

# ASSESSING HEIGHT PRECISION OF LASER ALTIMETRY DEMS

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### ABSTRACT:

The new Dutch national digital elevation model (DEM), acquired with laser altimetry, is almost complete. The Dutch Survey Department fulfils an intermediary function between laser data suppliers and final users of the DEM. One of the most important tasks of the Survey Department is to guarantee the quality of the delivered laser DEMs to the users. For this purpose, a new height error description scheme is developed, allowing to quantify error effects at different scales. Such a new error description scheme was necessary because the former quality description of DEMs was insufficient. The error behaviour of laser altimetry data, acquired by a complex system of different sensors, cannot be expressed by solely two parameters: a bias and a standard deviation. The former and the new error description scheme will be addressed together with methods to quantify the differently scaled error components. Among them are cross correlation techniques, empirical covariance function analysis from geostatistics and 1d strip adjustment. The contractual demands for maximal allowed error amplitudes, derived from real data, will be presented. The benefit of the error description scheme for DEM users will be illustrated by propagating the error components to the height precision of derived products. Summarizing, it can be stated that this paper deals with *assessing* the height precision of laser altimetry DEMs and *quantifying* the effects of the different error components on the measured heights. This paper does however not aspire to give methods for eliminating or minimizing these errors.

## 1. INTRODUCTION

Since 1996, the new Dutch national digital elevation model, the AHN (Actual Height model of the Netherlands), is being acquired using laser altimetry. The new DEM is expected to be completed in 2002. This project was initiated to meet the demand for detailed and actual information about elevation from water boards, provinces and "Rijkswaterstaat" (Ministry of Transport, Public Works and Water Management). The former elevation model, over 40 years old, has become obsolete. Moreover, it has a much lower point density than the AHN: 1 point per 10.000 m<sup>2</sup> in opposition to 1 point per 16 m<sup>2</sup> in average.

The Dutch Survey Department (Meetkundige Dienst Rijkswaterstaat) is co-ordinating the acquisition of the AHN. This task comprises the contracting of companies for the laser altimetry flights as well as for the processing of the original data to get X, Y, Z terrain co-ordinates and for the filtering. In addition, the quality of the delivered laser data is checked at the Survey Department and standard products, e.g. 5m x 5m grids, are derived and distributed to the participants such as the local water boards and provinces. The Survey Department also performs research to improve the AHN quality and to investigate further applications of laser altimetry ([Huising and Gomes-Pereira 1998], [Crombaghs et al. 2000], [Brügelmann 2000]).

In the very beginning of AHN acquisition, the laser data was not delivered in individual strips. Controlling the height precision consisted of checking the height differences at some horizontal and flat ground control regions ('control points') spread over the laser block. This way only a fairly limited sample test was possible for small parts of the whole DEM.

With increasing insight into the occurring errors in laser altimetry data, we realized that strip adjustment could strongly improve the laser DEM. Furthermore, strip adjustment provided the possibility to get a thorough and comprehensive insight into the quality of the delivered laser altimetry data. A 1D strip adjustment procedure was developed at the Survey Department correcting for strip tilts and height offsets [Crombaghs et al. 2000]. This requires, however, the delivery of the laser data in individual strips. Also further correction tools were created in order to eliminate possible occurring cross strip parabolic effects and to handle strip torsions or periodic effects.

This way, the Survey Department began processing and correcting the laser data itself on a large scale. This appeared to be a time consuming task. Furthermore, this did not correspond with our ambition to be a controlling and quality guaranteeing authority. Thus, the Survey Department encouraged the laser scanning companies to perform strip adjustment themselves by, among other things, placing the strip adjustment software at their disposal. In addition, some companies also did further efforts to improve the accuracy of laser data, e.g. by more thorough calibration procedures and additional control mechanisms.

Now that the first version of the AHN is almost complete, the Survey Department wants to retire from the data correcting task (which is expected to be done by the laser scanning companies) and to return to the originally aspired data certifying task. For this purpose, a new height error description scheme is developed for DEMs acquired by laser altimetry, allowing to quantify error effects at different scales. Such a new error description scheme was necessary because the former quality description of DEMs was insufficient for describing the whole error budget of a single laser point or of

mean heights of certain areas. Furthermore, it was not clearly formulated.

The former and the new error description scheme will be described in the following paragraphs, together with methods to quantify the differently scaled error components. The chosen thresholds for every error component, derived from real data, will be presented. Finally, the benefit of the error description scheme for the AHN user will be illustrated by propagating the error components to the height precision of derived products.

Thus, this paper deals with *assessing* the height precision of laser altimetry DEMs and *quantifying* the effects of the different error components on the measured heights. This paper does however not aspire to give methods for eliminating or minimizing these errors, or for quantifying the errors themselves, e.g. roll, pitch and heading errors of the INS, such as Schenk [2001] did.

## 2. FORMER WAY OF DESCRIBING LASER DEM HEIGHT PRECISION

The demanded height precision for the AHN is strict: 5 cm systematic error and 15 cm standard deviation. It turned out that these requirements were not always achievable. In addition, a fundamental drawback of this formulation is its ambiguity: several interpretations are possible. The region size, for which the thresholds for bias and standard deviation are valid, is missing. Does the 5 cm bias apply for, for example, 100 m<sup>2</sup> areas or for 1 km<sup>2</sup> or for 10 km<sup>2</sup>? Or is this maximal bias valid for all these areas? And what about controlling this, as such large ground control fields can hardly be measured?

Another disadvantage of this height error description is that not all the occurring error types of current laser data are taken into account. The error behaviour of laser altimetry data, acquired by a complex system of different sensors, cannot be expressed by only two parameters: a bias and a standard deviation. These two parameters do not suffice for describing the height precision of a laser altimetry DEM.

A more sophisticated approach for comprehensively describing the height quality is required. This new approach must take into account the specific scale of each error type. Some errors are stochastic for a single laser point. Others are systematic for a small area or for an entire laser strip, but stochastic as we focus on a large number of these small areas or strips. In the following paragraph, these different error components will be described including their technical causes.

## 3. NEW HEIGHT ERROR DESCRIPTION MODEL

In our opinion the total error budget of laser altimetry data can be divided into four components with different amplitudes and with different spatial resolution [Crombaghs et al. 2000]. These errors, which are illustrated in figure 1, are:

1. *Error per point.* Due to the measuring uncertainty of the laser scanner each laser point is affected with a random error. This error is also called 'point noise'.

2. *Error per GPS observation.* Every GPS observation as well is affected with a random error. This error, however, is constant (systematic) for all laser points measured during this second. Usually, these points are lying in a strip-wide area of about 100 m in length. This depends on flying speed and GPS observation interval.
3. *Error per strip.* GPS and INS sensors are needed to measure the position and orientation of the aircraft along the flight path. The GPS/INS-system introduces systematic errors in strips, like vertical offsets, tilts in along- and across-track direction and periodic effects with a period of several kilometres.
4. *Error per block.* Terrestrial reference measurements (ground control 'points') are used to transform blocks of laser measurements into the national height system. Errors in these control 'points' result in height deviations which affect entire blocks of laser strips. This influence depends on the block configuration (position and number of strips, cross strips and control 'points') and on the correction procedure (strip adjustment).

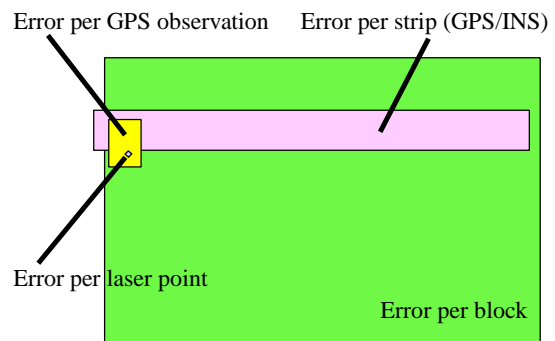


Figure 1. Different scaled error components.

At the Survey Department strip adjustment techniques are developed to minimize error components 3 and 4 [Crombaghs et al. 2000] whereas for error components 1 and 2 it is impossible to correct for. The amplitudes of the four error components differ per project and depend on hardware, software, measurement setup (block configuration) and measurement procedures (e.g. calibration). The next paragraph describes how the amplitude of the error components can be determined.

## 4. DETERMINATION OF ERROR COMPONENTS

In order to quantify the different error amplitudes, various methods are used, such as cross correlation techniques, analysis of empirical covariance functions and 1d strip adjustment. In the following paragraphs these techniques will be described.

### 4.1 Error per point

The amplitude of this error cannot be determined by simply taking the standard deviation of the laser data because this standard deviation covers the total error budget of a single laser point. To obtain the pure point noise, cross correlation techniques are used. Flat areas of 50m x 50m without vegetation and buildings are selected. The height of each

laser point in these areas is prognosticated by interpolating the heights of the neighbouring points (unweighted mean interpolation). Comparison of the interpolated height with the originally measured height yields a height difference. The standard deviation of all differences  $dH_i$  in the area is a suitable measure for the laser point noise:

$$\sigma_{point\ noise} = \sqrt{\sum_{i=1}^n \left( dH_i - \left\{ \frac{1}{n} \sum_{j=1}^n dH_j \right\} \right)^2} \quad (1)$$

with:

$n$  = number of laser points in 50m x 50m flat area  
 $dH_i$  = cross correlated height difference

In each laser dataset a large number of flat areas (about 100) have to be analyzed in order to eliminate the influence of the terrain roughness.

#### 4.2 Error per GPS observation

Flat areas of 50m x 50m are also used for the computation of the amplitude of this error type. In this case, the areas are selected in strip overlaps (see figure 2). Note that these areas are used as tie points for the strip adjustment as well. For every tie point the height difference between both concerned strips is calculated in the following way. At the location of every laser point in the 50m x 50m area in strip 1, an interpolated value is computed by interpolation of the neighbouring laser points in strip 2. The measured height of the laser point in strip 1 is subtracted from the interpolated value of strip 2. The mean value of all height differences in a 50m x 50m area is calculated yielding the height difference between adjacent strips for this tie point. Due to the averaging process, the error per laser point (point noise) is almost absent in this height difference.

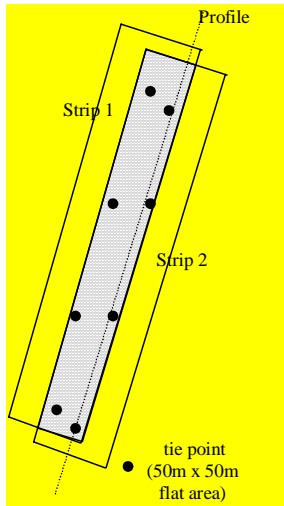


Figure 2. Horizontal, flat areas (tie points) in strip overlap.

The height differences calculated at all tie points along a strip overlap yield a profile (see figure 2). These profiles are analyzed for systematic effects. This can be done with empirical covariance functions, a tool from spatial statistics which is closely related to the (semi)-variogram from geostatistics. The empirical covariance function  $C(s)$  describes the correlation between the signal of sample points

as a function of the distance  $s$  between these points in time or in space.

$$C(s) = C(0) - \frac{1}{2n_s} \sum_{i=1}^{n_s} (x_i - x_{i'})^2 \quad (2)$$

with:

$C(0)$  = variance  
 $n_s$  = number of used point pairs for distance  $s$   
 $x_i$  = signal at point  $i$  (in this case:  $x_i = \Delta h_i$ )  
 $i'$  = point belonging by point  $i$  at distance  $s$

All possible point pairs lying at a distance  $s$  apart from each other are selected, and the covariance for the signal of these point pairs is calculated. This is repeated for increasing distances up to the largest chosen distance  $s_{max}$ . Usually, the signals of sample points at larger distances from each other are less correlated than the signals of sample points close to each other. The covariance values can be plotted in a graph. An example is given in figure 3. The numbers in the graph indicate the number of point pairs used to produce the covariance value. In this example  $s_{max}$  equals 15 km.

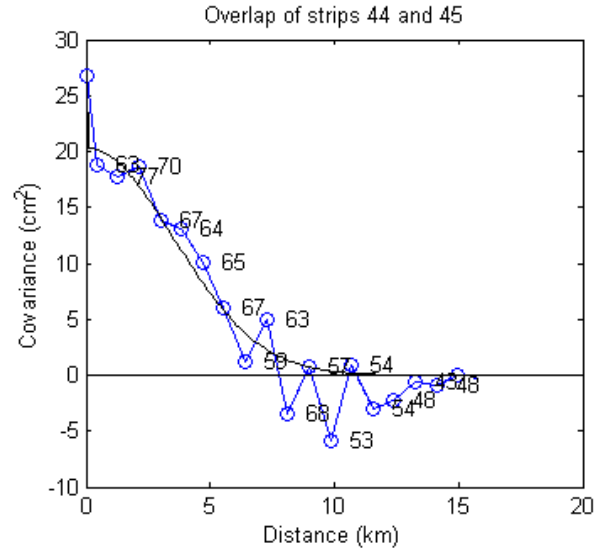


Figure 3. Empirical covariance function with fitted curve.

Usually, an analytical function is fitted through the empirically determined data points. In our case, a Gaussian function is chosen because it is representative for the behaviour of the calculated empirical covariance functions. From this curve three characteristic parameters can be determined (figure 4):

- *The nugget.* This is the variance of measurement errors combined with that from spatial variation at distances shorter than the sample spacing.
- *The sill.* This parameter gives the largest occurring covariance at smallest possible distance between sample points (here about 1 km).
- *The range* (also called correlation distance). The signal values from sample points at this distance and farther away from each other are no longer correlated.

In our height precision assessment the sample points are the tie points along a profile. The signal consists of the height

differences between adjacent strips measured at these tie points. The distance between the tie points in a profile is usually approximately 1 km. The distance between two successive GPS observations is approximately 100 m. Therefore, it can be assumed that the nugget can be related to the variance of the GPS observations corresponding with the random GPS errors. For each strip overlap in a laser dataset (block) the nugget can be determined for two profiles (see figure 2).

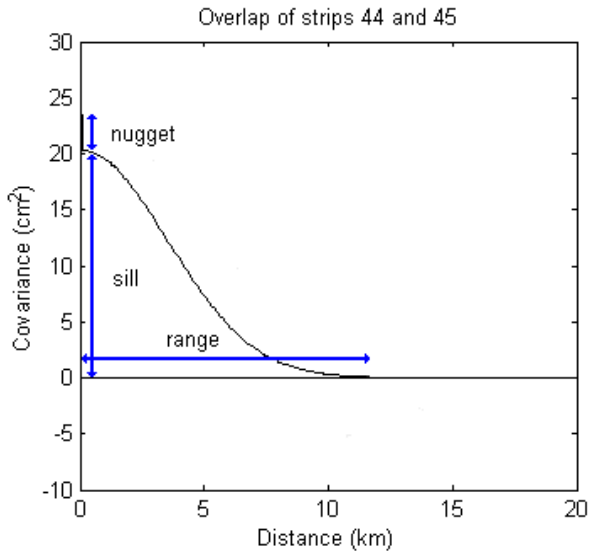


Figure 4. Characteristic parameters of a covariance function.

### 4.3 Error per strip

The introduced parameters, sill and range, of the fitted covariance function (see figure 4) describe systematic effects per strip in along-track direction. This could, for example, be a periodic effect of several kilometres length or an along-track tilt of the entire strip. Figure 5 shows the principle of such a long-time periodic error affecting groups of GPS observations (or generally spoken parts of strips).

The depicted behaviour of the signal due to GPS-noise (see figure 5) is an approximation and simplification of reality because each laser position is determined by interpolation between at least two GPS measurements and, moreover, we have to deal with two interfering signals (from the two overlapping) strips. But actually we found systematic effects in the strip overlaps at distances much smaller than the sample spacing of 1 km (forming the nugget in figure 3).

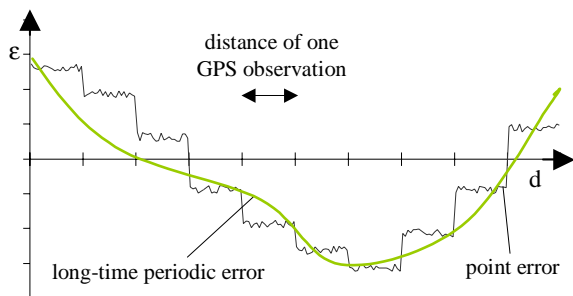


Figure 5. Height error  $\epsilon$  as function of along-track distance  $d$

For each strip overlap in a laser dataset two values (from two profiles) for the sill and for the range of the corresponding covariance function are determined as well. These values represent the errors per strip in along-track direction.

Note that vertical offsets and tilts in across-track direction are not manifest in the covariance function. Vertical offsets are, however, described by the fourth error component as well. There are alternative methods to detect and visualise tilts in across-track direction (like hill shades and profiles). The errors per strip can be decreased with suitable strip adjustment procedures.

### 4.4 Error per block

One of the results from the strip adjustment described in Crombaghs et al. [2000] is a vertical offset correction for every laser strip. The size of these offsets are a measure for the height deviations with respect to the national height system. Assuming that the laser scanning companies already executed a strip adjustment, these offsets must nearly be zero when executing a further strip adjustment for controlling purposes at the Survey Department.

Besides, the standard deviations of these offsets (a further output of the strip adjustment) indicate the height precision of the offset parameters and are therefore measures for the height precision of each individual strip. The estimated precision of the height offsets depends on the configuration of the block: position and quantity of strips, cross strips and control points.

It is common practice to use height differences with the available ground control points to say something about the quality of a laser DEM. In our opinion, it is, however, by far preferable to use the estimated strip offsets themselves and the corresponding standard deviations to get a thorough insight into the height precision of each strip and the entire block.

The used observations in the strip adjustment are height differences between strips ('measured' at tie points), between strips and ground control points, and between strips and cross strips. In order to get realistic values for the standard deviations of the strip offsets, it is necessary to use realistic values for the covariance matrix of the observations in the strip adjustment. This can be achieved by using realistic values for error components 2 and 3. These errors influence the precision of the 'measured' height differences. This has to be taken into account in the stochastic model of the observations. The elements at the main diagonal of the observations covariance matrix originally comprised values for point noise and uncertainties due to interpolation errors. These main diagonal elements were increased for error component 2 and for the sill of error component 3. Because of error component 3, there are also correlations between neighbouring height differences (at distance  $s$ ). They are modelled in the off-diagonal elements with a Gaussian function and the parameters from a representative covariance function derived from several datasets:

$$\text{cov}[cm^2] = \text{sill}[cm^2] \cdot e^{-\left(\frac{s[km]}{\text{range}[km]}\right)^2} \quad (3)$$

If this is done correctly, the overall model test of the least squares strip adjustment (F-test) will be approximately equal to 1, indicating an appropriate model

A fully filled covariance matrix for the observations implies a much longer processing time for the strip adjustment. From several datasets we learned that the resulting standard deviations of the strip offsets are about a factor 1.3 higher when taking into account the off-diagonal elements. In order to save time, the adjustment is therefore done with a diagonal covariance matrix for the observations, taking into account error component 2 and the sill of error component 3, but neglecting the off-diagonal elements (correlations). To correct for this, the resulting strip offset standard deviations are multiplied by a factor of 1.3.

In figure 6 strip offset standard deviations are visualised per strip. It is apparent that the standard deviations of the offsets are smaller in areas with relatively more ground control points, see dots, (and also more tie points, not visible in figure 6) and in areas with more cross strips. The right part of the block has 3 cross strips in opposition to the left strips with only a single cross strip.

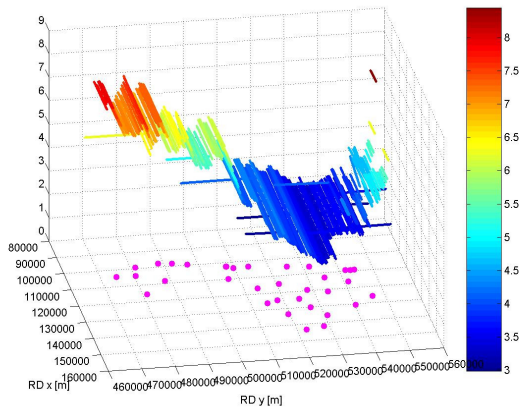


Figure 6. Standard deviations of strip offsets.

From the standard deviations of the strip offsets, a single standard deviation for the precision of a complete laser dataset (entire block) can be calculated by applying the propagation law of variances:

$$\sigma_{dataset}^2 = \frac{\sum_{i=1}^{i=n} (\sigma_{a_i}^2 + \sum_{j=1}^{j=n} \sigma_{a_i, a_j})}{n \cdot n} \quad (4)$$

with:

- $i$  and  $j$  = numbers of adjacent strips
- $\sigma_{a_i}$  = standard deviation of offset  $a$  of strip  $i$
- $\sigma_{a_i, a_j}$  = covariance between offsets of strip  $i$  and strip  $j$
- $n$  = total number of strips in the laser dataset

## 5. RESULTS OF REAL DATA ANALYSIS

The Dutch Survey Department is in the fortunate position to have laser altimetry data available for almost the complete

country. Therefore the amplitude of the different error components can be calculated for large and numerous data sets (approximately the size of a province). Table 1 gives typical values for the amplitudes of the different error components. Note that outliers occur in practice.

error per		mean	max	dimens.
1	point	0.08	0.10	m
2	GPS observation: $\sqrt{\text{nugget}}$	0.03	0.05	m
3a	strip: $\sqrt{\text{sill}}$	0.04	0.05	m
3b	strip: range	9	15	km
4a	block: $\sigma_{\text{strip offsets}}$	0.03	0.08	m
4b	block: strip offsets	0.03	0.10	m

Table 1. Typical values for the error components

## 6. UTILISATION OF ERROR DESCRIPTION

The new height error description will be used for two purposes. The new quality demands for laser DEMs, delivered by laser scanning companies, are based on this description. On the other hand, the customers (DEM users) will be provided with this extended quality description.

### 6.1 Towards the laser scanning companies

The Dutch Survey Department intends to use the new error model with suitable amplitude requirements in the contracts with the laser scanning companies. The calculation methods described in section 4 enable fast and (nearly) automatic tools for the assessing of the data delivered in strips from the contractors.

Table 2 shows the maximal allowed error amplitudes per error type. For every error component the mean value and the maximal occurring value (max) for an entire dataset (project area) has to be below these thresholds. Note that the demanded amplitude for error component 4b (strip offsets) is quite strict. The reason is that this error (systematic per strip) can easily be corrected by strip adjustment. We expect that the companies already have executed a strip adjustment.

error per		mean	max	dimens.
1	point	0.12	0.24	m
2	GPS observation: $\sqrt{\text{nugget}}$	0.03	0.06	m
3a	strip: $\sqrt{\text{sill}}$	0.05	0.08	m
3b	strip: range	10	30	km
4a	block: $\sigma_{\text{strip offsets}}$	0.05	0.08	m
4b	block: strip offsets	0.05	0.13	m

Table 2. Contract demands for maximal error amplitudes

### 6.2 Towards the users

Moreover, the new height error model will be used for the description of the precision of laser altimetry deliveries to customers. With the given error amplitudes per laser dataset, customers (or DEM users) are able to compute the precision of derived products from the laser data, e.g. volumes or mean



heights of differently sized areas. In the following, some examples are given for the determination of the height precision of derived entities. The total error budget for the height of a single laser point is:

$$\sigma_{laserpoint} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_{3a}^2 + \sigma_{4a}^2 + \sigma_{4b}^2}$$

All error components are present in a single laser point. The total error budget for the mean height of an area of 25m x 25m is:

$$\sigma_{25m \times 25m} = \sqrt{\sigma_1^2/40 + \sigma_2^2 + \sigma_{3a}^2 + \sigma_{4a}^2 + \sigma_{4b}^2}$$

It is assumed that there are 40 laser points in this area. The point noise is reduced by the averaging process. The total error budget for the mean height of an area of 500m x 500m is:

$$\sigma_{500m \times 500m} = \sqrt{\sigma_1^2/15,000 + \sigma_2^2/5 + \sigma_{3a}^2 + \sigma_{4a}^2 + \sigma_{4b}^2}$$

In this case the point noise is almost vanishing due to averaging such a large number of laser points. It is assumed that the data in this area is covered by 5 GPS observations. Error component 2 is therefore reduced by factor 5. The area of interest is still lying within a single strip. The total error budget for the mean height of an area of 5km x 5km is:

$$\sigma_{5km \times 5km} = \sqrt{\sigma_1^2/1,500,000 + \sigma_2^2/4,000 + \sigma_{3a}^2/20 + \sigma_{4a}^2 + \sigma_{4b}^2}$$

It is assumed that the data in this area covers 4000 GPS observations and 20 strips (overlap has to be taken into account). Error components 1 and 2 are averaged out (almost) completely, and error component 3a is reduced by factor 20.

For simplification purposes we assume that error components 4 are constant within a block (even though there are varying errors 4a and 4b per strip). Therefore for error components 4 cannot be reduced by averaging, except if the mean height of several blocks is determined. The larger the area of interest, the more dominant this error component becomes. The averaging effect of the error components for different sized areas is visualised in figure 7.

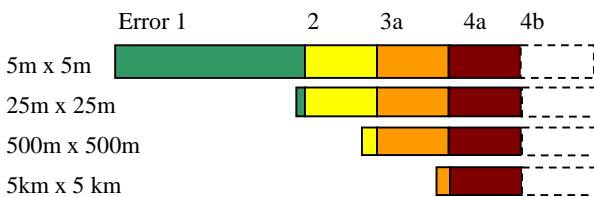


Figure 7. Averaging out of error components

Note that the total error budget refers to the absolute height precision with respect to the national height system. For some applications of laser data the relative error between mean heights of neighbouring regions are more important. These relative errors are smaller than the absolute errors.

## 7. CONCLUSIONS

Describing height precision of laser altimetry DEMs with solely two parameters, a bias and a standard deviation, is

ambiguous and, above all, not sufficient. Due to the integration of different sensors (GPS, INS, laser scanner) in a complex measuring system, the height error budget of single laser points or regions of a certain size is a combination of several error components. These errors can be characterized by amplitude and spatial resolution. With regard to the affected area, these errors can roughly be divided into four components: an error per point, strip section (covered during one GPS observation), strip and entire block.

Deriving typical values for the error amplitudes from numerous datasets enabled the Survey Department to formulate new demands for maximal error amplitudes with regard to the contractors (laser scanning companies). Besides, the new height error description model and the developed tools for determining the amplitude of each error component from a specific dataset provide a useful instrument to get a thorough insight into the height quality of the delivered datasets. Finally, the knowledge of such a comprehensive height error description per dataset is very useful to the users. They are enabled to determine height precision of derived products in a reliable way.

Up to now, the usual strip adjustment is 1d (height adjustment). A complete 3d strip adjustment, such as proposed in Burman [2000] or Vosselman and Maas [2001] could increase the height precision of laser altimetry DEMs due to a better modelling of the occurring errors. It is desirable to use a 3d strip adjustment in the near future.

## REFERENCES

- Brügelmann, R., 2000. Automatic breakline detection from airborne laser range data. In: *International Archives of Photogrammetry and Remote Sensing*, vol. 33, part B3/1, pp. 109-115.
- Burman, H., 2000. Adjustment of Laserscanner Data for Correction of Orientation Errors. In: *International Archives of Photogrammetry and Remote Sensing*, vol. 33, part B3/1, pp. 125-132.
- Crombaghs, M.J.E., Brügelmann, R., de Min, E.J., 2000. On the adjustment of overlapping strips of laseraltimeter height data. In: *International Archives of Photogrammetry and Remote Sensing*, vol. 33, part B3/1, pp. 224-231.
- Huisig, E.J., Gomes Pereira, L.M., 1998. Errors and accuracy estimates of laser data acquired by various laser scanning systems for topographic applications. *ISPRS Journal of Photogrammetry and Remote Sensing* 53 (5) 245-261.
- Schenk, T., 2001. Modeling and recovering systematic errors in airborne laser scanners. In: *Proceedings OEEPE workshop on Airborne Laserscanning and Interferometric SAR for Detailed Digital Elevation Models*. OEEPE Publication no. 40, pp. 40-48.
- Vosselman, G. and H.-G. Maas, 2001. Adjustment and filtering of raw laser altimetry data. In: *Proceedings OEEPE workshop on Airborne Laserscanning and Interferometric SAR for Detailed Digital Elevation Models*. OEEPE Publication no. 40, pp. 62-72.