and a field test.

- The proposed model can achieve higher accuracy than the conventional model on long distance observation.
- More than three overlapped images are required for accurate adjustment with the proposed model.
- Principal distance c can be self-calibrated with the proposed model.
- Initial values of orientation parameters are not necessary for adjustment with the proposed model. They can be estimated with a small number of control points.

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