REDUNDANT IMUS FOR PRECISE TRAJECTORY DETERMINATION

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ABSTRACT:

A redundant inertial measurement unit (IMU) is an inertial sensing device composed by more than three accelerometers and three gyroscopes. This paper analyses the performance of redundant IMUs and their potential benefits and applications in airborne remote sensing and photogrammetry. The theory of redundant IMUs is presented through two different algorithmic approaches. The first approach is to combine the inertial observations, in the observation space, to generate a "synthetic" non-redundant IMU. The second approach modifies the INS mechanization equations so that they directly account for the observational redundancy. The paper ends with an empirical assesment of the concept. For this purpose, redundant IMU data was generated by combining two IMUs in a non-orthogonal configuration and flying them. Preliminary results of this flight are presented.

1 INTRODUCTION

The use of redundant IMUs for navigation purposes is not new. From the very early days of the inertial technology, the inertial navigation community was aware of the need and benefits of redundant information. However, to the best knowledge of the authors, the focus of the research and development efforts was fault detection and isolation (FDI). In the early days, the idea was to make use of the redundancy in order to support fault-safe systems. A fault-safe system detects that a sensor —i.e., an angular rate sensor or an accelerometer— is not working properly and shuts the system down. A fault-tolerant system is able not only to detect a defective sensor, but also to isolate it. After isolating a defective sensor, the system may keep working as a fault-tolerant or a fault-safe system depending on the number of remaining sensors. By means of voting schemes (Pejsa, 1973), it can be shown that a minimum of four sensors are needed to devise a fault-safe system and a minimum of five to devise a fault-isolation one (compare to the parallel development in photogrammetry (Förstner, 1985)). Sensor configuration for optimal state estimation and optimal FDI was, as well, a topic of research in the early works.

In (Sturza, 1988b) and (Sturza, 1988a) a comprehensive analysis of the optimal spatial configuration of sensors for FDI applications is provided together with FDI algorithms. In addition, the performance for fail-isolation systems in case a sensor is removed due to failure. is analyzed.

In the literature, usually, two general geometries for redundant sensor configurations are considered. Assume that there are n sensors. The first geometric configuration distributes the sensors on a cone of half angle α in a way that there is a constant solid angle between any two consecutive sensors. This type of geometry is referred as Class I. In the second geometric configuration, named Class II, $n-1$ sensors are evenly distributed on a cone with half angle α and the remaining is in the cone axis. For these geometries there are different values for α that maximize the amount of information captured by the sensors and hence, allow for an optimal state estimation. Then, by means of hypothesis testing and maximum likelihood estimation, FDI is performed. For a detailed derivation of the optimal values for α as a function of the number of sensors, as well as for sensor FDI algorithms, the reader is referred to (Sturza, 1988b, Sturza, 1988a). More recent results on the use of redundant inertial sensors for FDI can be found in (Sukkarieh et al., 2000) and (Lennartsson and Skoogh, 2003). The former is mainly concerned with the use of skewed redundant configurations for unmanned air vehicles while the latter focuses in guidance, navigation and control of underwater vehicles. The two references are good examples of the wide range of applications for skewed redundant configurations that are currently under research.

The approach to and applications of redundant inertial sensors proposed in this paper are different. The approach taken is the geodetic one; i.e., use redundancy as a fundamental strategy to asses the quality of the navigation parameters and, together with an appropriate mission design, to calibrate the instrument systematic errors. The application is focused on the precise, accurate and reliable INS/GPS trajectory determination for airborne photogtammetry and remote sensing (APRS). This includes, among others, the improvement of heading determination and the detection of gross errors.

The current INS/GPS approach to APRS applications is that of a single IMU combined with one or two GPS receivers. Satellite positioning contributes the long wavelength information, while inertial positioning contributes the short wavelength information. Although this approach has brought remarkable progress to APRS, it has some limitations so there is need for further research and room for further improvement. These limitations range from unobservable heading drifts in long strips to defective sensors going undetected for years. One possible cause of this limitations is the lack of redundant inertial data. This paper explores the possible benefits of the Skewed Redundant IMU (SRIMU) concept. In a previous paper (Colomina et al., 2003) the authors already presented a general overview of the SRIMU concept for APRS and some preliminary tests.

2 POTENTIAL ADVANTAGES FOR AIRBORNE PHOTOGRAMMETRY AND REMOTE SENSING

2.1 Realistic noise estimation

Redundancy allows for intrinsic noise estimation and therefore input precision for inertial measurements is realistic provided that good calibration of the sensors is achieved. Realistic noise estimates eliminate the need for adaptive Kalman filtering and improve the performance of robust Kalman filtering techniques.

2.2 Overall navigation performance improvement

Overall navigation improvement is to be expected as there is more input information. By simply adding a sensor to the normal 3-axis configuration, an increase by 33% in the amount of information is achieved (Sukkarieh et al., 2000). Through increased redundancy, a noise reduction in the navigation output parameters is expected.

In airborne surveys, heading determination accuracy relies on the performance of the vertical angular rate sensor and the horizontal accelerometers. Long, straight and constant speed flight lines do not allow to calibrate these three sensors properly. Redundant configurations may change this situation.

Redundancy allows for de-contextualization of the calibration process. In contextual calibration, there is only a limited control on the physical correctness of the calibration states. In other words, an apparently correct calibration at the system level does not necessarily correspond to a correct calibration at the sensor level. This may result in unpredictable navigation performance when the context changes as it uses to be the case in terrestrial navigation.

2.3 Reliability and integrity improvement

Redundancy is on the basis of hypothesis testing for error detection and isolation. In INS/GPS navigation there is no redundancy with the exception of the coordinate update observations. With redundant inertial measurements, reliability aspects may be approached at the IMU level in a way that the detection of defective sensors or spurious signals can be detected. The authors are aware of defective IMU sensors going undetected and being operated for years.

Recall that INS navigation integrity is many times achieved by means of multiple-sensor configurations and that in civil aviation, airplanes are frequently equipped with more than one IMU.

3 ALGORITHMIC APPROACHES TO SRIMU NAVIGATION

As mentioned before, the use of redundant inertial measurements may provide some benefits for INS/GPS trajectory determination in APRS. However, correctly blending the redundant measurements together is of vital importance for the performance of the system. There is not a unique way to tackle this issue and all of them have their own pros and cons.

Before seizing the data blending issue, it has to be noted that when dealing with redundant configurations two options arise: the use of a genuine SRIMU (more than 3 + 3 sensors assembled in a single box) and the use of two or more standard inertial units.

The first option might be the optimal one in that the device has been designed to provide redundant data. However, nowadays, this kind of devices are not being produced at a very high rate for commercial purposes and, to the best knowledge of the authors, they are mainly used in research or space projects.

The second option is interesting in the sense that it is enough to arrange two IMUs in such a way they are skewed with respect to each other to simulate a SRIMU. As mentioned, the difficulty of purchasing a genuine SRIMU and the usual high cost of ownership of nonstandard product, makes of this kind of simulations a very practical, attractive and economical solution. These configurations will be referred to as *dual IMU configurations*.

The next section discusses these two options, SRIMU and dual IMU, in detail.

3.1 ONE SKEWED REDUNDANT IMU

In this section a single skewed redundant IMU with n angular rate sensors and m accelerometers is assumed (n, $m > 3$). As the focus of this paper is not on SRIMU design optimization, a fixed orientation of the $n + m$ inertial sensors is assumed to be given through the direction cosine vectors of each sensor sensing axis. Let b be an instrumental reference frame. Further, let ℓ_ω^b be the angular rate observation vector produced by the n gyroscopes and ℓ_a^b the linear acceleration vector produced by the m accelerometers. If the instrument is subject to angular rates and linear accelerations described by the vectors ω^b and a^b respectively, then it holds that

$$\begin{pmatrix} \ell_\omega^b \\ \ell_a^b \end{pmatrix} = \begin{bmatrix} A_\omega & 0 \\ 0 & A_a \end{bmatrix} \begin{pmatrix} \omega^b \\ a^b \end{pmatrix} \qquad (1)$$

where A_ω and A_a are $n \times 3$ and $m \times 3$ matrices. The rows of A_ω (resp. A_a) contain the direction cosine vectors of the n angular rate sensor axes (resp. of the m accelerometers). Note that in equation 1 the errors of the inertial sensors have been neglected.

At this point, two avenues can be explored for the optimal exploitation of the redundant inertial information contained in the observation vector ℓ^b where $\ell^b = (\ell_\omega^{b^T}, \ell_a^{b^T})^T$: either redundancy is dealt with in the observation space or in the state space (also called the parameter space). The next two sections explore the two possibilities.

3.1.1 Dealing with redundancy in the observation space: synthetic 3D axis IMU generation. Note that the A_ω and A_a matrices transform data from the actual sensor axes to the three orthogonal axes of the predefined body frame b. Thus, one could think of an imaginary non-redundant IMU aligned to the b frame axes and centered at the b frame origin. This imaginary IMU will be called *synthetic*. Equation 1 can be rewritten as an error equation for the measured $n+m$ amounts by introducing the corresponding residual vectors $v_{\ell_\omega^b}$ and $v_{\ell_a^b}$.

$$\begin{pmatrix} \bar{\ell}_\omega^b \\ \bar{\ell}_a^b \end{pmatrix} + \begin{pmatrix} v_{\ell_\omega^b} \\ v_{\ell_a^b} \end{pmatrix} = \begin{bmatrix} A_\omega & 0 \\ 0 & A_a \end{bmatrix} \begin{pmatrix} \omega^b \\ a^b \end{pmatrix}. \qquad (2)$$

Equation 2 is the basis for the transformation of the actual redundant observations ℓ^b into a standard set of $3+3$ observations of the synthetic IMU. All what has to be done is to solve for ω^b and a^b in equation 2. Considering the redundant nature of the problem, ω^b and a^b can be estimated by least-squares in the usual way. This would lead to two orthogonal projectors, Π_ω for the angular rates and Π_a for the linear accelerations, that blend the raw redundant data into the synthetic IMU through the equations

$$\omega^b = \Pi_\omega \ell_\omega^b \text{ and } a^b = \Pi_a \ell_a^b \qquad (3)$$

where

$$\Pi_\omega = \left({A_\omega}^T C_{\ell_\omega^b \ell_\omega^b}^{-1} A_\omega \right)^{-1} {A_\omega}^T C_{\ell_\omega^b \ell_\omega^b}^{-1} \qquad (4)$$

and

$$\Pi_a = \left({A_a}^T C_{\ell_a^b \ell_a^b}^{-1} A_a \right)^{-1} {A_a}^T C_{\ell_a^b \ell_a^b}^{-1} \qquad (5)$$

where $C_{\ell_\omega^b \ell_\omega^b}$ and $C_{\ell_a^b \ell_a^b}$ are the covariance matrices of the raw inertially sensed vectors $\bar{\ell}_\omega^b$ and $\bar{\ell}_a^b$ respectively.

The above simple [plain least-squares] procedure has some advantages. It allows the use of off-the-shelf existing INS and INS/GPS software as the non-standard redundant IMU output is converted into the usual IMU output. It allows for the epoch-by-epoch, realistic estimation of sensor noise and of covariance matrices for the synthetic IMU angular rate sensor and accelerometer triads respectively. It allows for defective sensor detection, identification and isolation depending on the number and distribution of sensors by standard geomatic data-snooping and gross-error detection techniques based on hypothesis testing in linear models. And it eliminates the need for adaptive Kalman filtering, and other dangerous mathematical acrobacies.

However, the procedure has its drawbacks. Calibration of the synthetic IMU is certainly possible and is, to a large extent, acceptable, as linear combinations of the systematic errors can be estimated by the Kalman filter for the synthetic IMU. But they cannot be back projected into the single actual sensors and the noise estimates may be inflated by these unknown errors. The situation can be dealt with in a number of ways, but it is not the ideal one.

3.1.2 Dealing with redundancy in the state space: extended INS mechanization equations. In order to overcome the limitations of combining the redundant raw observations in the observation space the well known inertial mechanization equations (Jekeli, 2001) can be easily modified to accommodate the redundant data. The following set of extended inertial mechanization equations include all redundant sensors and calibration states that, for the sake of simplicity, are limited to biases (o^b and a^b) obeying a Gauß-Markov first order stochastic process model. They are written in a cartesian geocentric coordinate system for an Earth-Centered-Earth-Fixed (ECEF) type of reference system.

$$\begin{aligned} \dot{x}^e &= v^e + w_v \\ \dot{v}^e &= R_b^e \Pi_a (\ell_a^b + a^b + w_{\ell_a}) - 2\Omega_{ie}^e v^e + g^e(x^e) \\ \dot{R}_b^e &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b (\Pi_\omega (\ell_\omega^b + o^b + w_{\ell_\omega})) \right) \\ \dot{o}^b &= -\beta o^b + w_o \\ \dot{a}^b &= -\alpha a^b + w_a \end{aligned} \qquad (6)$$

In equation 6 above, $\alpha, \beta > 0$ and w_v, w_f, w_ω, w_o and w_a are white noise generalized processes.

The above modeling allows for the calibration of the actual sensors at the external aiding epochs provided that the geometric and dynamic properties of the motion guarantee the observability of the system. Moreover, it can be combined with the procedure of section 3.1.1 as the predicted systematic errors can be eliminated from the raw sensed data from the outset.

The main drawback of the procedure is that it requires the modification of the INS and INS/GPS software. Whether this is a problem or not, depends on non technical factors that are out of the scope of the paper.

3.2 TWO INERTIAL MEASUREMENT UNITS

One, not yet mentioned, problem of SRIMUs is that, from a practical market standpoint, they do not exist. Or almost. Some laboratories have built their own SRIMU and the company L3 manufactures and sells them. However, if the use of redundant inertial observations proves to be of practical interest, a straightforward way to go is to use two independent, standard inertial units. But this is easier said than done. Time synchronization becomes critical, the relative orientation (position and attitude) of the units must be measured and/or calibrated precisely. And these geometrical constrains have to be transferred to either the system dynamic observation model —the equations of motion— or to the static observation model —the measurement equations.

As with the SRIMUs, various modeling approaches are possible. One approach, working in the observational space, combines the two units into a single one. A second approach, in the state space, is to define a single inertial unit containing all the sensors of the two units. A third approach, in the state space, is to navigate the two units and to impose geometrical constrains to the navigation results. In the next sections the first and, particularly, the third approach are discussed. For this purpose, assume: that a dual configuration with two inertial units, named one and two rwspectively, is given; that the inertial units body frames are $b1$ and $b2$ respectively; that unit i measurements are ω_i^{bi} and f_i^{bi}, for $i = 1, 2$; that their relative orientation (u_{b1}^{b2}, R_{b1}^{b2}) does not change with time and that it is known through direct measurements or through calibration.

3.2.1 Dealing with redundancy in the observation space: synthetic 3D axis IMU generation. In principle, if the relative orientation between the two inertial units is known, it is possible to reduce all inertial measurements to a single synthetic IMU. That is, if the inertial unit 1 is chosen as a reference for the synthetic unit, then the original measurements of unit 2, ω_2^{b2} and f_2^{b2} must be transformed —"corrected"— to ω_2^{b1} and f_2^{b1}. While the transformation for angular rates is straightforward $\omega_2^{b1} = R_{b2}^{b1}\omega_2^{b2}$ the transformation for linear accelerations is a bit more complex because of the combined effect of the u_{b1}^{b2} lever-arm and vehicle motion. The correction to be applied is similar to the "size-effect" correction (Savage, 2000) and can be obtained after some algebraic manipulations and some simplification assumptions. Once the correction is applied the situation is similar to that of section 3.1.1 for the synthetic generation of a standard non-redundant IMU and the discussion is not repeated here.

3.2.2 Dealing with redundancy in the state space: geometrically constrained dual navigation. Geometrically constrained dual navigation is the navigation of the two units, subject to the geometric constrains of their constant relative orientation. The dynamic observation model for dual navigation is composed of the inertial mechanization equations of the two inertial units plus the dynamic observation model for the relative orientation parameters or states between the inertial units. The static observation model for dual navigation is, essentially, a set of three vector equations describing the relative orientation between the inertial units. The two models are given in the next two paragraphs.

Dynamic Observation Model. The following equations are the standard inertial mechanization equations for the two inertial units extended with two sets of differential equations for u_{b2}^{b1} and R_{b1}^{b2}. The relative orientation parameters between the IMUs are modeled as random constants in case their direct measurement is not accurate enough.

$$
\begin{aligned}
\dot{x}_1^e &= v_1^e + w_{v_1} \\
\dot{v}_1^e &= R_{b1}^e(f_1^{b1} + a_1^{b1} + w_{f_1}) - 2\Omega_{ie}^e v_1^e + g^e(x_1^e) \\
\dot{R}_{b1}^e &= R_{b1}^e\left(\Omega_{ei}^{b1} + \Omega_{i\,b1}^{b1} + \Omega^{b1}(o_1^{b1}) + w_{\omega_{i\,b1}}\right) \\
\dot{o}_1^{b1} &= -\beta_1\, o_1^{b1} + w_{o_1} \\
\dot{a}_1^{b1} &= -\alpha_1\, a_1^{b1} + w_{a_1} \\
\dot{R}_{b2}^{b1} &= 0 \\
\dot{u}_{b2}^{b1} &= 0 \\
\dot{x}_2^e &= v_2^e + w_{v_2} \\
\dot{v}_2^e &= R_{b2}^e(f_2^{b2} + a_2^{b2} + w_{f_2}) - 2\Omega_{ie}^e v_2^e + g^e(x_2^e) \\
\dot{R}_{b2}^e &= R_{b2}^e\left(\Omega_{ei}^{b2} + \Omega_{i\,b2}^{b2} + \Omega^{b2}(o_2^{b2}) + w_{\omega_{i\,b2}}\right) \\
\dot{o}_2^{b2} &= -\beta_2\, o_2^{b2} + w_{o_2} \\
\dot{a}_2^{b2} &= -\alpha_2\, a_2^{b2} + w_{a_2}
\end{aligned}
\tag{7}
$$

In equation 7, $\beta_1, \alpha_1, \beta_2, \alpha_2 > 0$ and the terms w_{v_1}, w_{f_1}, $w_{\omega_{i\,b1}}$, w_{o_1}, w_{a_1}, w_{v_2}, w_{f_2}, $w_{\omega_{i\,b2}}$, w_{o_2} and w_{a_2} are white noise generalized processes.

Static Observation Model. In addition to the measurements provided by the external navigation aids, dual navigation exploits the following relative orientation relationship.

$$
\begin{aligned}
0 + v_r &= R_{b1}^e - R_{b1}^{b2} R_{b2}^e \\
0 + v_x &= x_1^e + R_{b1}^e u_{b2}^{b1} - x_2^e \\
0 + v_v &= \dot{x}_1^e + R_{b1}^e\left(\Omega_{ei}^{b1} + \Omega_{ib}^{b1} + \Omega^{b1}(o_1^{b1})\right) u_{b2}^{b1} - \dot{x}_2^e
\end{aligned}
\tag{8}
$$

In equation 8, v_r, v_x, v_v are zero-mean normally distributed random variables.

The processing strategy for the above dynamic and static observation models is the usual one. The state of the system is predicted by solving the stochastic differential equation 7 and the static observation model is used to feed the system with the relative orientation constrains. Note that, in principle, the filter corresponding to equation 7 can be applied after each prediction step. Note, as well, that, although not discussed here, any other external navigation aids can be integrated in this model, at their own frequency and in the usual way.

3.3 Comparative analysis

In the preceding sections, two algorithmic approaches have been proposed for each one of the redundant configurations considered. One approach combines the redundant data in the observation space (synthetic IMU generation) and and the other in the state space (extended INS navigation equations and geometrically constrained navigation of dual IMU configurations).

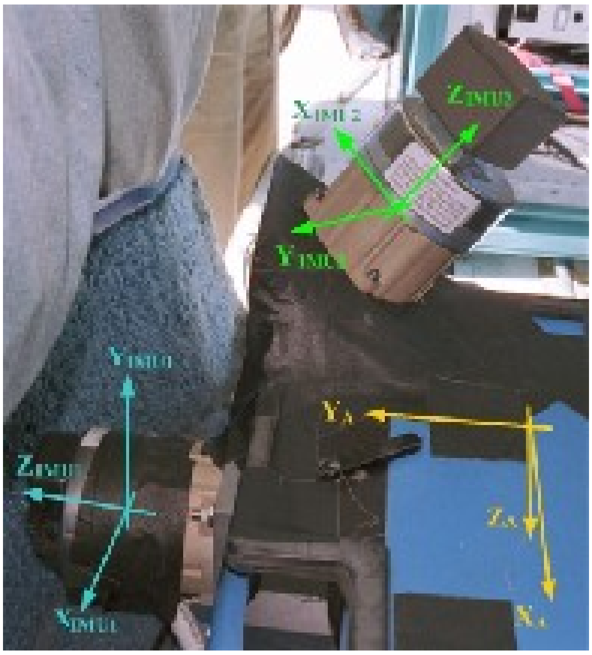

Figure 1: Dual IMU configuration of the test flight.

From a strict technical point of view, the authors believe that the optimal solution is a combination of both approaches where the dominant procedure is the integration at the state space level (sections 3.1.2 and 3.2.2) for the purpose of navigation and sensor calibration. The procedure at the observation level plays a secondary role; estimating actual noise figures. (This statement is subject to change as empirical validation as the analysis of actual redundant data sets is still in progress.)

4 DUAL IMU TEST FLIGHT

In order to validate the concepts and models described so far, in July 2003 a test flight was jointly conducted by the Institute of Geomatics and StereoCARTO. In the test, two similar IMUs were flown. As the goal of this paper is not to report on the test flight and its results (still undergoing analysis) only a short summary of it and some related results will be given.

4.1 Description of the experiment

The test was performed in the outskirts of Madrid with a Cessna 207 aircraft owned by HIFSA and consisted of a photogrammetric-like flight in which two similar IMUs (Northrop Grumman LN-200) were placed in a non-orthogonal configuration using an special mount designed for this experiment (see figure 1). The IMUs were some 15 cm apart from each other.

The inertial units used in the experiment had different control units and output rates. One of the inertial units was connected to an Applanix POS/AV 410 owned by StereoCARTO that delivered data at 200 Hz. The other LN-200 unit belongs to the IG, was connected to the TAG system and delivers data at 400 Hz. Other details about this flight can be seen in (Colomina et al., 2003).

	ω_x	ω_y	ω_z	a_x	a_y	a_z
		(deg/h)			(m/s^2)	
μ	-1.19	-1.25	0.81	-.005	.000	.002
σ	165	177	157	.251	.268	.215

Table 1: Consistency of the dual-IMU configuration: residuals of inertial observations after IMU frame-to-frame transformation.

4.2 Consistency analysis of the dual IMU data

In (Colomina et al., 2003) a preliminary analysis of the consistency of the two IMU data sets was reported. There, the simplest possible comparative analysis was carried out. After synchronizing/interpolating the two data streams to a common discrete time scale at 200 Hz, the total (vector's norm) angular rates and linear accelerations sensed by the inertial units were compared. In this paper a similar comparative analysis is done for each one of the sensors. For this purpose, the rotation matrix R_s^i between the two inertial units, i and s was computed by means of the overdetermined set of equations

$$\begin{aligned} \ell_\omega^i + \upsilon_\omega^i &= R_s^i(\ell_\omega^s + \upsilon_\omega^s) \\ \ell_a^i + \upsilon_a^i &= R_s^i(\ell_a^s + \upsilon_a^s) \end{aligned}$$

where ℓ_ω^i, ℓ_ω^s are the angular rate and ℓ_a^i, ℓ_a^s are the linear acceleration observations of the i and s inertial units respectively. υ_ω^i, υ_ω^s, υ_a^i and υ_a^s are the residuals of the observation equations as usual. In the above equations, the amounts of interest are the precision of the R_s^i determination and, above, all the residuals.

The rotation matrix R_s^i was parametrized by a sequence of Euler angles that were estimated with precisions of 4×10^{-5}, 5×10^{-7} and 5×10^{-5} degrees respectively which is better than enough for the purpose of the test.

Some residuals' series (x- and z-accelerometers and z-angular rate sensor) and their Fourier transforms are depicted in figure 2. The results, as clearly seen from the graphics cannot be more encouraging as they can be interpreted as white noise (left column) perturbed with engine vibrations at 20 Hz and its harmonics. Moreover, the mean (μ) and empirical standard deviation (σ) of the residuals after the estimation of the rotation matrix R_s^i for each pair of homologous sensors in the domain space were computed. The μ and σ values are depicted in table 1. They are consistent with the figures provided by the LN-200 vendor (1 deg/h angular rate sensor bias, 0.003 m/sec^2 accelerometer bias, 62 deg/h (1-σ) angular rate sensor noise and 0.34 m/sec^2 (1-σ) accelerometer noise, at 200 Hz). Therefore, the residuals are close to the uncertainty that characterize the sensors and the dual set of inertial observations can be considered valid to pursue further research in redundant inertial information as intended.

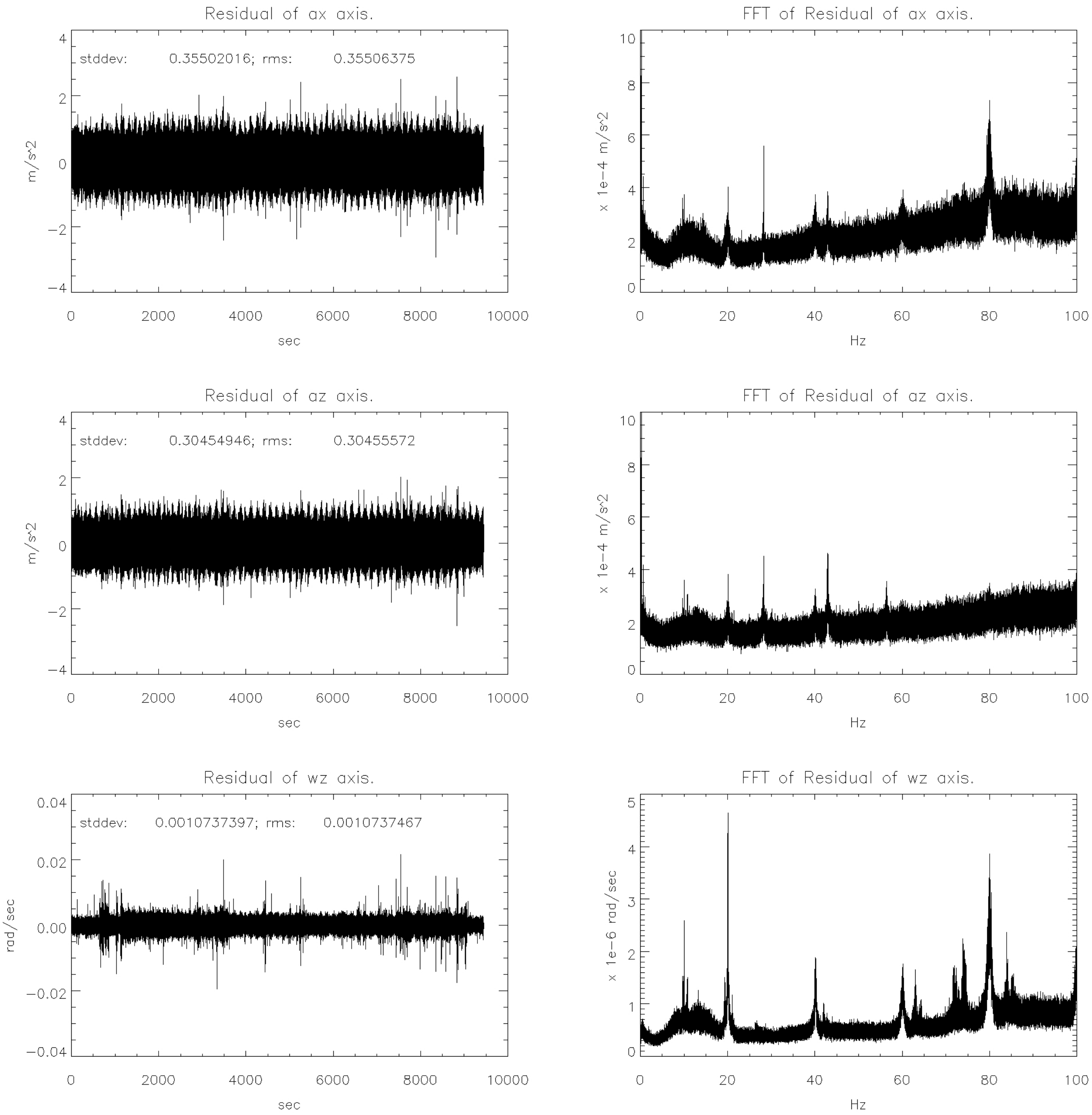

Figure 2: IMU-to-IMU residuals and their PSD function.

5 SUMMARY AND FURTHER RESEARCH

The potential benefits of redundant inertial observations for airborne surveying have been analyzed and the theory for their processing has been presented for skewed redundant IMUs and for dual IMUs configurations. A test flight with a dual IMU redundant configuration with two Northrop-Grumann's LN-200 IMU has been realized and presented. The consistency of the two data sets has been analyzed and proven. Future research will concentrate on the processing of the dual LN-200 data set according to the theory discussed in this paper.

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