

DIGITAL ANALYSIS AND BASIC SHAPE RELIEF EXTRACTION FROM DTM

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ABSTRACT :

Geomorphometry, by its analytical approach, contributes towards understanding landscape shapes and processes that generate them. The automation of relief's feature extraction is in a transition period and has now to lead the users to practical solutions in their systems. This corresponds also to the emergence of a great variety of acquisition modes for DTM, and a fast-growing production of elevation data. The scope of this study aims to provide a practical illustration of geomorphometric descriptive parameters extraction. First, on the basis of simple criteria linked to the digital representation of the altitude, for which a hypothesis of modelling by polynomial regional so-called "translated functions" is introduced, the various slopes and curvatures will be represented analytically. From this base, more complex and pertinent criteria for geomorphometric features will be suggested. The survey will provide an analytical foundation for the description of the relief aimed at all those who study relief evolution processes. From the geographer point of view, this approach confirms past evolutionary processes leading to present landscape shapes and explains the role of these shapes in current and ongoing erosion phenomena by using digital data. The analytical geometric analysis provided and the objects recognized help the settlement of environmental database. Therefore, the DTM finds its place as an operational cornerstone data in the GIS, dedicated to the standard geo-spatial applications, and joins the tools involved by the resource and environmental monitoring.

1. FROM GEOGRAPHY TO GEOMORPHOMETRY

Geography is the scientific study of the distribution of physical, biological and anthropogenic phenomena on the surface of the Earth. Traditionally determinist, recent trends have led to the development of more and more quantitative tools. An optimal analytical method should reconcile these two approaches, quantitative analyses being more adapted when there is a need for greater objectivity and precision in the perception of these phenomena in relation to the relief of the terrain (Depraetere, 1984a & 1984b). Geomorphology is the science which studies the relief of the terrain and the geological formations which shape it (Derrau, 1974; Dewolf, 1982; Joly, 1997...). The geomorphologist describes the relief of the terrain in view of his experience and his specialized discipline : historic, dynamic, structural or climatic geomorphology. This natural subjectivity combined with observations in the field can easily be standardized with digital data, particularly in hydrology. Thus, with the help of modern means of calculation, the Digital Terrain Model (DTM) has taken on a preponderant role

The description of the landscape can be based on various types of classification ; one of them is the parametric approach, which divides and classifies the terrain in terms of particular attributes based on sampling. The DTM is of this type, especially since a careful choice of the grid and its spacing is essential to be consistent with the characteristics and scale of the features to be studied. In the case of geomorphology, this parametric approach

is thus called geomorphometry.

The first part offers a possible mathematical model of the terrain's surface. Parametric indicators will next be defined to determine the relief. The following chapters are largely inspired by the work of H.M. Dufour (1988a & 1988b) and C. Depraetere (1984a), who have both profoundly influenced modern geomorphometry ; their contributions will therefore not be systematically mentioned, but the objective is to present a global synthesis of their works.

2. POLYNOMIAL DESCRIPTION OF A SURFACE

The representative function of altitude must take into account the different discontinuities in the terrain :

- Talus, cliffs directly related to altitude,
- Breaks in slope lines (concave, convex),
- Breaks in curved lines.

Mathematically, linear functions can be used to describe inclined or horizontal planes ; slope and orientation are thus constant and the vertical curves of slope lines or the horizontal curves of contour lines are zero. Quadratic functions produce more varied shapes, which can be used to better describe topographic shapes, although the latter derived remain constant. The surfaces thus modelled will therefore have constant curves, which is doubtless not the usual case. Cubic functions may lend themselves to represent surfaces whose contour lines and slope

profiles are variable.

The method presented here (Dufour, 1988a) is based on the hypothesis that the terrain can be modelled with the help of Taylor's equations (1). It will be limited to the second order here to allow us to describe a sufficiently large number of products. This function, of the $z=H(x,y)$ type has two branches : the first is linear, the second is quadratic.

$$z = H(x, y) = h_0 + ax + by + \frac{1}{2}(cx^2 + 2dxy + ey^2) + \varepsilon \quad (1)$$

where $x=X-X_0$ and $y=Y-Y_0$ are small and $a = \frac{\partial H}{\partial x}$, $b = \frac{\partial H}{\partial y}$, $c = \frac{\partial^2 H}{\partial x^2}$, $d = \frac{\partial^2 H}{\partial x \partial y}$, $e = \frac{\partial^2 H}{\partial y^2}$ are the interpolation coefficients to be determined. The equations to determine the polynom's coefficients depend on the grid chosen (figure 1).

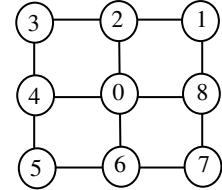


Figure 1 : around a current central position (X_0, Y_0) with an elevation of H_0 , the neighboring points, having altitudes H_1, H_2, \dots, H_8 are numbered as shown

The equations presented can be used and are fully applicable to the vicinity in which the calculations of the altitudes have been based. This method of calculating by « pieces » supplies the numerical values which thus lead to discontinuities ; each grid node in fact generates a surface with distinct coefficients. It is possible to extend the expression by a third cubic term or even others of higher degree. This hypothesis is justified by the fact that it can be used to calculate as many topographic surfaces in this case, as the number of terms increases.

For this square configuration of 3 lines x 3 columns, the coefficients h_0, a, b, c, d and e of the Taylor polynom calculated by least squares in this vicinity (2) (Dupéret, 1989).

$$\begin{aligned} h_0 &= \frac{5H_0}{9} + \frac{2}{9}(H_2 + H_4 + H_6 + H_8) - \frac{1}{9}(H_1 + H_3 + H_5 + H_7) \\ a &= \frac{1}{6}(H_1 + H_7 + H_8 - H_3 - H_4 - H_5), \quad b = \frac{1}{6}(H_1 + H_2 + H_3 - H_5 - H_6 - H_7) \\ c &= \frac{1}{3}(H_8 + H_4) - \frac{2}{3}(H_2 + H_6) + \frac{1}{3}(H_1 + H_3 + H_5 + H_7) - \frac{2}{3}H_0 \\ d &= \frac{1}{4}(H_1 + H_5 - H_3 - H_7) \\ e &= \frac{1}{3}(H_2 + H_6) - \frac{2}{3}(H_8 + H_4) + \frac{1}{3}(H_1 + H_3 + H_5 + H_7) - \frac{2}{3}H_0 \end{aligned} \quad (2)$$

3. THE REPRESENTATION OF BASIC GEOMORPHOMETRIC VARIABLES

The quantitative description of a topographic surface requires taking into account at least 5 basic parameters which are :

altitude, slope, orientation and vertical and horizontal curvatures. Whenever necessary, an analytic expression will be given for each of them based on the terrain surface model presented at the beginning of this publication.

3.1 The altitude

The altitude is a fundamental variable upon which the analysis is derived. It is considered as a function $Z(x, y)$ which presents a dual aspect, simultaneously random and structured. The altitude can be considered as a random variable whose behavior can be studied with familiar concepts. It can be assigned values in the zone under investigation, each of them corresponding to an event whose frequency of occurrence can be used to define a law of probability. Increases are directly associated with effects like a reduction in atmospheric pressure or temperature to which they are linked by a simple linear equation. It contributes towards the complex determination of thresholds between morphogenic systems, the staging of these latter not representing a mere succession of altitudinal strips.

It is practical to represent altitude by shading, which corresponds to a scaled combination of a normal topographic surface vector and a light vector, the calculation capable of being parameterized via a formula (3) linking the shading value to the azimuth Azi_e and the zenith distance zi_e from each of the light sources used.

$$Shad. = \sum_{i=1}^{\# \text{ suns}} k_i(-\text{asin}zi_e \times \sin Azi_e - \text{bsin}zi_e \times \cos Azi_e + \cos zi_e) \quad (3)$$

The most direct representation is the graph in which altitudes are represented in the X-axis while the totals in number of points or links appear in the Y-axis for each altitude value. This representation can be declined by replacing the totals in number of links (or pixels) by the corresponding surface covered (the surface of the DTM's link unit being known). If the criteria for the Y-axis is standardized, dividing by the total number of pixels (or by the total area of the zone), the function represented becomes the altitude probability density function, or more precisely, the probability distribution, since the function is discrete. In the case of a standardized representation in the Y-axis, the study of graph modes can be used to represent the altitudinal spectrum of the zone, in the probabilistic sense of the word.

Generally, and especially when the zone studied presents remarkably terraced surfaces, their characteristics, identifiable in the altitudinal spectral modes, can be quantified. Each one of them has three characteristic level zones (figure 2) corresponding to a terrace : the upper zone presenting the proportion of surface elements being incorporated into the level studied, the actual level and the lower zone being incorporated or transferred into the lower terrace level. This study can only be carried out when the slope of the altitude density function is not too sharp in relation to the zone studied : in this case, the overlap is such that no differentiation is possible from the altitudinal spectrum.

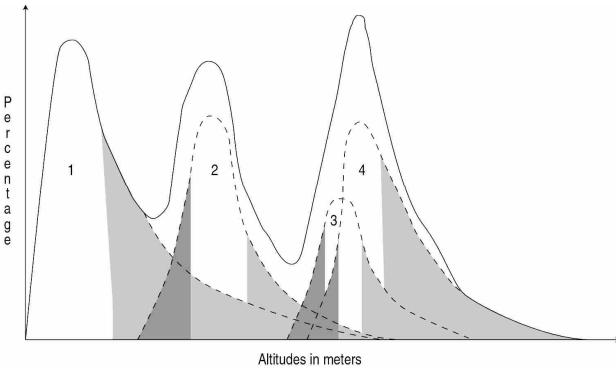


Figure 2 : theoretical case of an altitudinal spectrum representing a system of terraces with several surfaces (Depraetere 1984)

As for any discrete random variable, it is possible to define the distribution function of the altitude from the law of distribution defined above. The accumulated probabilities are copied onto the Y-axis, the altitudes still being represented on the X-axis. Altitude values are usually standardized along the X-axis, as well as an interchange between the two axes ; the curve produced is called an hypsometric curve (figure 3). It is a descriptive statistical figure well adapted to the study of a slope. Its shape provides a standard description of the organization of rock volumes and reflects the evolution of the landscape. The calculation of the area located beneath the curve gives a value called an hypsometric integral. For a given altitudinal magnitude, the potential energy contained in the rock mass in relation to the base level can be estimated. This estimation is important for the evaluation of transport processes. Like pressure and temperature in chemistry, hypsometric curves and integrals can be considered as variables of state.

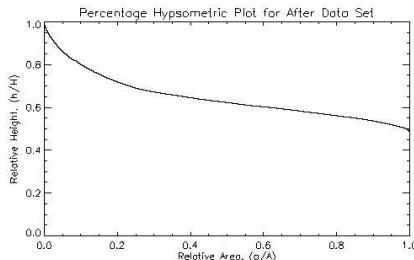


Figure 3 : hypsometric curve (right) of Mount Saint Helens after the eruption. The calculated value of the hypsometric integral indicates a change from 0.58 to 0.64 after the eruption.

A convex curve signifies a young landscape with an important energy potential of rock mass in relation to base level, suggesting tendencies like perching throughout the region, the presence of entrenchment valleys, sharp slopes with probable active transport processes. Inversely, a concave curve signifies the development of surface erosion adjusted to the base level of the zone. The energy potential is transformed into mechanical and kinetic energy leading for example to the erosion of interfluvial zones.

3.2 The slope

The vector is represented by the equation $\vec{P} = \overline{\|grad(Z)\|}$

whose coordinates are thus $\begin{cases} a + cx + dy + \varepsilon' \\ b + dx + ey + \varepsilon'' \end{cases}$ for the

polynomial model proposed. If the norm on the surface of the terrain and the vertical at this site form an angle α , the module of the slope vector ,usually called slope, is equal to the tangent of the angle α which can be considered equal to $\sqrt{a^2 + b^2}$. The slope can be assimilated to a force field, whose exploitation is fundamental in order to model the dynamics of the sloped basin.

As for altitude, certain representations can be fruitful, especially to study dynamic factors of the local geomorphology. By the study of representational modes on the clinographic curve (figure 4) and the equivalent of hypsometric curve slope angles for altitudes, identification of the process functioning threshold is possible. The estimation of lower limit, characteristic and higher limit angles of the modes are potentially associated with the nature of each dominant rock in the zone studied. The superimposition of clinographic curves characteristic of each of these rocks with the general clinographic curve of the zone can be used to estimate the role of each of the rocks in the relay process which exists between them, and therefore, on the general dynamics of the zone; however, taking the morpho-structural context, structural stage of evolution and the vegetation cover into account can make this type of study rather complex.

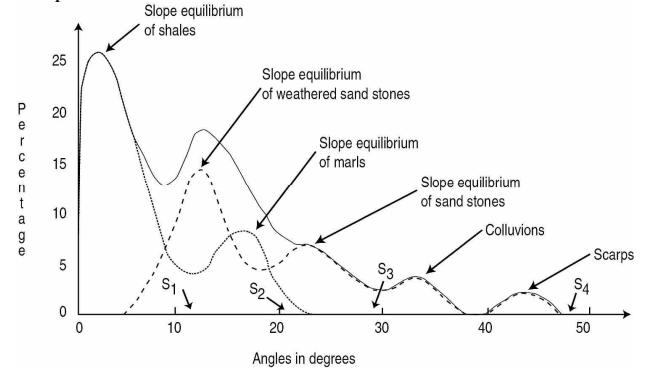


Figure 4 : theoretical case of a clinographic curve in a lithologically contrasted region (Depraetere 1984)

The spectral distribution of slopes can also be calculated, as is the case for orientations. It must be recalled that the quantitative diagnosis made is not necessarily the determining factor in relation to other elements of the landscape such as the type of rocks, the vegetation cover, man-made modifications... The combined study of hypsometric and clinographic curves provides a better understanding of links between the shapes and processes in a given zone. A comparison with studies made on the characteristic elements of the relief is easy.

The fusion between the two indicators of altitude and slope provides effective statistical data on the curvature of the slope faces, by the representation of the clino-hypsometric curve (figure 6) ; altitudes are traditionally represented on the X-axis and slopes on the Y-axis. The cloud of points thus produced makes variable densities typical of curvature in the zone visually appear. The cloud of points reveals the profile of an average slope face indicative of the general layout of curvatures in the zone.

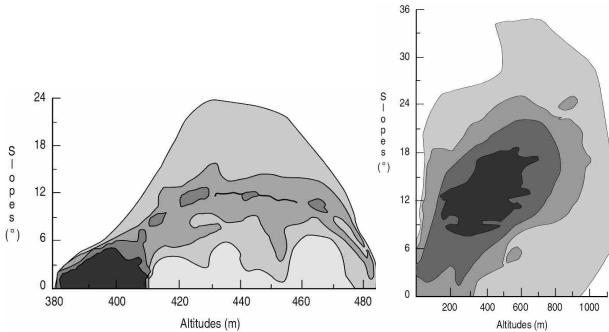


Figure 5 : statistical representation of the relationship between altitudes and slope in two zones located in Oklahoma (left) and the Ferro basin in Italy (right) (Depraetere 1984).

On the graph, zones where the average slope profile is positive helps to identify the distribution of altitudes in concave zones, while convex zones are identified by negative slope sectors of the average sector. The dominant statistical tendencies in the profiles of the slope faces can thus be quantitatively estimated.

3.3 The orientation

The orientation of a slope determines the quantity of solar radiation received on its surface. Associated with the slope, it plays a fundamental role in differentiating the contrasts between the slopes, also subject to the geographical latitude of the site (this latter intervenes through the manner in which the shadows are cast onto the ground). The radiative contrasts generated can vary very quickly and substantially, emphasizing the effects of external processes and influencing the vegetation on ground level. The expression of the orientation with a polynomial model seen previously is given below (4).

$$\begin{cases} \arccos\left(\frac{-a}{\sqrt{a^2 + b^2}}\right) & \text{if } a < 0 \\ -\arccos\left(\frac{-a}{\sqrt{a^2 + b^2}}\right) & \text{if } a > 0 \end{cases} \quad (4)$$

The northern and southern slope faces of a valley are exposed differently ; they are not subjected to the same conditions of sun exposure and therefore of erosion. In temperate climates, regions thus develop differently provoked dissymmetries. The orientation is an indicator that can distinguish between these two slope faces (figure 5).

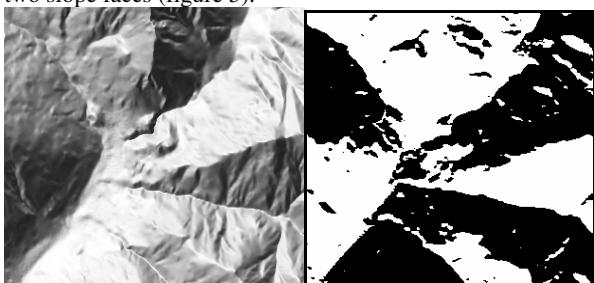


Figure 6 : shaded DMT of Briançon (left), data from IGN's BD TOPO® (step=25m) ; orientation representation thresholded at 90° and 270° (right) , in black, orientation to the north; in white, orientation to the south.

This kind of image can be used to study the fine structure of the surface; the white zones in the middle of the black slope faces (and vice-versa) can easily be seen ; the slope contains the relief oriented differently from its general orientation, these are thus clues to understanding its structure.

3.4 Vertical convexity

The vertical convexity of the ground surface plays a role in the acceleration of transport speed of matter as slope values, whose variations it reflects, increase. In brief, the increase in competence and capacity of superficial runoff in convex zones can thus be described , as well as their slowing down in concave zones, with all the consequences implied with these phenomena such as the deposit of sedimentary loads. This given leads to a preliminary association between preferential zones of terrain erosion and convex zones of the terrain, and those runoffs coming together in concave zones without neglecting the effects of other factors such as the roughness of the ground. The expression of the vertical curvature with a polynomial model seen previously is given below (5).

$$V = \frac{ca^2 + 2abd + eb^2}{(1 + a^2 + b^2)^{3/2}} \quad (5)$$

The sign of the vertical curvature of slope lines reveals the convex or concave nature of the terrain, just like the horizontal curvature line of level lines.

3.5 The horizontal curvature of contour lines

The horizontal convexity of the ground surface is associated with the solid contributions to a zone due to the convergence or divergence of runoff. This curvature of the surface is a characteristic element of the type and evolution of a relief zone and will often be closely associated with zones of thalwegs or ridges. If the vertical curvature can be modelled into two dimensional profiles, it becomes absolutely essential to study surfaces before horizontal curvatures. It takes on very strong and negative values in the thalwegs, remaining weak in the regular slope faces and becoming very strong and positive along ridges. The expression of horizontal curvature of contour lines with a polynomial model seen previously is given below.

$$\Gamma_N = \frac{2abd - cb^2 - ea^2}{(a^2 + b^2)^{3/2}} \quad (6)$$

The calculation of the curvatures of level lines is one approach to the modelling of the thalweg network and the ridge network which are the sites of extreme negative and positive values in the curvature of the level line. The final network is not compact, points not belonging to the network appear after all. It is thus merely an informative result, which is logical given the simplicity of the method used.

3.6 The horizontal curvature of slope lines

The horizontal curvature of the lines of greatest slope can be calculated in the same way as the horizontal curvature of the contour lines. The line $\Gamma_p=0$ is of particular interest as is that of lines $\Gamma_h=0$ because it defines the topographic surface zones

- the left-hand zones where $\Gamma_p<0$, for which an observer who follows a slope line towards the valley constantly moves diagonally towards his right,
- the right-hand zones where $\Gamma_p>0$, for which an observer who follows a slope line towards the valley constantly moves diagonally towards his left.

It can be expressed as (7) :

$$\Gamma_p = \frac{d(a^2 - b^2) + ab(e - c)}{(a^2 + b^2)^{3/2}} \quad (7)$$

These geomorphometric parameters must lead to the description of how the morphogenic systems function even if other factors such as vegetation or soils intervene in the interrelations between shapes and processes. In addition, the establishment of more complex adapted indicators becomes possible to describe the topographic surface.

4. REPRESENTATION OF COMPLEMENTARY GEOMORPHOMETRIC VARIABLES

4.1 Other various curvatures

Several magnitudes indicate the manner in which the topographic surface bends in vertically, but this time independently from the line of the greatest slope. The results obtained with these indicators may even contribute information to a geological study of the area.

- The Laplacien's equation can be expressed as $\Delta H = c + e$,
 - The average quadratic curvature is equal to $C_{mq} = \sqrt{c^2 + 2d^2 + e^2}$,
 - The total curvature is $C_t = d^2 - ce$,
 - Altitude magnitude, sometimes called relief, is the difference between the highest altitude and the lowest altitude in the 3*3 mobile. It is assigned to the central node . This indicator is very close to the slope: it is simply more concrete.
 - In a 3*3 convolution window, the entrenchment is the sum weighted by the distance in levels between a grid link and its eight neighbors ; its expression is
- $$Entrenchment = \sum_{i=1}^8 (H_i - H_0) \times d_i$$

A geomorphological typology can thus be defined within a family of terrain shapes : horizontal plains (without significant relief), inclined slopes, entrenched sites (valleys, depressions, basins), dominant sites (ridges, summits), passes.

4.2 Indicators derived from slopes

The study of this slope value graph reveals significant indicators :

- Minimum, maximum, average and absolute magnitude (max-min) slopes
- Quantiles Q5, Q10,... and relative magnitudes (Q95-Q5, Q90-Q10,...)
- Graph symmetry and histogramm curtosity.

For example, these indicators have been used to compare two types of relief: one of recent erosion ("young" relief in Briançon, the other, older, in the Vosges (figure 7). A study of the function of slope distribution can also be carried out, as was the case for altitude.

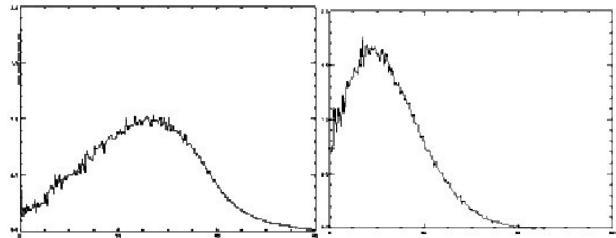


Figure 7 : slope graph : DTM of Briançon (left), data from IGN's BD TOPO® 2003 ; DTM of the Vosges (right), data from IGN's BD ALTI®, 2003 - X-axis scale : from 0° to 60°, Y-axis scale : from 0% to 2% of the number of points in the DTM

4.3 Indicators derived from orientation

- #### 4.3.1 Average slope orientations N / NE / E / SE / S / SW / W / NW :
- As an example, average slope orientations indicate the following : during the ice ages, water froze on the slopes of the relief. The *ubac* (slope face with northern exposure) is not subject to the warmth of the sun and the water remains frozen. Inversely, the *adret* (slope face with southern exposure) is reheated every day by the sun, which leads to a daily freeze/thaw cycle. During the thaw, water flows off; it digs at the slope which increases the inclination. The slopes with northern exposure are thus not as steep as slopes with southern exposure in the countries with temperate climates located in the northern hemisphere. This phenomenon is similar to slopes with western exposure, heated by the afternoon sun, more eroded than the slopes with eastern exposure, heated by the cooler morning sun.

- #### 4.3.2 Orientations spectrum :
- The analysis of the orientations spectrum of a zone is used to assess the manner in which they are distributed. This indicator was tested on a zone in the Vosges (figure 8). The pronounced dissymmetry of a spectrum can be explained by the fact that certain zones of the relief resemble a rooftop : western slope faces are « longer » than eastern slope faces, which explain why there are more points with western exposure than with eastern exposure. The spectrum is thus typical of the particularity of the zone. Furthermore, average SE slope orientations are much less sharp, but the dissymmetry stops there.

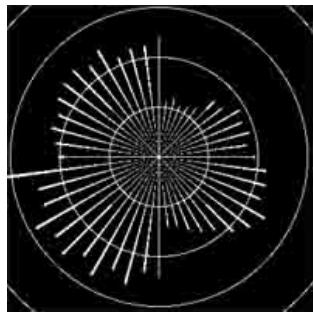


Figure 8 : orientations spectrum for DTM of the Vosges (Data from IGN's BD ALTI®, 2003) ; each circle represents 1% of the total number of points in the DTM, steps of 7.5°.

4.3.3 Orientation modes : It is interesting to know the mode(s) of orientation maps (figure 9) ; a visual study can provide an approximate determination. To complete the study of a graph, let us visualize the distribution function of the indicator (cumulative graph).

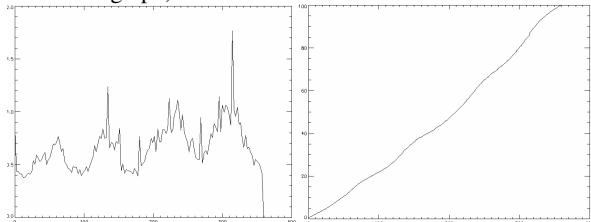


Figure 9 : DTM of Briançon– Orientation graphs (left), data from IGN's BD TOPO® 2003. This function has a very clear profile ; the mode(s) correspond(s) to the maximum number of slope points ; distribution function of the orientations (right)

5. CONCLUSION

There exists a dynamic equilibrium which links the shapes of the terrain to the geomorphological processes which give rise to them ; thus, slopes measured in the field are simultaneously the result of a long chain of previous evolutionary processes as well as the result of actual processes currently in progress. Terrain shapes and evolutionary processes of slopes interact with to such a degree that it is difficult to disassociate them ; The present study is designed to justify the usefulness of geomorphometry as an interdisciplinary field rooted in hydrology, engineering science and geomorphology, from the social sciences.

If the interpretation of terrain shapes cannot be disassociated from vegetation cover, climate and the nature of the ground, others seem more independent of this interface, such as geology and climate. In any case, this interface is associated with a surface that is studied for its own sake for the role it plays. This manner of proceeding requires the identification of irregularities on the basis of reliable indicators, constructed from models satisfactorily representing the surface of the ground.

Geomorphometry does not pretend to understand the shapes of the relief. It can contribute valuable information due to the complex measurements it can provide with the objectivity of a statistical method. Nevertheless, for a measurement to be useful, the reality to be measured must first be defined and then be measurable with an acceptable approximation in relation to the objects studied. This involves sampling theory which is based on Shannon's theorem. The difficulty in geomorphometry

consists in avoiding surplus information by adopting grid links which are too small, susceptible to interpolation static, which may also shrink the number of samples in the zones studied and may impoverish the statistical properties linked to the terrain shapes.

The authors have begun to program numerous indications around the Research Systems Incorporated's Rivertools software. The task undertaken will be prolonged to result in a complete set of geomorphometric indicators which can be used in geographic information systems. They will be added to the data already available in IGN's Large Scale Referential, called RGE, to complete other environmental or thematic indicators and to promote the development of a new geography based on traditional human foundations and numerical indicators such as those presented above.

References and Selected Bibliography :

- Deffontaines B., 1990. Développement d'une méthodologie morphostructurale et morphonéotectonique ; Analyse des surfaces enveloppes, du réseau hydrographique et des modèles numériques de terrain ; Application au Nord Est de la France. Thèse de Géologie structurale et de Télédétection, Université Paris VI, 1991, 194 p.
- Deffontaines B., Chorowicz J., 1991. Principle of drainage basin analysis from multisource data, Application to the structural analysis of the Zaire Basin, *Tectonophysics*, 194 : 237-263.
- Depraetere, C., 1984a. Etudes géomorphométriques comparatives en Afrique du Sud : applications hydrologiques et géomorphologiques, Thèse de 3^{ème} cycle en géographie, Université Paris-Sorbonne, PIV.
- Depraetere, C., 1984b. Exemples d'Analyses Géomorphométriques dans les Appalaches à partir de Modèles Numériques de Terrain, *Physio-Géo*, n°17, pp: 49-76.
- Derrau M., 1974. *Précis de géomorphologie*, Masson et Cie, 6^{ème} Edition, Paris, 453 p., 171 fig., 62 pl. h.t., (chapitre XI: pp.: 117-122).
- Dewolf Y., 1982. Cours de géomorphologie, Univ. Paris VII, 90p.
- Dufour, H. M., et Abgrall, F., 1988a. Eléments Remarquables du Relief : Définitions Numériques Utilisables, *Bulletin CFC*, n° 95.
- Dufour, H. M., 1988b Quelques idées générales concernant l'établissement et l'amélioration des Modèles Numériques de Terrain, *Bulletin. d'information. de l'I.G.N.* 88/1.
- Dupéret, A., Contribution des MNT à la Géomorphométrie, Rapport de DEA SIG, Ecole Nationale des Sciences Géographiques et CNRS, Septembre 1989, 54p.
- Joly F., 1997. *Glossaire de géomorphologie, Base de données sémiologiques pour la cartographie*, Coll. U, Armand. Colin Ed., 325p.