# 3D RECONSTRUCTION OF THE SOLID OF ROTATION LACKING IN TEXTURE


Jianqing Zhang [a, *], Jun Tao [a], Zuxun Zhang [a]

[a] Wuhan University, College of Remote Sensing and Information Engineering,
#129 Luoyu Road, Wuhan, Hubei, P.R.China
Jqzhang@supresoft.com.cn, martintao@etang.com, zxzhang@supresoft.com.cn



**KEY WORDS:** Reconstruction, Three-dimensional, Image, Texture, Camera, Platform, Adjustment, Photogrammetry



**ABSTRACT:**

The paper proposed an approach to reconstruct the solid of rotation lacking in texture from two-dimensional images. The approach only requires a digital camera, an ordinary slide projector and a rotated platform. The camera and projector are required to be calibrated in advance. The rotated platform is functioned as a control ground and the target object is put on it. The projector projects the slide points onto the solid of rotation so that the texture is supplied to the surface of it. The digital camera takes sequential images with these points when the platform rotates continually. For each space feature point projected on the surface of the solid of rotation, there are two corresponding 2D points existing. One is an image point in one of the image sequences and another is a point within the slide points. Gotten the two 2D coordinates of the points, the 3D coordinates of the space points on the surface of the solid of rotation can be calculated by the collinear equations. The sequential images are taken from different directions of the solid of rotation. Using the correspondence of whole adjustment and the inherent structure characteristic of the solid of rotation, the 3D coordinates of all space points projected on the whole surface of the solid of rotation can be computed out entirely. The 3D model of this solid of rotation is acquired by connecting all neighbour space points. This approach is flexible and practical, which are confirmed by the results of the reconstructing experiments.


## 1. INTRODUCTION

### 1.1 3D Reconstruction

3D reconstruction is referred as a process of the recovery of three-dimensional model from the two-dimensional digital camera images. During the image formation process of the camera, explicit 3D information about the scene or objects in the scene is lost. Therefore, 3D model or depth information has to be inferred implicitly from the 2D intensity images. The key of the traditional method lies in the matching of the corresponding features in the images. When there is no feature or lack of feature in the objects, or when the features cannot be matched correctly at all, the main problems appear in the process of the 3D reconstruction

The paper provides a method to resolve the problems above. It is an ordinary slide projector that can supply any feature what you want to the target objects. These features are easy to be controlled and to be extracted out, which has paved the convenient path for the matching of them. According to the demand and real condition, the features can be changed or adjusted.

The paper explains clearly how to use the ordinary slide projector and how to apply it to the applications of the 3D reconstruction, too.

### 1.2 Definition of the Solid of Rotation

In the real world, there exist many things being the solid of rotation or being component of several solids of rotation, such as: Ball, Cup, flowerpot, jug, pencil and cap, even dome. So the solid of rotation is a common thing and it is necessary to research its 3D reconstruction.

General definition of the solid of rotation is that the main body of the target objects can be formed by a curve rotating around a fixed axis.

The paper mainly researches the 3D reconstruction of the main body. Namely, the target object is only a simple solid of rotation and it has no other component with it. This kind of object has an obvious characteristic that each vertical section of the fixed axis is a circle.

### 1.3 Overview of the Paper

In the last few years the interest in three-dimensional object reconstruction from two-dimensional images has dramatically increased in the field of photogrammetry. The main difficulties lie in the texture feature extraction and the model acquisition with time-consuming and expensive process, especially the reconstructed object lacking in texture. An approach is proposed in this paper, which avoids most of the problems mentioned above. The target object needed reconstructing is given emphasis on the simple solid of rotation.

When three positions of the camera, the slide projector and the rotating platform are adjusted to the suitable place, the solid of rotation is put on the centre of the rotating platform. The projector projects a feature slide onto the solid of rotation so that the texture is supplied to the surface of it. According to the different demand and condition, the feature slide can be designed differently. Then the digital camera takes sequential images with projected feature slide when the platform rotates by an angle continually.


[*] jqzhang@supresoft.com.cn; Tel: 86-27-87561116; Fax: 86-27-87561011

For each space feature point projected on the surface of the solid of rotation, there are two corresponding 2D points existing. One is an image point in one of the image sequences and another is a point within the slid. Using the image processing method, the 2D coordinates of the image points of each image can be worked out completely. At the same time, the slide projected is designed first so that the 2D coordinates of the point within the slide are calculated by the known data. Hence, the 3D coordinates of the space points on the surface of the solid of rotation can be calculated by the collinear equations.

The sequential images are taken from different directions of the solid of rotation. Using the correspondence of whole adjustment and the inherent structure characteristic of the solid of rotation, the 3D coordinates of all space points projected on the whole surface of the solid of rotation can be computed out entirely. The 3D model of this solid of rotation is acquired by connecting all neighbour space points. By this time, the 3D reconstruction of the solid of rotation is completed entirely.

## 2. METHODOLOGY AND ALGORITHM

### 2.1 Background

**2.1.1** The calibration of the digital camera
The digital camera needs to be calibrated in advance, because the late computation of the coordinates of the space feature requires the intrinsic and extrinsic parameters of the digital camera. Therefore, the calibration of the digital camera is an important preceding step.

Direct Linear Transformation (DLT) is a well-known method used in close-range photogrammetry because of it's no need for initial value of camera intrinsic and extrinsic parameters. The existed camera calibration techniques are studied thoroughly. The restricting condition among 2D-DLT parameters is worked out using the correspondence of collinear equation and 2D-DLT. The decomposition of initial values of camera intrinsic and extrinsic parameters using 2D-DLT is detailed. Planar-scene camera calibration algorithm with collinear equations is addressed. So the calibration of the digital camera is accomplished entirely and the intrinsic and extrinsic parameters of the digital camera are ready to be used in the future. [Zuxun Zhang, and Yongjun Zhang, 2002.]

If the camera is not an ordinary one but a special digital measuring-camera, the calibration could be cancelled because the intrinsic parameters of the digital camera can be gotten as the known data. However, the extrinsic parameters of the digital camera are still required to calculated out first.

**2.1.2** The calibration of the ordinary projector
In this approach proposed in the paper, the slide projector makes the same function as a digital camera. So the ordinary slide projector also desires to be calibrated in advance. Its intrinsic and extrinsic parameters are also applied to the computation process.

The algorithm with 2D direct linear transformation (2D-DLT) and collinear equations is used to calibrate the projector. The algorithm is addressed systematically and entirely as following. First, the image coordinates of the projector are designed carefully and the space coordinates of the projector are computed by the image data and the intrinsic and extrinsic parameters of the digital camera. Then, the decomposition of initial values of the projector intrinsic and extrinsic parameters using the correspondence of 2D-DLT and collinear equation is deduced. Finally, the projector calibration parameters are worked out by the whole adjustment. By this time, the intrinsic and extrinsic parameters of the ordinary slide projector are ready to be used in the next program. [Jianqing Zhang, Jun Tao, Zuxun Zhang, 2003.]

**2.1.3** The design of the feature slide
In order to adapt to the different cases, the design of the feature slide is convenient to be changed and adjusted. The program is coded to control the formation of the feature slide. For example, there are four design formations shown in the Figure 1.

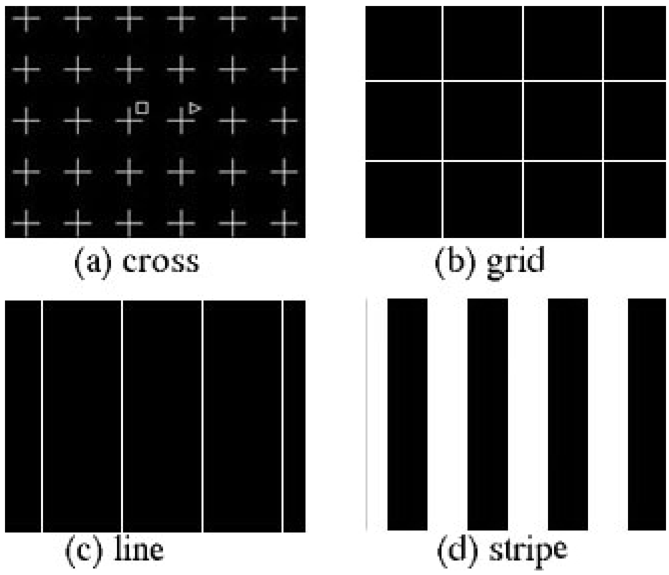

Figure 1. Four designs of the feature slide

Every feature slide is sure to be designed by one regulation which makes it possible and easy to compute out the 2D coordinates of the feature.

**2.1.4** The control of the rotating platform by computer
In order to control conveniently and exactly the rotating angle every time, a computer and a controlling program are used to operate the rotating platform. It can rotate continually by a fixed angle or the same degree of angle. In this way the calculation of degree of angles is simplified and time consumption is reduced remarkably.

When the computer controls the platform rotating and the camera taking photos at the same time, the efficiency of the whole approach can enhance heavily.

### 2.2 Equipment and Method

**2.2.1** The main equipment
A digital camera, an ordinary slide projector and a rotating platform are mainly required in this approach. The rotating platform is functioned as a control ground so that there are many control points on it. By using these points, the digital camera and the slide projector can be calibrated correctly and exactly. At the same time, the space coordinates of the space features are entirely based on the coordinate system of these points. The slide projector mainly provides the texture onto the surface of the solid of rotation, so that the slide design is done carefully in advance. Because the slide projector functions as

another camera, the binocular stereo system is made up of it and the real digital camera. It is pave the smooth path for the calculation of the space coordinates. The digital camera is the main collector of the image data. From the information of these images, the space model is only able to be computed out and be formed up.

**2.2.2** The steps of the method

First, the three positions of the digital camera, the slide projector and the rotating platform are adjusted suitably according to the size of the target solid of rotation. Then the digital camera and the slide projector are focused respectively. So the texture feature projected is shown clearly on the surface of the solid of rotation and the images of the solid of rotation taken are all in focus, too.

Second, the target solid of rotation is put on the centre of the rotating platform. The slide projector projects a feature texture slide onto the solid of rotation with reference to the different condition, such as points, lines or grid. The digital camera takes the sequential images of the solid of rotation with the texture feature from the different orientations when the platform is controlled to rotate by the fixed angle.

Third, for each space feature point projected on the surface of the solid of rotation, there are two corresponding 2D points existing. One is an image point in one of the image sequences and another is a point within the slide. Using the image processing method, the image points of each image can be extracted out completely and the 2D coordinates of them can be computed out correctly. At the same time, the slide projected is designed first so that the 2D coordinates of the points within the slide are gotten by the known data.

Fourth, how to get the homologous points from the slide and the image? Known the intrinsic and extrinsic parameters of the digital camera and the slide projector and the 2D coordinates of a point in the slide or the image, the homologous epipolar lines can be computed out. So the homologous point of this known point can be calculated. Hence, the 3D coordinates of the space points on the surface of the solid of rotation can be worked out by the collinear equations when two 2D coordinates of the two corresponding 2D points are gotten already.

Fifth, The sequential images are taken from different directions of the solid of rotation. So there are the feature textures projected on the every aspect of the solid of rotation. Using the correspondence of whole adjustment and the inherent structure characteristic of the solid of rotation, the 3D coordinates of all space points projected on the whole surface of the solid of rotation can be computed out entirely.

Finally, The 3D model of this solid of rotation is acquired by connecting all neighbour space points. By this time, the 3D reconstruction of the solid of rotation is achieved ultimately.

### 2.3 Algorithm

The collinear equations are:

$$x - x_0 = -f\frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \qquad (1)$$

$$y - y_0 = -f\frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)}$$

where $x_0, y_0, f$ = the intrinsic parameters of the projector

$X_S, Y_S, Z_S$ = the coordinates of the projector centre

$X, Y, Z$ = the space coordinates of points

$x, y$ = the image coordinates of the relative points

$R = \{a_i, b_i, c_i, i = 1,2,3\}$ = the rotated matrix made up of rotated angles $\phi, \omega, \kappa$

$Z$ = 0

From the formula (1), the formulas of the space resection and the space forward intersection can be deduced correctly. Considering to these formulas known well already, their list is omitted here. It is very easy to find them in the books about photogrammetry. [Deren Li, 1992.]

Because of the same reason, the formulas of the homologous epipolar lines are not listed here, either. They can be found easily in the books about the digital photogrammetry. [Zuxun Zhang, and Jianqing Zhang, 2000.]

According to the inherent structure characteristic of the solid of rotation, every section of it parallel with horizontal surface is vertical with its fixed axis. The section is just a circle. In the same horizontal section, the equation of the circle is:

$$(x - x_0)^2 + (y - y_0)^2 = R^2 \qquad (2)$$

where $x_0, y_0$ = the coordinates of the centre of the circle

$x$ , $y$ = the coordinates of the points on the circle

$R$ = the radius of the circle

Considering to the same coordinates of the centre of circle in the every horizontal section, the formula (3) is from the two different horizontal sections. For example, one is from the top horizontal section of the solid of rotation and another is from the bottom horizontal section of the solid of rotation. Then:

$$-2(x_1 - x_2)x_0 - 2(y_1 - y_2)y_0 - R_1^2 + R_2^2 + (x_1^2 - x_2^2 + y_1^2 - y_2^2) = 0 \qquad (3)$$

where $x_0, y_0$ = the coordinates of the centre of the circle

$x_i, y_i$ = the coordinates of the points on the circle $i$

$R_i$ = the radius of the circle $i$

From the formula (3), the equation of the whole adjustment is:

$$X = (A^T A)^{-1} A^T L \qquad (4)$$

$$A = \begin{vmatrix} -2(x_{11}-x_{21}) & -2(y_{11}-y_{21}) & -1 & 1 \\ -2(x_{11}-x_{22}) & -2(y_{11}-y_{22}) & -1 & 1 \\ \ldots\ldots & \ldots\ldots & .. & .. \\ -2(x_{1i}-x_{2j}) & -2(y_{1i}-y_{2j}) & -1 & 1 \\ \ldots\ldots & \ldots\ldots & .. & .. \\ -2(x_{1m}-x_{2(n-1)}) & -2(y_{1m}-y_{2(n-1)}) & -1 & 1 \\ -2(x_{1m}-x_{2n}) & -2(y_{1m}-y_{2n}) & -1 & 1 \end{vmatrix}$$

$$L = \begin{vmatrix} x_{11}^2 - x_{12}^2 + y_{11}^2 - y_{21}^2 \\ x_{11}^2 - x_{22}^2 + y_{11}^2 - y_{22}^2 \\ \ldots\ldots \\ x_{1i}^2 - x_{2j}^2 + y_{1i}^2 - y_{2j}^2 \\ \ldots\ldots \\ x_{1m}^2 - x_{2(n-1)}^2 + y_{1m}^2 - y_{2(n-1)}^2 \\ x_{1m}^2 - x_{2n}^2 + y_{1m}^2 - y_{2n}^2 \end{vmatrix}$$

$$X = \begin{vmatrix} x_0 \\ y_0 \\ R_1^2 \\ R_2^2 \end{vmatrix}$$

where $x_0, y_0$ = the coordinates of the centre of the circle

$x_{1i}, y_{1i} (i = 1...m)$ = the coordinates of $m$ points on the circle whose radius is $R_1$

$x_{1j}, y_{1j} (i = 1...n)$ = the coordinates of $n$ points on the circle whose radius is $R_2$

$A$ is a coefficient matrix whose size is ($m \times n$)×4

$L$ is a constant matrix whose size is ($m \times n$)×1

$X$ is a unknown number matrix whose size is 4×1

After resolving the $X$, the coordinates of the fixed axis of the solid of rotation can be worked out obviously. At the same time the distance from the every space point projected on the surface of the solid of rotation to its fixed axis can be computed out by the formula (2).

## 3. DATA AND EXPERIMANTAL RESULTS

### 3.1 Design Data

The size of the rotating platform is 60cm×60cm. A planar grid is fixed upon it, which supplies the controlling points and the coordinates system. There are 18×18=324 controlling points in the planar grid. The interval of these points is the same and is 30mm. Each point has its own serial number which is exclusive. By using the coordinates of these points, the digital camera and the slide projector can be calibrated correctly and the extrinsic parameters of both are also calculated out entirely.

The size of the feature slide designed is 1024 pixels×768 pixels. Because the target solid of rotation in the experiment is a convenient cup that is relatively small, the texture feature is designed as a line which is better to show the edge of the solid of rotation. The line is located in the centre of the slide so that its equation is expressed as:

$$X = 512 \tag{5}$$

where $X, Y$ = the coordinates of the points on the line

$Y = \{0...767\}$ pixel unit

When the line is projected on the surface of the solid of rotation, the feature texture on it becomes a curve. This curve is sure to be corresponding to one section of the line in the slide. The homologous points can be gotten by the crossing between the homologous epipolar lines and the curve and the crossing between the homologous epipolar lines and the section of the line.

### 3.2 Image Sequences

When the positions of the camera and the projector are adjusted well and fixed relatively, both need to be focused respectively. The distance from the digital camera and the slide projector to the rotating platform is about 1.5 meters. Then a convenient cup is put on the centre of the rotating platform, which is the target solid of rotation. The slide projector projects the line in the slide onto the surface of the cup. It is ascertained that the projected line is through the whole body of the cup, which is very important and necessary. Then the camera is used to take sequential images of the convenient cup with the slide projector illuminating when the rotating platform rotates continually. In the experiment the camera takes images from 4 orientations and there are 4 images in total as the image data shown in the Figure 2. The size of each image is 1300pixels×1030pixels.

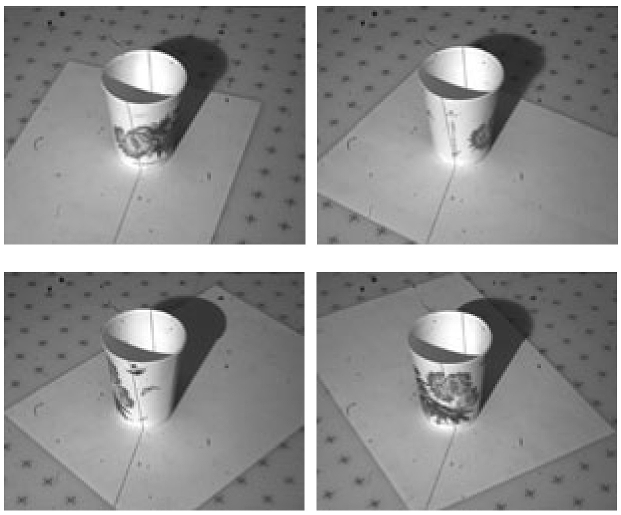

Figure 2. The sequential images with the feature

### 3.3 Experimental Results

For each image, it is exactly key that the projected line is extracted from this image. In the experiment, the curve is extracted out by the image processing skill. According to the characteristic of the image and the regulation of the coordinates movement of the extracted curve, the section of the curve on the body of the convenient cup is separated from the whole curve. For example, the experiment processes the first image in the Figure 2. The experimental results are shown as two images of the Figure 3.

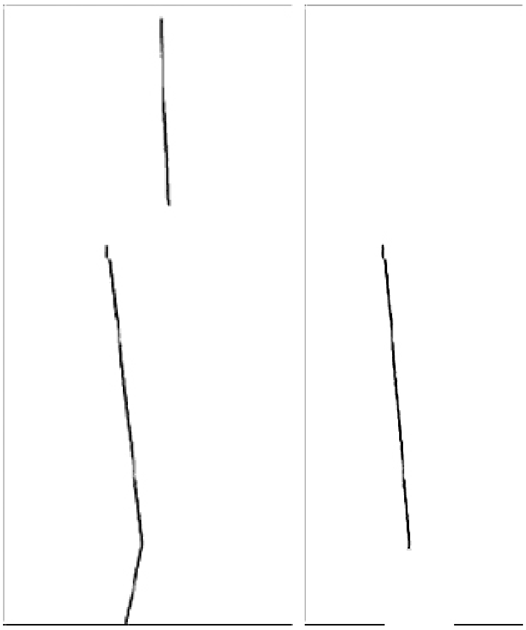

Figure 3. The whole curve and the section of the curve

The points on the section of the extracted curve are available for the next calculation. The coordinates of these points are expressed by the unit of pixel. There are separately 284 points, 294 points, 297 points and 249 points in the first image, the second image, the third image and the fourth image of Figure 2. Every point on the four sections of the curve has its own homologous point on the line of the projected slide through the homologous epipolar lines. So the space coordinates of four sets of space points can be computed out by the space forward intersection. The two end points of each set of the space points is picked out and their coordinates are used in the whole adjustment. The results of the whole adjustment are shown in the Table 1.

| $x_0$ (mm) | $y_0$ (mm) | $R_1$ (mm) | $R_2$ (mm) | $H$ (mm) |
|---|---|---|---|---|
| -0.0954 | 5.7235 | 27.9223 | 23.9147 | 80.7226 |

Table 1. The results of the whole adjustment

$(x_0, y_0)$ is the coordinates of the centre of circle. $R_1$ is the radius of circle of the mouth of the convenient cup and $R_2$ is the radius of circle of the bottom of the convenient cup. $H$ is the height of the convenient cup.

Known the coordinates of the centre axis, there is a radius of circle in every horizontal level according to the coordinates of the points of the extracted curves. The whole model of the convenient cup is made up of many levels of the circles. The view of the final model is shown in the Figure 4.

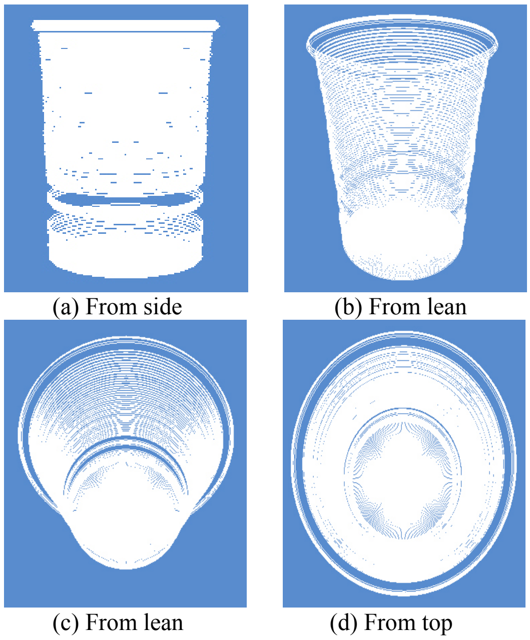


Figure 4. The model of the target cup

## 4. CONCLUSIONS

### 4.1 Achievements of the Paper

The approach proposed in the paper is confirmed to be proper and applicable from the results of the experimental data. This approach only requires a digital camera, an ordinary slide projector and a rotating platform. These equipments are easy to be ready for the applications.

The paper also deduces the detail algorithm of this approach which is understandable and relatively simple. Using the algorithm provided, the correct model results can be gotten from the image data. To sum up, the approach with this algorithm is effective and effectual.

According to the size of the object, the distance of the slide projector from the rotating platform and the point density of the slide can be changed and adjusted, so that this approach is suitable for many kinds of applications. Moreover, it is hardly affected by the space factor or time factor. The approach provided by the paper is flexible and practical.

### 4.2 Limitations and Future Work

Because of the tight time and the restricted experimental environment, there are a few limitations appearing in this approach.

- According to the size of the rotating platform and the intensity of the light of the slide projector, the distance from the slide projector to the rotating platform is not too far and the solid of rotation is not bigger than the rotating platform. So this set of the system and this approach are applicable for the relatively small target object.

- The reflection of the projecting light from the target object is complex and variable with different colour or material of the solid of rotation. If the reflection is not clear or is more than one, it would happen that the extraction of the feature becomes difficult or is reduplicate. So the reflection from the solid of rotation need be good enough.

- The experimental results of the model is not added the real texture. So the collection of the real texture of the target object is demanded. Then the real texture should add to the model.

- The experimental target of the paper aims at the simple solid of rotation. So the approach and the algorithm need to be consummated and meliorated for applying to the complex solid of rotation.

To these limitations above, the future work that will be done is as following.

- The feature projected should be designed carefully and in detailed according to the different target object. It is its size, its shape, its colour, its brightness and so on that should be considered entirely. It is a trial and error procedure.

- When the digital camera takes sequential images with feature projected, it also takes another sequential images with no feature at the same time. These images are functioned as the data of the real texture of the target object. The 3D model of the solid of rotation is acquired by the image data and the texture is achieved form relative images with no projected feature.

- The 3D reconstruction of the complex target object is divided into two main steps. The main body of the target object is reconstructed first as a solid of rotation. Then, affiliation of the target object is located onto the model of the main body. The latter is the key work of the next.

To sum up, the future work is put forward for the next research on the basis of the approach and the algorithm proposed by the paper.

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