

# MULTISTATION ESTIMATING OF GPS SIGNAL ATMOSPHERIC DELAYS BY UNDIFFERENCED OBSERVATIONS

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## ABSTRACT:

Precise RTK positioning is usually performable only on short length bases, to limit distance correlated biases, as atmosphere and ephemerides. However, it is possible to recover some residual biases also over long distance using appropriate area models, improving the performances of long base RTK positioning up to 100 km base length. Permanent reference station networks are used to estimate the bias model in surveying type applications requiring centimeter accuracy, reducing the worth of traditional single baseline methods. The well known advantages provided by reference station network information include improved modeling of the residual tropospheric, ionospheric and orbit biases. The system state vector, including the parameters of the bias model, is evaluated via Kalman filtering from the undifferenced observation equations. On the rover side, corrections are applied to the observations as single differences, then the model estimates and recovers the undifferentiated residual biases.

## 1. MOTIVATION

Continuous operating reference stations, tracking GPS or GLONASS, and in the future also Galileo, can be coordinated in static and also real time networks. Real time networks provide the support for surveying type applications, as for reference frame realization and maintenance, weather and space-weather forecasting, physical and geophysical type applications. The network properties can be defined under different aspects:

- Provided products and services and their broadcasting infrastructure.
- Network extension, density and distribution of the reference stations.
- Network architecture, number and interconnection of the master stations, communication network.
- Mathematical models and algorithms.
- Integration of different source data, as meteo, radar, VLBI, laser ranging, etc.

The paper focus on bias estimating and surveying type applications. To overcome the limitations of standard RTK systems, reference networks of permanent operating GPS stations can provide Network RTK (NRTK) services, broadcasting network based state parameters to the mobile user, who is then enabled to operate in Multi Reference Station or Virtual Reference Station mode.

Network parameter broadcasting is limited in the RTCM 2.x standards, but is now included in the new RTCM 3.0 standard. Standardisation discussion within RTCM in the new version design, has targeted the interoperability between reference station systems and rover receivers from various manufacturers. However, the obstacles in network parameters broadcasting with RTCM 2.x are overcome by the manufacturers by means of proprietary formats, sometimes largely accepted and used as FKP corrections included in RTCM message type 59, sometimes available in proprietary solutions only. The use by different manufacturers of the same proprietary formats, is due to the fact that interoperability is needed urgently. Moreover, the

interoperability issues, are related to the creation and proper description of the models used for deriving the biases.

The paper presents some remarks about NRTK techniques, taking account of results obtained by professional dual frequency and also low cost single frequency receivers.

Data processing simulates NRTK procedures, using a self made software named ALARIS. This software performs one-way estimation of biases, real time static or kinematic positioning, by using different Kalman filtering techniques.

## 2. DEFINITIONS

In NRTK a master station estimates atmospheric biases over a network of permanent reference stations; then the rover station applies to his observations the corrections broadcasted by the master station. This positioning technique will be largely diffused in future for many applications in engineering, cartography, cadastre, GIS and so on. For all this applications, that usually don't require very high precisions, the use of low cost single frequency receivers is quick and economic (Cina, Manzano, Roggero, ISPRS VI/3, 2003).

The proposed technique can be applied mainly in real time positioning and satellite navigation, but also in weather forecasting, ionospheric and tropospheric tomography on regional scale, time transfer, signal propagation and geomagnetic activity monitoring.

To perform real time multistation estimating of GPS biases, the parameterisation in the state space domain is applied. For the state space approach the use of undifferenced observations is recommended, since this allows the estimation of clock parameters as well as the separation of atmospheric delays. It also makes the modelling of error processes easier, compared to approaches based on double differences. However, using undifferenced observations we must correctly model some physical parameters, such as the corrections related to the receiver measurement style (cross-correlation or Z-tracking), satellite and receiver antenna phase centre, constant and periodic

relativistic corrections, polar motion and tidal earth. State space vector is estimated by Kalman filtering and include the receiver dependent parameter (receiver clock) and the satellite dependent parameters (carrier phase ambiguities, ionospheric and tropospheric delays). Due to the low redundancy of the system, some parameters as orbit errors, satellite clock estimation errors and multipath are neglected.

### 3. EQUATIONS FOR BIASES CORRECTION

Undifferenced equations are suitable for use in bias estimation, necessary to build the bias model and then broadcast the network parameters, to correct the observations in the rover station. A refined way to calculate the GPS signal biases is to estimate every single bias from undifferenced equations: all the error sources that is possible to model, are estimated in the reference station by Kalman filter, applying deterministic and stochastic models in the observation and in the parameter space. This procedure is named *one way biases estimation*. The observation equations in the reference station, identified by the subscript  $k$ , are:

$$\begin{aligned} P_{1k}^j &= \mathbf{r}_k^j - E_k^j + \dot{\mathbf{r}}_k^j \cdot \Delta T_k + c \cdot (\Delta T_k - \Delta t^j) + I_k^j + Tr_k^j + M_{P_{1k}^j} + \mathbf{d}_{P_{1k}^j} + e_{P_{1k}^j} \\ P_{2k}^j &= \mathbf{r}_k^j - E_k^j + \dot{\mathbf{r}}_k^j \cdot \Delta T_k + c \cdot (\Delta T_k - \Delta t^j) + \mathbf{n} I_k^j + Tr_k^j + M_{P_{2k}^j} + \mathbf{d}_{P_{2k}^j} + e_{P_{2k}^j} \\ \Phi_{1k}^j &= \mathbf{r}_k^j - E_k^j + \dot{\mathbf{r}}_k^j \cdot \Delta T_k + c \cdot (\Delta T_k - \Delta t^j) - I_k^j + Tr_k^j + M_{\Phi_{1k}^j} + \mathbf{d}_{\Phi_{1k}^j} + e_{\Phi_{1k}^j} + \mathbf{I}_1 N_{1k}^j \\ \Phi_{2k}^j &= \mathbf{r}_k^j - E_k^j + \dot{\mathbf{r}}_k^j \cdot \Delta T_k + c \cdot (\Delta T_k - \Delta t^j) - \mathbf{n} I_k^j + Tr_k^j + M_{\Phi_{2k}^j} + \mathbf{d}_{\Phi_{2k}^j} + e_{\Phi_{2k}^j} + \mathbf{I}_2 N_{2k}^j \end{aligned} \quad (1)$$

- $\mathbf{r}_k^j$  Geometric range between the receiver and the satellite antenna reference point.
- $\dot{\mathbf{r}}_k^j$  Topocentric range rate.
- $E_k^j$  Ephemeris geometric radial error.
- $\Delta T_k$  Receiver clock error.
- $\Delta t^j$  Satellite clock error.
- $I_k^j$  Ionospheric delay.
- $Tr_k^j$  Tropospheric delay.
- $M_{O_{nk}^j}$  Multipath affecting the observation  $O_n$ .
- $\mathbf{d}_{O_{nk}^j}$  Hardware delay.
- $e_{O_n}$  Observation noise.
- $N_{nk}^j$  Integer ambiguity.
- $\mathbf{I}_1, \mathbf{I}_2$  Wavelengths of the L1 and L2 signals.
- $c$  Light speed in the vacuum.
- $\mathbf{n} = \frac{f_1^2}{f_2^2}$  Rate between the two squared frequencies.

Subtracting the terms known from the ephemeris, the range  $\mathbf{r}$ , the topocentric range rate and the satellite clock error, we obtain for every observation:

$$\begin{aligned} B_{P_1} &= P_{1k}^j - \mathbf{r}_k^j - \dot{\mathbf{r}}_k^j + c \cdot \Delta t^j = [c \cdot \Delta T_k + Tr_k^j] + I_k^j + [E_k^j + c \cdot \mathbf{d} \mathbf{x}^j + M_{P_{1k}^j} + \mathbf{d}_{P_{1k}^j}] \\ B_{P_2} &= P_{2k}^j - \mathbf{r}_k^j - \dot{\mathbf{r}}_k^j + c \cdot \Delta t^j = [c \cdot \Delta T_k + Tr_k^j] + \mathbf{n} I_k^j + [E_k^j + c \cdot \mathbf{d} \mathbf{x}^j + M_{P_{2k}^j} + \mathbf{d}_{P_{2k}^j}] \\ B_{L1} &= \Phi_{1k}^j - \mathbf{r}_k^j - \dot{\mathbf{r}}_k^j + c \cdot \Delta t^j = [c \cdot \Delta T_k + Tr_k^j] - I_k^j + \mathbf{I}_1 N_{1k}^j + [E_k^j + c \cdot \mathbf{d} \mathbf{x}^j + M_{L1k}^j + \mathbf{d}_{L1k}^j] \\ B_{L2} &= \Phi_{2k}^j - \mathbf{r}_k^j - \dot{\mathbf{r}}_k^j + c \cdot \Delta t^j = [c \cdot \Delta T_k + Tr_k^j] - \mathbf{n} I_k^j + \mathbf{I}_2 N_{2k}^j + [E_k^j + c \cdot \mathbf{d} \mathbf{x}^j + M_{L2k}^j + \mathbf{d}_{L2k}^j] \end{aligned} \quad (2)$$

In these equations, the ephemeris error can be negligible by using appropriate ephemeris, such as ultra rapid IGS available in real time, or can be estimated by modelling over large networks. In

ultra rapid IGS ephemeris the satellite clock error is available with  $\sim 5$  ns = 1.5 m RMS, and the geometric error as  $\sim 10$  cm RMS. The terms in these equations are ordered as geometric biases (receiver clock error and tropospheric delay), frequency dependant biases (ionospheric delay and phase ambiguities) and negligible biases (ephemeris errors, multipath and hardware delays). The geometric terms are separable by modelling the clock offset and drift, and the zenithal tropospheric delay; their estimation may be conditioned by model uncertainties. Moreover, the negligible terms are mainly absorbed by the residuals, however they affect the parameter estimation. Simplifying, we can write:

$$\begin{aligned} B_{P_1} &= [c \cdot \Delta T_k + Tr_k^j] + I_k^j \\ B_{P_2} &= [c \cdot \Delta T_k + Tr_k^j] + \mathbf{n} I_k^j \\ B_{L1} &= [c \cdot \Delta T_k + Tr_k^j] - I_k^j + \mathbf{I}_1 N_{1k}^j \\ B_{L2} &= [c \cdot \Delta T_k + Tr_k^j] - \mathbf{n} I_k^j + \mathbf{I}_2 N_{2k}^j \end{aligned} \quad (3)$$

By using a good oscillator, such as a rubidium clock or better, the clock error can be modelled linearly with

$$c \cdot \Delta T_k = a_0 + a_1 (t_n - t_0) \quad (4)$$

where  $a_0$  is the clock offset and  $a_1$  the clock drift. The zenithal tropospheric delay can be estimated introducing a mapping function  $m$ :

$$\begin{aligned} B_{P_1} &= [c \cdot (a_0 + a_1 (t_n - t_0)) + m(e_k^j) \cdot Tr_k^j] + I_k^j \\ B_{P_2} &= [c \cdot (a_0 + a_1 (t_n - t_0)) + m(e_k^j) \cdot Tr_k^j] + \mathbf{n} I_k^j \\ B_{L1} &= [c \cdot (a_0 + a_1 (t_n - t_0)) + m(e_k^j) \cdot Tr_k^j] - I_k^j + \mathbf{I}_1 N_{1k}^j \\ B_{L2} &= [c \cdot (a_0 + a_1 (t_n - t_0)) + m(e_k^j) \cdot Tr_k^j] - \mathbf{n} I_k^j + \mathbf{I}_2 N_{2k}^j \end{aligned} \quad (5)$$

The state vector and the design matrix will be:

$$\mathbf{x}_k = [a_0, a_1, Tr_k^j | I_k^j, \dots | \mathbf{I}_1 N_{1k}^j, \dots | \mathbf{I}_2 N_{2k}^j, \dots] \quad (6)$$

$$\mathbf{A} = \begin{array}{c} \begin{array}{ccc|ccc} 1 & \mathbf{d} & t & m(e_k^j) & 1 & \ddots & & & 0 & & \\ & & & & & & 1 & & & & \\ \hline 1 & \mathbf{d} & t & m(e_k^j) & \mathbf{n} & \ddots & & & 0 & & \\ & & & & & & \mathbf{n} & & & & \\ \hline 1 & \mathbf{d} & t & m(e_k^j) & -1 & \ddots & & & 1 & \ddots & \\ & & & & & & & -1 & & & 1 \\ \hline 1 & \mathbf{d} & t & m(e_k^j) & -\mathbf{n} & \ddots & & & 1 & \ddots & \\ & & & & & & & & & & -\mathbf{n} & & & & 1 \end{array} \end{array} \begin{array}{l} \leftarrow P1 \\ \leftarrow P2 \\ \leftarrow L1 \\ \leftarrow L2 \end{array} \quad (7)$$

The total number of parameters to be estimated is:

$$p = 3 + 3 \cdot sat \quad (8)$$

that are 21 parameters with 6 tracked satellites.

#### 3.1 Weighting observations

The Signal to Noise Ratio (SNR) is a function of antenna gain pattern and of atmospheric diffraction; it decrease with the

satellite elevation, and the observation standard deviation can be modelled as an elevation dependant function. Many authors have proposed an exponential function as:

$$\mathbf{s} = \mathbf{s}_0 + \mathbf{s}_1 \exp(-e/e_0) \quad (9)$$

where  $e$  is the satellite elevations in units coherent with  $e_0$ . The function shape is given by the parameter  $\mathbf{s}_0$ ,  $\mathbf{s}_1$  and  $e_0$ , that depend on the observation type, the receiver and the antenna type. The approximate zenithal standard deviation is  $\mathbf{s}_0$  (if  $e_0 \ll 90^\circ$ ) and the horizontal standard deviation is  $\mathbf{s}_0 + \mathbf{s}_1$ . By using the same values of  $\mathbf{s}_0$ ,  $\mathbf{s}_1$  and  $e_0$  estimating the  $\mathbf{s}$  for every observation, we obtain

$$\mathbf{s}(e) = 0.08 + 4.5 \cdot \exp(-e/10) \quad (10)$$

where the numerical parameters depend mainly on the antenna type. Observation weights are given by

$$w_{obs} = \frac{0.2}{\mathbf{s}(e) \cdot \mathbf{s}_{obs}} \quad (11)$$

where  $w_{obs}$  is an a priori variance. Observation weighting leads to other advantages, mainly to reduce multipath effects and mapping function errors, more evident for low elevations.

### 3.2 Pseudo Range Correction estimation

The *Pseudo Range Correction* (PRC) value can be computed with the estimated biases, and then broadcasted with the RTCM signal to the rover user. The PRC parameter must include the atmospheric delays and corrections to the observations derived from other sources of error; the PRC is the sum of:

$m(e_k^j) \cdot Tr_k$	tropospheric delay correction
$I_k^j$	ionospheric delay correction
$\mathbf{r}_{k,B}^j - \mathbf{r}_{k,P}^j$	radial orbit error correction, or radial difference between broadcast and predicted ranges, referred to the satellite antenna reference point
$c(\Delta t_B^j - \Delta t_P^j)$	satellite clock error correction
$\mathbf{r}_{k,dar1}^j - \mathbf{r}_{k,dar2}^j$	radial datum difference, between user and satellite datum; includes reference antenna phase centre variations

The application of the network corrections on the rover side, leads to cancel satellite dependant errors, but residual atmospheric biases are not negligible for long baselines. Residual bias also can be estimated and recovered by parameterisation in the state space domain.

## 4. STATE SPACE ESTIMATION

State model will be defined in terms of state vector  $x$ , transition matrix  $F$  and covariance matrix of the system noise  $C_{ek}$ . In the Kalman filter formulation, the prediction is given by:

$$\mathbf{x}_k = F_{k-1} \cdot \mathbf{x}_{k-1} + \boldsymbol{\varepsilon}_k \quad (12)$$

$$\mathbf{Q}_{k|k-1} = F \mathbf{Q}_{k-1|k-1} F^T + C_{ek} \quad (13)$$

where  $\mathbf{Q}_k$  is the variance-covariance matrix of the state vector;  $\boldsymbol{\varepsilon}_k$  is the system noise and  $C_{ek}$  its a priori variance matrix. Then, introducing the observation vector  $b_k$  and the design matrix  $A_k$ , the correction is given by using the gain matrix

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k (b_k - A_k \hat{\mathbf{x}}_{k|k-1}) \quad (14)$$

$$\hat{\mathbf{Q}}_k = (F_k \hat{\mathbf{Q}}_{k-1} F_k^T + C_{ek}) - K_k A_k (F_k \hat{\mathbf{Q}}_{k-1} F_k^T + C_{ek}) \quad (15)$$

where  $K_k$  is the Kalman gain matrix defined by

$$K_k = (F_k \hat{\mathbf{Q}}_{k-1} F_k^T + C_{ek}) \cdot A_k^T \cdot [A_k (F_k \hat{\mathbf{Q}}_{k-1} F_k^T + C_{ek}) A_k^T + C_{ek}]^{-1} \quad (16)$$

The recursive solution is locally optimal. Several strategies have been proposed in the state space estimation:

KF	standard Kalman filter
KFA	augmented Kalman filter
AKF	adaptive Kalman filter to estimate system parameters
2SKF	two stage Kalman filter to estimate constant biases

### 4.1 Adaptive Kalman filter

State estimation given by Kalman filtering depends on system and observation noise variance, usually defined a priori and as constant. Adaptive algorithms are designed to refine this a priori knowledge, analysing the stochastic properties of the residuals, following the dynamic variations in the system stochastic model (Mohamed and Schwarz, 1999). Adaptive filtering leads to faster convergence and produce self-calibrating covariance matrices. Introducing the observation noise  $e_k$  and its a priori variance matrix  $C_{ek}$ , the vector of predicted residuals and its covariance matrix are given by

$$\bar{v} = b_k - A_k \cdot x_{k|k-1} \quad (17)$$

$$V_k = \text{Var}(\bar{v}_k) = A_k \mathbf{Q}_{k|k-1} A_k^T + C_{ek} \quad (18)$$

In adaptive filtering, the a priori knowledge of the noise variance matrix can be refined by using the estimated covariance matrix of the predicted residuals vector, given by

$$\hat{V}_k = \frac{1}{k-1} \sum_{i=1}^k (v_i - \bar{v}_i)(v_i - \bar{v}_i)^T \quad (19)$$

$$\bar{v}_i = \frac{1}{i} \sum_{j=1}^i v_j \quad (20)$$

that can be recursively updated with

$$\hat{V}_k = \hat{V}_{k-1} \cdot \frac{k-1}{k} + \frac{(v_k - \bar{v}_k)(v_k - \bar{v}_k)^T}{k} \quad (21)$$

$$\bar{v}_k = \bar{v}_{k-1} \cdot \frac{k-1}{k} + \frac{v_k}{k} \quad (22)$$

Finally, the noise variance matrix can be estimated with

$$\hat{C}_{ek} = \hat{V}_k - A_k \mathbf{Q}_{k|k-1} A_k^T \quad (23)$$

substituting the a priori  $C_{ek}$ .

## 4.2 Two stage Kalman filter

The state equation and the observation equation can be modified for the presence of a constant bias vector  $b_k$

$$x_k = F_k x_{k-1} + B_k b_{k-1} + e_{k-1} \quad (24)$$

$$y_k = A_k x_k + C_k b_k + h_k \quad (25)$$

A new state vector can be defined as

$$z_k = \begin{bmatrix} x_k \\ b_k \end{bmatrix} \begin{matrix} \Downarrow n \\ \Downarrow r \end{matrix} \quad (26)$$

so the state equation and the observation equation become

$$z_k = Z_k z_{k-1} + G x_k \quad (27)$$

$$y_k = L_k z_k + h_k \quad (28)$$

where

$$Z_k = \begin{bmatrix} \begin{matrix} \underline{n} & | & \underline{r} \\ F_k & | & B_k \\ \hline 0 & | & I \end{matrix} \begin{matrix} \Downarrow n \\ \Downarrow r \end{matrix} \end{bmatrix} \quad (29)$$

$$G = \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{matrix} \Downarrow n \\ \Downarrow r \end{matrix} \quad (30)$$

$$L_k = \begin{bmatrix} \underline{n} & | & \underline{r} \\ A_k & | & C_k \end{bmatrix} \Downarrow s \quad (31)$$

The observation equation also can be transformed. The optimal estimator and its covariance matrix are given by

$$\hat{z}_k = Z_k \hat{z}_{k-1} + K_k (y_k - L_k Z_k \hat{z}_{k-1}) \quad (32)$$

$$Q_{k|k} = Z_k [I - K_k L_k] Q_{k|k-1} Z_k^T + G C_{e,k} G^T \quad (33)$$

The variance equation can be modified partitioning the variance-covariance matrix

$$Q_k = \begin{bmatrix} \underline{n} & | & \underline{r} \\ \hline Q_{x,k} & | & Q_{xb,k} \\ Q_{xb,k}^T & | & Q_{b,k} \end{bmatrix} \begin{matrix} \Downarrow n \\ \Downarrow r \end{matrix} \quad (34)$$

This leads to decouple the estimation of the unbiased state vector and the bias vector, involving smaller matrices. Moreover, in many cases the bias vector can be estimated at lower frequency than the state vector, and this can be applied in GPS biases estimation to avoid excessive processor loading. Carrier phase ambiguities represent a special case of constant bias in GPS signal, but two stage Kalman filter can also handle slow varying biases and noisy biases, although this implies more complex equations. Discontinuous biases, such as carrier phase ambiguities that are discontinued by cycle slips, can be handled integrating quality control procedures in the two stage Kalman filter, as will be underlined in the following.

## 4.3 Quality control

The main error sources in GPS observations are clock jumps in the receiver clock, cycle slips, outlier and quasi random errors

(mainly multipath, diffraction, ionospheric scintillation). Real time state space estimation requires robust quality control procedures, to be applied both to observation space and to parameter space, testing the correctness of the a priori stochastic model. The appropriate test statistic can be formulated in terms of predicted residuals, that have been already defined by the equations (17) and (18). To handle the various alternative hypothesis we make use of three steps, as suggested by (Teunissen, 1998): detection, identification and adaptation.

- **Detection:** a global model test is performed on the whole observation set at a given epoch. In case of global model test failures, the identification step is performed.
- **Identification:** is used to identify the potential error source.
- **Adaptation:** after identification is possible to cancel the detected and identified bias, correcting its effects in the state estimation.

The DIA procedure can be designed for batch and for recursive solutions too. The recursive (epoch by epoch) form can be integrated with recursive estimators as Kalman filter. To test a null hypothesis against an alternative hypothesis, the detection test value with  $\chi^2$  distribution is given by

$$T_D = \tilde{V}_k^T Q_{\tilde{V}|k,k}^{-1} \tilde{V}_k \quad (35)$$

and depends on predicted residuals and their covariance matrix. In local identification the test value is

$$t_I = \frac{(s Q_{\tilde{V}|k,k}^{-1} \tilde{V})^2}{(s^T Q_{\tilde{V}|k,k}^{-1} s)} \quad (36)$$

where  $s = (0, 0, \dots, 1, \dots, 0)$  is a flag vector used to identify the observation to test. The Minimum Detectable Bias is given by

$$MDB_i = \frac{ds_{b_i}}{\sqrt{r_{i|k}}} \quad (37)$$

where  $r_{i|k}$  is the redundance, or the trace of the redundance matrix

$$R_k = Q_{V|k,k} Q_{B|k,k}^{-1} \quad (38)$$

After identification, the detected bias is compared with the MDB value, that is a threshold value used to identify meaningful biases. If the bias candidate has a value less than the MDB, the observation is accepted. Otherwise the procedure continues with the adaptation step. In adaptation the state vector and its covariance matrix are corrected with

$$\hat{x}_{k,k}^a = \hat{x}_{k,k}^0 - K_k s_k \hat{B}_k \quad (39)$$

$$Q_{x,k,k}^a = Q_{x,k,k}^0 + K_k s_k \mathbf{S}_{\hat{B}_k}^2 s_k^T K_k^T \quad (40)$$

where  $\hat{B}_k$  is the least square estimated bias vector and  $\mathbf{S}_{\hat{B}_k}^2$  its variance, given by

$$\hat{B}_k = \frac{sQ_{\tilde{V}|k,k}^{-1} \tilde{V}}{s^T Q_{\tilde{V}|k,k}^{-1} s} \quad (41)$$

$$s_{\hat{B}_k}^2 = \frac{1}{s^T Q_{\tilde{V}|k,k}^{-1} s} \quad (42)$$

The DIA procedure can be integrated in two stage Kalman filter, to estimate and correct cycle slips, as in (Lu, 1991).

## 5. THE SOFTWARE ALARIS

Several functions described in this paper and others are implemented in the software ALARIS, mainly:

- Reference station:
  - Raw biases estimation
  - One way biases estimation, clock offset and drift, atmospheric delays
  - PRC and RRC estimation
  - VRS generation
- Rover station:
  - Static positioning
  - Kinematic positioning
  - Single or double frequency data processing
  - Combined or non combined observation models
  - Code observation processing for RT DGPS
  - Residual bias estimation and recovering
- Real time quality control (DIA procedure) as described in (Teunissen 1998, Tiberius 1998).
- Clock jump and cycle slip correction
- On The Fly ambiguity fixing to the nearest integer
- Satellite tracking with broadcast or precise ephemerides

The software ALARIS is developed in FORTRAN90, and is intended to perform kinematic positioning using a Multi Reference Station approach. State space estimation is based on Kalman filtering approach, by using standard, adaptive or two stage Kalman filter algorithms.

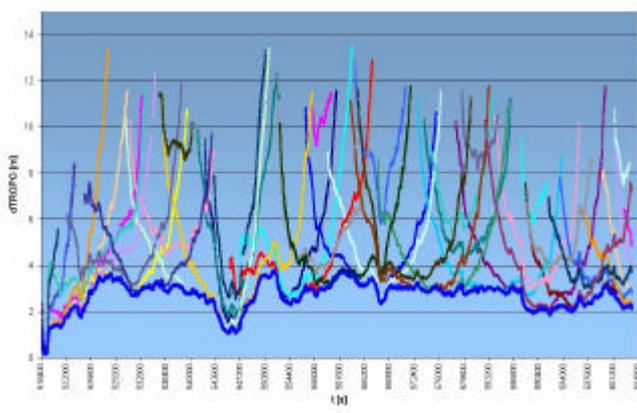


Figure 1. Tropospheric delay estimated over 24 hours observations, collected by TORI reference station.

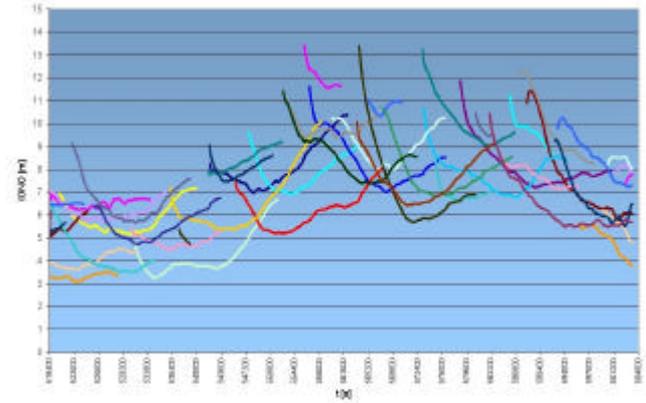


Figure 2. Ionospheric delay estimated over 24 hours observations, collected by TORI reference station.

## 6. CONCLUSIONS

The proposed algorithms are implemented in the software ALARIS, that includes functions for real time applications or data post processing, data quality control, bias estimation, for static or kinematic positioning, differential or absolute point positioning, in single or double frequency. Real time bias estimation as been deeply investigated, mainly for atmospheric biases reduction, in a multi station environment. The same state space estimation approach has been used in the implemented positioning algorithms, making possible the residual biases estimation and recovering.

Undifferenced techniques are sensitive to the presence of errors in the data, so is necessary to set up robust and noise adaptive quality control procedures. The effects of outliers, cycle slips and clock jump are been taken in account; moreover the local detection test value can be used also as model correctness test. These algorithms are then suitable to give good results in permanent stations integrity monitoring and also in post processing applications. The estimated biases have been used to build an RTCM-like message. RTCM 2.x complete compatibility is now under development.

The proposed methodology for biases estimation has at the moment practical application problems, due to the necessity of good ephemeris in real time and of a good oscillator to supply an external frequency input to the reference station receiver. The real time implementation has required the solution of problems related to data transfer between the hardware components, slave, master and rover stations, not yet completely concluded. The real time estimation procedures have been optimized for memory and processor requirements and for numerical stability.

The undifferenced approach take advantages from the improvements in continental or global network real time products, such as ephemeris or atmospheric models. On the other side, can contribute to this improvement, that will be faster with the upcoming modernized GNSS, leading also to better performances in SBAS (Satellite Based Augmentation Systems) and, as consequence, to better performances in real time also with low cost equipments.

Finally, the real time GPS bias estimation can be useful in weather and space weather forecasting, and in time transfer applications.

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