QUALITY MEASURES FOR DIGITAL TERRAIN MODELS

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COMMISSION II, WG II/2

KEY WORDS: DEM/DTM, Quality, Laserscanning, LIDAR, Photogrammetry

ABSTRACT:

In the last few years a lot of new possibilities for the acquisition of terrain data were developed. Next to new developments in the area of digital photogrammetry due to automated image matching techniques, laserscanning has revolutionized the capturing of topographic data. For digital terrain models (DTMs) derived from "traditional" photogrammetric techniques accuracy measures, which were derived from theoretical but also from practical studies, are in use. With the upcoming of the new technologies the question of quality "How accurate is the DTM?" has to be studied new. This paper gives several solutions for the computation of different quality parameters. On the basis of these measures two different methods are presented. First, the empirical stochastic approach, which is mainly dependent on the point density of the data set, the root mean square error (RMS) and the local curvature of the DTM, is introduced step-by-step. Afterwards, a geometrically based approach is shown. It allows the computation of the DTM accuracy based on the local curvature of the DTM and the distance from each grid point of the DTM to the original terrain point next to it. The last quality measure helps to determine areas with unreliable surface description. The presented theories are independent from the data source, the modelling process and the software. For the verification of the theory the quality measures were practically tested with a typical airborne laserscanner data set. Additionally, the usability of the developed method was checked with photogrammetric data. A final section gives a short summary and a short outlook on future research work.

1. INTRODUCTION

The idea of digital terrain models (DTMs) has been proposed nearly 50 years ago by C. Miller of the Massachusetts Institute of Technology, Boston, USA. In this realm, the decades to follow have been characterized by searching for technologies of (photogrammetric) data acquisition, and by developing adequate software. Currently, DTMs constitute a fundamental data base for geographical information systems (GISs). In recent years airborne laserscanning (ALS), a new data acquisition technique, gained special importance in digital terrain modeling.

The importance and responsibility of DTM applications makes it inevitable to provide DTMs with adequate quality measures. Practical rules of thumb are nowadays available in a more or less adequate and tested form (see, e.g., Kraus, 2004). Furthermore, provided that the model has been computed by software applying least squares algorithms, rules for error propagation based on variances and co-variances can be applied to estimate the accuracy of the points as interpolated from the original terrain data. Applying this method has however the disadvantage for users of applying a kind of a black box: the user obtains detailed accuracy measures without any information on individual factors of influence.

However, there is very little literature on deriving detailed accuracy measures for DTMs in a post-processing phase, i.e. at the stage of their application. Below, methods for deriving a composite accuracy measure gradually, step-by-step will be presented. All these steps and aspects are readily comprehendible and well suited for individual visualization. They can be applied to any DTMs existing beforehand, independent of the methods applied in creating them. In the description of the method we will make use of a DTM with

hybrid grid data structure (i.e. a grid augmented by meshed breaklines) for deriving the quality measures.

2. THE EMPIRICAL STOCHASTIC APPROACH

It is assumed that the set of original data, as applied for computing the DTM, is still available, with any blunders in data acquired by photogrammetry eliminated; and in data from ALS, all points not belonging to the terrain surface – i.e. all points on trees or buildings – filtered out of the set.

In this approach, the height accuracy of the original set of data will be estimated empirically, based on the set itself. Providing an overall rough estimate of the accuracy of this set, $\sigma_{a\text{-priori}}$, is also useful.

2.1 Density $\bar{n}$ of the Original Set of Data

Terrain point density in the original set of data represents a parameter fundamentally influencing the accuracy of DTMs derived of them. We derive this density $\bar{n}$ in overlaying the DTM with an analyzing grid; in counting the number of the acquired terrain points in each cell of the analyzing grid; and finally in dividing each of these numbers by the area of the analyzing grid cell. Figure 1 displays the density $\bar{n}$ of terrain points acquired by laserscanning; of these, a DTM has been derived. In the area shown, the maximum point density is 4.64 points/m². The high density of points is in the overlapping areas of the ALS strips. In the areas shown white there are no points: they have been removed by the program package SCOP as not belonging to the terrain surface, e.g. positioned on trees or buildings (compare Figure 9). (Literature to this method of filtering: Kraus, Pfeifer, 1998; information concerning SCOP: http://www.inpho.de,

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http://www.ipf.tuwien.ac.at/products/produktinfo/scop/scop_dt m_sheet.htm).
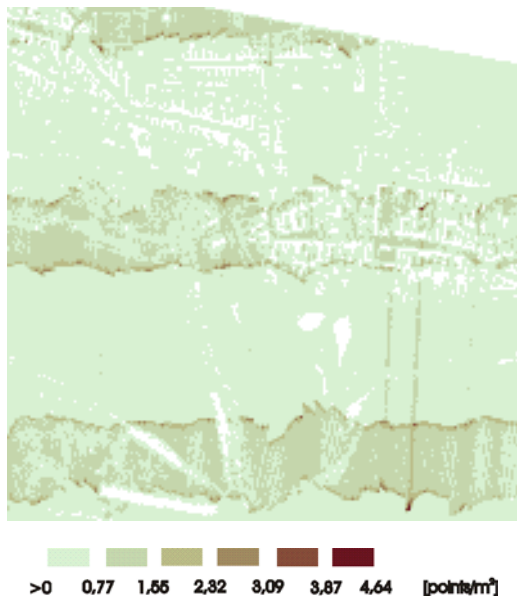

Figure 1: Density $\overline{n}$ of the original set of terrain points serving to derive the ALS-DTM (width of the analyzing grid: 5m)

2.2 Root Mean Square Error (RMS)

At the position XY of individual points of the original data set ("terrain points"), the elevation as measured is compared to the corresponding elevation of the DTM surface. Of these discrepancies, per cell of the analyzing grid, RMS (root mean square) values will be computed. It represents the accuracy of the *original* elevation values as measured by data acquisition – provided that a suitable analyzing grid size is chosen with regard to terrain surface characteristics and point density. This accuracy is varying depending on the quality of data acquisition. In case the RMS thus computed in some cell of the analyzing grid is smaller than the estimate $\sigma_{a\text{-}priori}$ as mentioned, it will be replaced by the estimate. Cases of such replacements occur in cells with few terrain points in them and/or if the algorithm applied in interpolating the DTM is without filtering of random errors (e.g. a triangulation of the given point set).

Figure 2 visualizes the RMS of the terrain points applied to compute a laser DTM with the program package SCOP. The maximum RMS value is 52cm; the quadratic mean of the RMS values equals 5.1cm. In 64% of the cells the RMS has been replaced by $\sigma_{a\text{-}priori}$ estimated as 5cm. The area shown in Figure 2 and in the figures to follow is identical with the area represented in Figure 1. In cells of the analyzing grid without original terrain points no RMS is indicated; these cells are marked black.
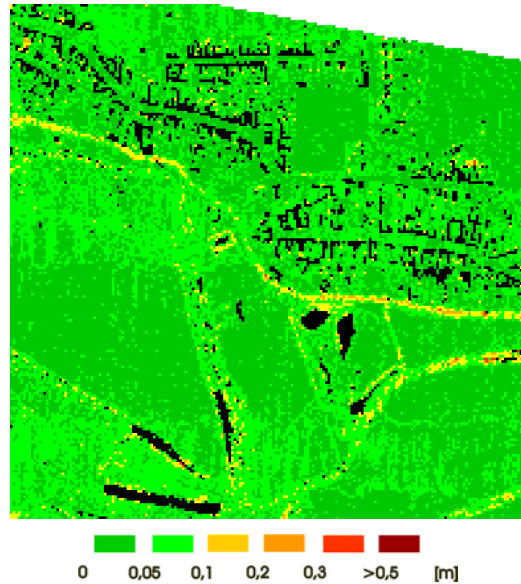

Figure 2: RMS of terrain points in relation to an ALS-DTM, black: no data available (width of the analyzing grid: 5m)

2.3 Curvature 1/r

DTM quality is highly influenced by local terrain curvature. Therefore, it is computed for all points of the DTM grid. The algorithm applied is based on differential geometry (Appendix A at the end of this paper), and it yields the maximum value of curvature; figure 3 visualizes these values. They vary between $-0.61m^{-1}$ and $0.78m^{-1}$. The curvature overlay thus computed yields a detailed visualization of the geomorphology of the terrain (Figure 9 shows the corresponding orthophoto). In agriculturally fields, curvature is very low; it is considerably larger in residential areas, and also in woods. Extreme curvature values are typical along break lines.
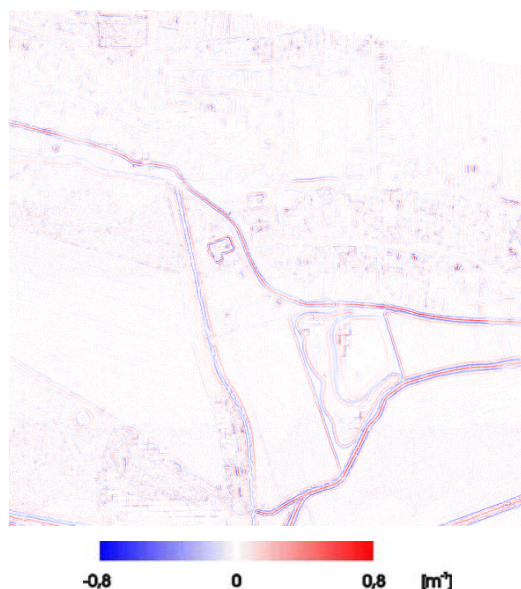

Figure 3: Curvature values 1/r in an ALS-DTM

2.4 Accuracy Measure $\sigma_{z,DTM}$ of each DTM Grid point

For estimating elevation accuracy in every DTM grid point it is adequate to apply any simple interpolation method[1]. Our procedure for this purpose is interpolating the elevation of individual grid points based on just those terrain points closely surrounding them; applied is a least squares fitting of a tilted plane. The elevation accuracy measure sought, $\sigma_{z,DTM}$, is computed of the RMS values as introduced in section 2.2, and of the number n of the surrounding terrain points:

$$\sigma_{z,DTM} = \frac{RMS}{\sqrt{n}} \qquad (1)$$

This rule is immediately understandable for horizontal planes, where the interpolation is nothing but computing the arithmetic mean; it is however also true for tilted planes, as shown in the Appendix B at the end of this paper. We have also to answer the question how to find the number n of the surrounding terrain points. The overlay as described in section 2.3 provides the curvature of the interpolated surface in all DTM grid points. With these values, the maximum size of a surrounding area will be computed, so to assure that within it deviations from a tilted plane will not exceed the accuracy of approximation as specified (e.g. Briese, Kraus, 2003). Denoting the size of this surrounding by A (Figure 4), and applying the point density measure $\bar{n}$ (Figure 1), the number n of the surrounding terrain points can be computed as
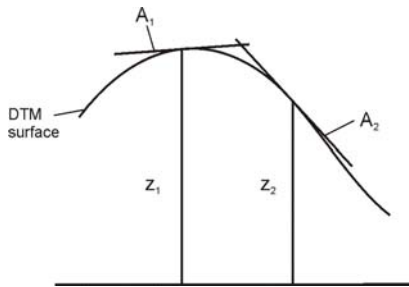
$$n = \bar{n} \cdot A \qquad (2)$$

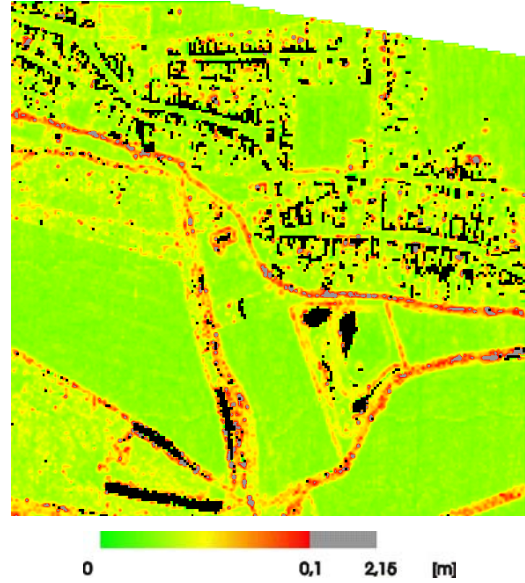Figure 4: Two adjacent DTM grid points with their approximating areas A

Figure 5: Accuracy of an ALS-DTM computed statistically, black: areas without original data

In our data set for testing, with the accuracy of approximation specified as 5cm, areas A vary between 2m^2 and 512m^2. With this area size information A, with density $\bar{n}$ of the terrain points (Figure 1), and with accuracy RMS of the individual terrain points, the accuracy $\sigma_{z,DTM}$ of the individual DTM grid points can be computed applying formulae (1) and (2). Figure 5 shows the results. Thus, these have been found by statistical methods. The values vary between 4mm and 2.15m. In those cells of the analyzing grid not containing any terrain points, the accuracy measures cannot be computed applying formula (2); these cells are marked black.

The "black" areas corresponding to gaps in terrain data are bridged over by DTM interpolation more or less acceptably. However, the endangered reliability of the interpolated surface in such areas without terrain data must be documented. In such areas, the empirical stochastic approach will be replaced by a geometrically based one.

3. THE GEOMETRICALLY BASED APPROACH

In this approach, one of the important parameters is the distance between each grid point of the interpolated DTM grid and the (original) terrain point next to it.

3.1 Distance between each Grid Point of the DTM and the Terrain Point Next to it

These shortest distances d_s can be determined efficiently by applying the Chamfer function from digital image processing (Borgefors, 1986). Figure 6 shows this information for our data set for testing. The maximum shortest distance is 19m. In the areas of the gaps in terrain data these values are particularly large.

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[1] Numerous comparative studies performed earlier have shown that the accuracy of DTMs depends very little on the method of interpolation. Various interpolation methods result, however, in widely differing geomorphologic quality of the interpolated surface and in the quality of the filtering process.
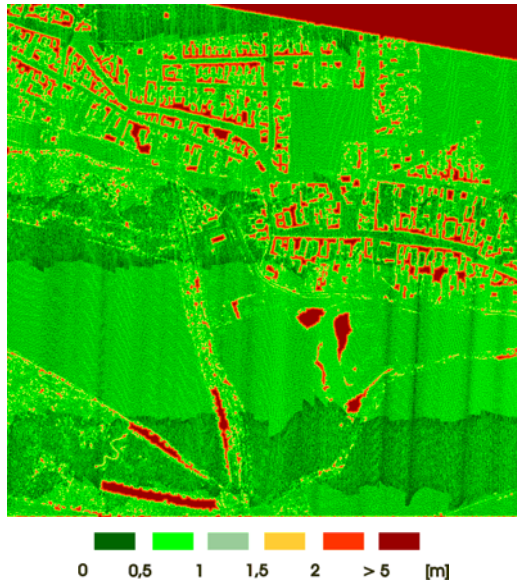
Figure 6: Distances between the points of the DTM grid and the terrain points next to them

Figure 8: Geometric accuracy of an ALS-DTM, white: areas where d_s is bigger than the radius of curvature

3.2 Interpolation Error dZ in Areas without Terrain Points

Figure 7 shows a special constellation occurring in areas without terrain points: the DTM software bridges big gaps between distant terrain points more or less straight. The terrain however shows on the left a curvature 1/r. This corresponds then to an interpolation error dZ in the DTM grid point, with the distance d_s to the next terrain point:
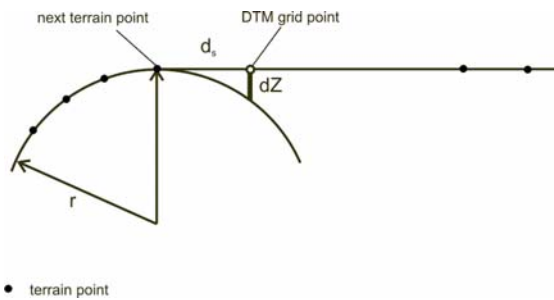
$$dZ \approx \frac{d_s^2}{2r} \qquad (3)$$

Figure 7: Interpolation error dZ in an area without terrain points

Applying formula (3) with information on local curvature (Figure 3), and with the information d_s (Figure 6), one arrives at results as displayed in Figure 8. The interpolation errors dZ vary between –8.52m and 7.12m. In areas of higher point densities, with point distances around 1m, interpolation errors dZ are negligible. In areas without terrain points, i.e. in areas where no statistic accuracy measure could be derived (Figure 5), the dZ values computed here are of great importance. Where the value d_s exceeds the radius r of the curvature, no error value should be displayed; rather, such areas must be marked as unusable (white areas in figure 8).
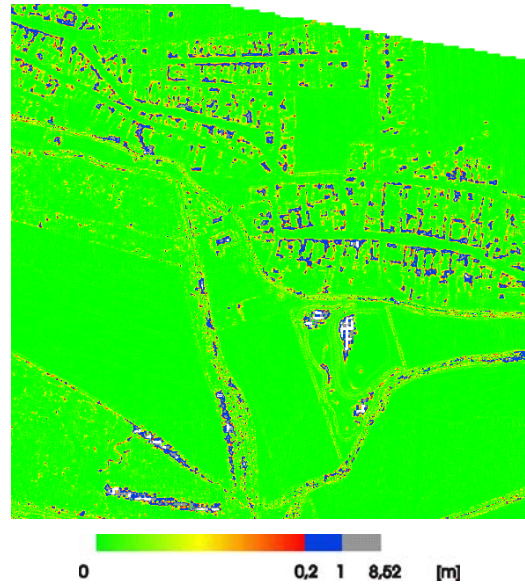
4. DTM FROM LASERSCANNER DATA

For DTMs derived from ALS data, the quality measures as described are of special importance. Airborne laserscanning delivers very dense sets of terrain point measurements – but nonetheless, concerning quality in small areas much is left to chance. There are also large areas without terrain points. Therefore, in creating ALS-DTMs a subsequent analysis of the data and of the DTM derived of them is inevitable.

Figures 1, 2, 3, 5, 6 and 8 are sections of an ALS project in Austria. The aim of the project was to model flood risk areas of the very flat valley Pulkau. The flying height was about 1000m. An Optech laserscanner was used for data acquisition.

Figure 9: Orthophoto

5. DTM FROM PHOTOGRAMMETRIC DATA

In the test area analytic photogrammetric measurements have been carried out, too, and a DTM was derived with SCOP. The aerial images, using a 15cm camera, have an image scale of 1:7500. With the practical rule of thumb (Kraus, 2004) this

leads to an accuracy of ±17cm for the measurement of terrain points.

The photogrammetric DTM was analyzed in the same way as the ALS-DTM. The point density is 0.0016 points/m² in the areas where primarily a raster of points has been measured. Additionally, breaklines were digitized manually, cf. Figure 10. The maximum RMS (cf. section 2.2) of the original elevation values is 70cm; the quadratic mean of the RMS values equals 15cm. The curvature values $1/r$ at the grid points of the DTM are roughly 20% lower than the curvature values of the ALS-DTM (reason: the lower point density of photogrammetry leads to a smoother DTM compared to laserscanning). The accuracy measure $\sigma_{z,DTM}$ is shown in Figure 10. The values range from 2.1cm to 1.9m; their median is 14cm. These values are significantly poorer than the values derived for the ALS-DTM in Section 2.4.
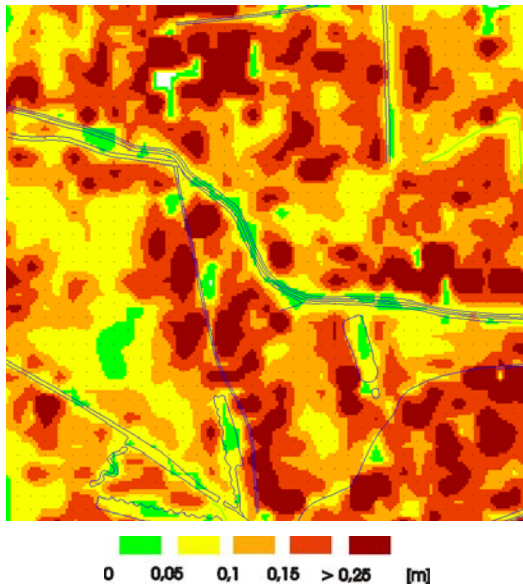

0   0,05   0,1   0,15   > 0,25   [m]

Figure 10: Accuracy $\sigma_{z,DTM}$ of a photogrammetric DTM including digitized breaklines, white: areas without original data

In the case, that the DTM is determined without the manually digitized breaklines and the accuracy is derived, the results shown in Figure 11 are obtained. The deterioration along the breaklines can be recognized clearly. The values vary between 2.5cm and 4.5m; their median is 18cm. Not only the accuracy measure $\sigma_{z,DTM}$ gets worse, but already the RMS worsens from 70cm to 1.7m (maximum value), and from 15cm to 21cm (quadratic mean), respectively. The curvature values for deriving the areas A were taken from the DTM computed with the breaklines.

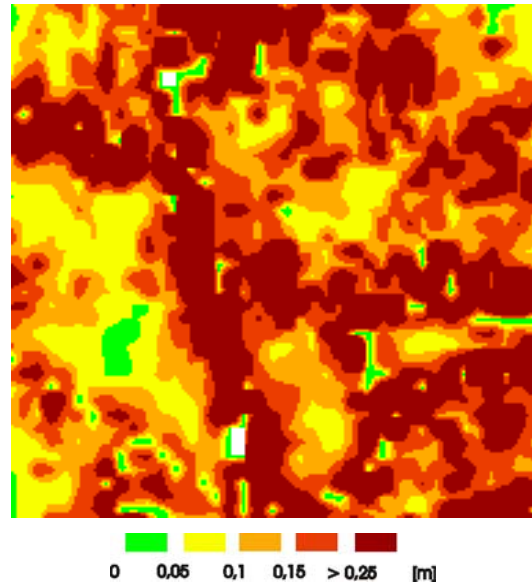0   0,05   0,1   0,15   > 0,25   [m]

Figure 11: Accuracy $\sigma_{z,\,DTM}$ of a photogrammetric DTM without breaklines, white: areas without original data

6. CONCLUDING REMARKS AND OUTLOOK

The aim of this publication was to present a method for the derivation of quality measures – especially accuracy measures – for DTMs. Applying the developed tools, the qualities of DTMs acquired by different methods can be specified and compared. It has to be noted critically, that the obtained quality measures specify in the first instance relative DTM accuracy. Furthermore, detail information not contained in the DTM used for deriving the curvature, does not show up in the presented accuracy measures.

The comparison of ALS-DTMs with photogrammetric DTMs started in Section 5 will be developed further in the coming months and presented in a separate publication (Kraus et al., 2004).

The intention is to provide in future in addition to the DTM data used so far, also the presented quality measures. Concerning this, an implementation in the software SCOP is planned.

The following questions have to be answered based on empirical tests:

- Appropriate size for the analyzing unit. It has to be estimated from the density of the original set of data (Section 2.1).
- Estimation of $\sigma_{a\text{-priori}}$ for different methods of data capturing (Section 2.2).
- Appropriate choice of the accuracy of approximation, which influences the areas A strongly (Section 2.4).
- Using the geometrically based approach, the areas exceeding the curvature radius were classified unusable (white areas in Figure 8). Presumably, the value for the critical distance d_s has to be set to a value significantly smaller than the radius r of the curvature, e.g. 0.5r.

The results obtained so far already demonstrate the capabilities of the method for the determination of quality measures for digital terrain models.

APPENDIX A: DETERMINATION OF THE MAXIMUM CURVATURE

The computation of the maximum value of the curvature at a certain position with the help of a 2.5D hybrid raster/grid surface model (Kraus, 2000) can be performed by differential geometry.

The 2.5D hybrid raster/grid surface can be described as a discrete function $z(x,y)$ which provides height information along the stored vector data and on each raster/grid position. In the first step, based on this hybrid surface description, the first (z_x, z_y) and second derivatives (z_{xx}, z_{xy}, z_{yy}) have to be computed taking into account the surface discontinuity on breaklines. Then the maximum value κ_{max} of the principle curvature can be determined with the help of the coefficients of the first (E, F, G) and second (L, M, N) fundamental form by solving the following quadratic equation:

$$\kappa^2 - \frac{EN - 2FM + GL}{EG - F^2}\kappa + \frac{LN - M^2}{EG - F^2} = 0 \qquad \text{(A-1)}$$

with:

$$E = 1 + z_x{}^2$$
$$F = z_x z_y$$
$$G = 1 + z_y{}^2$$

and

$$k = \frac{1}{\sqrt{1 + z_x{}^2 + z_y{}^2}}$$
$$L = k z_{xx}$$
$$M = k z_{xy}$$
$$N = k z_{yy}$$

APPENDIX B: ELEVATION ACCURACY OF A PLANE SURFACE ELEMENT DERIVED OF 3D POINTS

To fit a plane to 3D points by least squares, the following error equations are used:

$$v_i = a_0 + a_1 X_i + a_2 Y_i - Z_i \qquad \text{(B-1)}$$

X_i, Y_i, Z_i three dimensional coordinates of the reference points

a_0, a_1, a_2 parameters of the plane

v_i residuals (corrections) to occur when the number n of terrain points exceeds 3

Normal equations:

$$\begin{pmatrix} n & [X] & [Y] \\ [X] & [X^2] & [XY] \\ [Y] & [XY] & [Y^2] \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} [Z] \\ [XZ] \\ [YZ] \end{pmatrix} \qquad \mathbf{Nx = n} \qquad \text{(B-2)}$$

This system of equations becomes much simpler when the origin of the XY coordinates is moved to the points' center of gravity. In this case parameter a_0 becomes the elevation in that center; this can be readily seen in analyzing equation (B-1). The accuracy of this elevation is representative for DTM interpolation. To estimate this accuracy, one has to re-write the system N of normal equations (B-2) with the coordinates reduced to the weight center – denoted here by x and y:

$$N = \begin{pmatrix} n & 0 & 0 \\ 0 & [x^2] & [xy] \\ 0 & [xy] & [y^2] \end{pmatrix} \qquad \text{(B-3)}$$

This is an invertible hyper-diagonal matrix; this means, that by inverting the individual matrices N, the corresponding cofactor matrices Q carrying the weight coefficients can be computed. We are interested in inverting the element [1,1] only:

$$\sqrt{q} = \sqrt{\frac{1}{n}} \qquad \text{(B-4)}$$

This result is not surprising: in case when the plane is horizontal, the result of the modeling becomes the arithmetic mean. The accuracy of the arithmetic mean will be computed of the accuracy of the individual observations divided by the square root of their number.

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