

# HIGH PRECISION KINEMATIC POSITIONING USING SINGLE DUAL-FREQUENCY GPS RECEIVER

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## ABSTRACT:

Currently, high precision kinematic GPS positioning with centimetre level accuracy can only be carried out using differential GPS (DGPS) positioning techniques which require the deployment of base receiver stations. The requirement to deploy base receiver stations, however, spatially limits the operating range of the rover receiver to about 20 km from the base stations. As a result, it not only increases the operational costs of equipment and human resources but also significantly increases the logistical complexity for many applications such as land geodetic surveying and airborne geo-referencing and mapping. With the increased availability of precise GPS satellite orbit and clock data in real-time from the International GPS Service (IGS) and many other organizations, high precision kinematic positioning at centimetre to decimetre level is now possible using a single GPS receiver. Presented in this paper are the methods and algorithms that have been developed at the University of Calgary for high precision kinematic positioning using a single dual-frequency GPS receiver. Different from the conventional DGPS approach, the new system does not need a base station since the position determination is based on the processing of un-differenced GPS code and carrier phase observations. This eliminates the range limitation related to the conventional methods, resulting in instant advantages in field operations. A software system developed at the University of Calgary will also be described along with numerical results to demonstrate the obtainable positioning accuracy and its potential for various applications.

## 1. INTRODUCTION

Current carrier phase positioning with centimetre level accuracy requires the combination of observations from a minimum of two GPS receivers. At least one of these serves as the base station with known coordinates, and the others serve as rover stations whose position coordinates are to be determined relative to the base station(s). Drawbacks of this approach include the practical constraints imposed by the requirement that simultaneous observations need to be made at the rover and base stations, and that the rover station should be in the vicinity of the base station(s), typically within 20 kilometres. These constraints increase the operational cost and logistical complexity in the field. Airborne mapping is a typical example where direct geo-referencing using GPS currently requires the deployment of a number of base stations on the ground if the surveying and mapping is to be conducted over large areas. For remote and rough terrain regions, the difficulty level would be further increased. Direct geo-referencing using GPS without the need to establish ground base stations would be advantageous in such applications because it can reduce both equipment and labor costs and simplify the field operations.

This paper describes a positioning method using un-differenced code and carrier phase observations from a single dual-frequency receiver, aided by precise orbit and clock data. Since this positioning approach has no requirement for the deployment of base station(s), it is a global positioning approach capable of providing greater solution consistency with increased operational flexibility. A software package developed at the University of Calgary that achieves centimetre to decimetre level accuracy with a single GPS receiver will also be described.

The remainder of the paper is organized into four sections. The concepts of kinematic positioning using a single dual-frequency GPS receiver are first described in Section 2. Software developed at the University of Calgary specifically for un-differenced carrier phase processing is described in Section 3. Results obtained from the software are presented in Section 4, using various external solutions for reference. Concluding remarks are provided in Section 5.

## 2. CONCEPTS OF PRECISE POINT POSITIONING

Conventional Standard Point Positioning (SPP) is based on un-differenced GPS data processing and is subject to the influence of all error sources. Major error sources include those introduced by broadcast orbits and clocks, as well as atmospheric effects. Since SPP is only able to provide position solutions with an accuracy level of several metres, it is not suitable for applications that require higher positioning accuracy such as geo-referencing in airborne mapping.

With the advent of precise orbit and clock products with centimetre level accuracy, the two errors associated with the broadcast orbits and clocks can be significantly reduced. Once these errors are removed from the observations, higher positioning accuracy can be expected even when a single GPS receiver is used, provided that new data processing algorithms are developed. To date, there are many organizations, including the International GPS service (IGS), Natural Resources Canada (NRCAN) and Jet Propulsion Laboratory (JPL), which offer precise data in post-mission and real-time modes.

The method that derives high precision positioning solutions by processing un-differenced carrier phase observations from a

single GPS receiver assisted with precise orbit and clock products is called Precise Point Positioning (PPP). The word “precise” is used here to distinguish it from the conventional SPP method. The method developed at The University of Calgary is described in the following [Gao and Shen, 2002].

## 2.1 Observation Combination

The observation equations for code and carrier phase measurements on the  $L_i$  frequency ( $i = 1, 2$ ) are shown in Equations 1 and 2.

$$P(L_i) = \rho + c(dt - dT) + d_{orb} + d_{trop} + d_{ion/L_i} + d_{mult/P(L_i)} + \varepsilon(P(L_i)) \quad (1)$$

$$\Phi(L_i) = \rho + c(dt - dT) + d_{orb} + d_{trop} - d_{ion/L_i} + \lambda_i N_i + \lambda_i (\phi_r(t_0, L_i) - \phi_s(t_0, L_i)) + d_{mult/\Phi(L_i)} + \varepsilon(\Phi(L_i)) \quad (2)$$

where:

$P(L_i)$	– Measured pseudorange on $L_i$ (m)
$\Phi(L_i)$	– Measured carrier phase on $L_i$ (m)
$\rho$	– True geometric range (m)
$c$	– Speed of light (m/s)
$dt$	– Satellite clock error (s)
$dT$	– Receiver clock error (s)
$d_{orb}$	– Satellite orbital error (m)
$d_{trop}$	– Tropospheric delay (m)
$d_{ion/L_i}$	– Ionospheric delay on $L_i$ (m)
$\lambda_i$	– $L_i$ wavelength (m)
$N_i$	– Integer ambiguity on $L_i$ (cycle)
$\phi_r(t_0, L_i)$	– Initial phase of receiver oscillator
$\phi_s(t_0, L_i)$	– Initial phase of satellite oscillator
$d_{mult/P(L_i)}$	– Multipath effect in measured pseudorange on $L_i$ (m)
$d_{mult/\Phi(L_i)}$	– Multipath effect in measured carrier phase on $L_i$ (m)
$\varepsilon(.)$	– Measurement noise (m)

Note that the initial phase of the receiver and satellite oscillators, always less than half of the corresponding wavelength (Gabor, 2000), is commonly ignored in conventional carrier phase based double differenced systems. If it is combined with the integer phase components into a single term, Equation 2 can be rewritten as:

$$\Phi(L_i) = \rho + c(dt - dT) + d_{orb} + d_{trop} - d_{ion/L_i} + \lambda_i N_i' + d_{mult/\Phi(L_i)} + \varepsilon(\Phi(L_i)) \quad (3)$$

where  $N_i'$  is no longer an integer term if the initial phase value becomes significant.

In order to mitigate the ionospheric effect, which is the largest error source in GPS positioning after SA was turned off, the following ionosphere-free combinations can be formed:

$$P_{IF,L1} = 0.5[P(L1) + \Phi(L1)] = \rho - cdT + d_{trop} + 0.5\lambda_1 N_1' + 0.5d_{mult/P(L1)} + 0.5\varepsilon(P(L1) + \Phi(L1)) \quad (4)$$

$$P_{IF,L2} = 0.5[P(L2) + \Phi(L2)] = \rho - cdT + d_{trop} + 0.5\lambda_2 N_2' + 0.5d_{mult/P(L2)} + 0.5\varepsilon(P(L2) + \Phi(L2)) \quad (5)$$

$$\Phi_{IF} = [f_1^2 \cdot \Phi(L1) - f_2^2 \cdot \Phi(L2)] / [f_1^2 - f_2^2] = \rho - cdT + d_{trop} + \frac{cf_1}{f_1^2 - f_2^2} N_1' - \frac{cf_2}{f_1^2 - f_2^2} N_2' + d_{mult/\Phi(L1+L2)} + \varepsilon(\Phi(L1 + L2)) \quad (6)$$

Note the application of corrections from precise orbit and clock products have been applied in the above equations to eliminate the satellite orbit and clock error terms. A combination of Equations 4, 5 and 6 yields a new observation model for PPP. Different from the traditional ionosphere-free model, the new model is capable of estimating the ambiguities associated with  $L_1$  and  $L_2$  frequencies separately. This makes it possible to exploit the integer properties of both  $L_1$  and  $L_2$  ambiguities, which is essential for real-time kinematic positioning. The unknowns to be estimated in precise point positioning therefore include the position coordinates, receiver clock offset, troposphere, and ambiguity terms.

## 2.2 Error Mitigation

In equations (4) to (5), the ionosphere-free code and carrier phase combinations are used to mitigate the effect of the ionospheric error. The troposphere cannot be mitigated in this manner due to its non-dispersive nature. However, it can be modelled or estimated along with other parameters.

To facilitate high precision position determination, a number of unconventional error corrections have to be applied. These unconventional errors, related to un-differenced observations and precise satellite orbit/clock products, include satellite antenna phase centre, earth tide and ocean loading etc. The satellite antenna phase center correction is necessary for Block II/IIA satellites because the phase centers and centers of mass of these satellites do not coincide. Earth tide and ocean loading models are necessary because errors associated with them can reach several decimetres. Similarly, a satellite phase windup correction is necessary since the error can reach half a cycle. Note that these corrections are commonly ignored in double differenced positioning approaches because they can be cancelled out by the differencing procedure that is implemented between satellites and receivers. In the case of un-differenced code and carrier phase observations, however, these errors do not cancel out and their sizes are relatively large, influencing the accuracy of the point positioning solution.

### 2.3 Fast Ambiguity Convergence and Resolution

In precise point positioning, carrier phase ambiguities can be treated as float terms. However, the float ambiguity solutions may require long convergence times before centimetre-level accuracy can be obtained, ranging from several tens of minutes to several hours. Since convergence is a crucial issue for real-time applications, long convergence times may prevent the PPP approach from fulfilling the necessary accuracy requirements. As such, fast ambiguity convergence methods and algorithms should be developed.

Integer ambiguities must be treated as integers and subsequently must be resolved in order to fully realize the accuracy of carrier phase observations. Real-time centimetre level accuracy will be supported if integer values of the carrier phase ambiguities can be determined On-The-Fly (OTF) over short time intervals. The ionospheric-free observation combination models presented in Section 2.1 allow for the exploitation of the integer property and new ambiguity resolution methods are required with un-differenced GPS observations.

### 3. UNIVERSITY OF CALGARY'S P<sup>3</sup> SOFTWARE

A software package called P<sup>3</sup> has been developed at the University of Calgary to support precise point positioning using un-differenced GPS code and carrier phase observations. The software can be used to assess the performance of different data processing models as well as the influence of different error sources on positioning results.

Processing in P<sup>3</sup> can be done in post mission or in real-time, and the program can be run in either static or kinematic mode. Two point positioning modes are available: Single Point Positioning (SPP), which only makes use of code measurements, and Precise Point Positioning (PPP), which makes use of code and phase measurements along with precise satellite orbit and clock corrections. P<sup>3</sup> also supports forward and backward data processing.

The software lists various values for each processed epoch and displays a sky plot and a residual plot during processing. After processing is completed, a variety of graphs may be displayed, including the trajectory and velocity, the estimation of the receiver clock offset and zenith tropospheric delay, and the number of satellites and DOP values. A sample screenshot of the software during processing is shown in Figure 1.

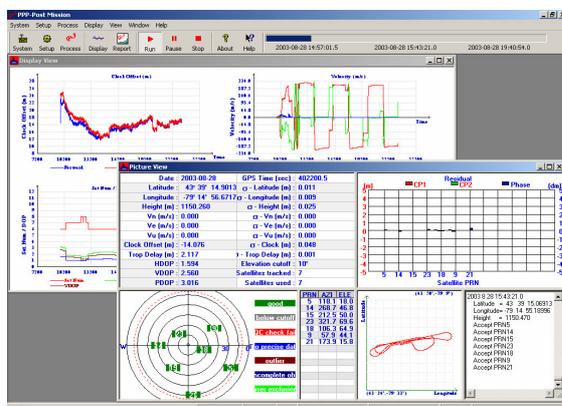


Figure 1: P<sup>3</sup> Software Sample Screenshot

### 4. RESULTS AND ANALYSIS

Two airborne kinematic data sets were post-processed using the P<sup>3</sup> software. A 10° elevation angle cut-off was used for all P<sup>3</sup> processing results.

#### 4.1 Aircraft – Low Dynamics (~300 km/h)

The first data set was flown on August 23, 2003, with a flight duration of approximately 4.75 h. Maximum aircraft speed did not exceed 310 km/h. Precise orbit and clock data was provided from JPL, at resolutions of 15 minutes and 1 second, respectively. The positioning results obtained using P<sup>3</sup> software were then compared to JPL's GIPSY/OASIS II solution, since it is the only software package that we found in the market that performs un-differenced carrier phase processing with a single receiver.

The number of satellites and PDOP, the altitude of the aircraft, and an estimation of the zenith tropospheric delay for the forward and backward pass is shown in Figures 2, 3 and 4, respectively. Finally, the position errors from backward processing (with respect to the GIPSY OASIS II solution) are shown in Figure 5. The position error statistics are listed in Table 1.

The estimation of the zenith tropospheric delay is relatively stable, as shown in Figure 4. This is because of the relatively constant flying altitude, shown in Figure 3. Figure 5 and the statistics in Table 1 show good agreement between the two PPP solutions, despite the remaining sinusoidal effect that still remains in the height component.

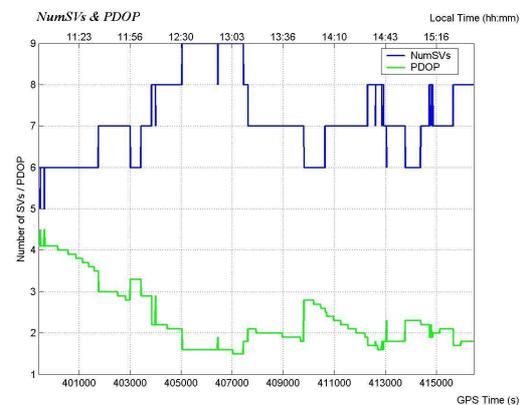


Figure 2: Number of Satellites and PDOP

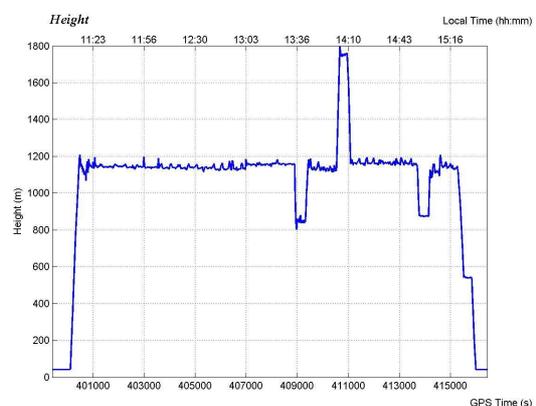


Figure 3: Altitude

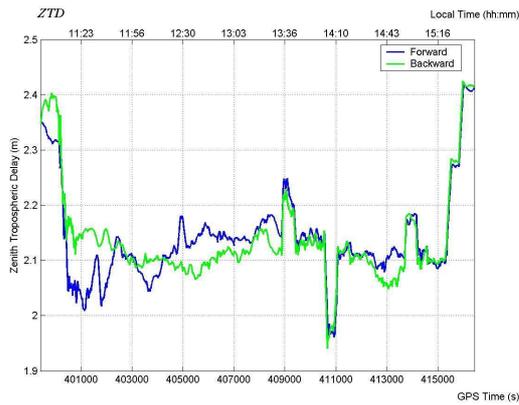


Figure 4: Zenith Tropospheric Delay

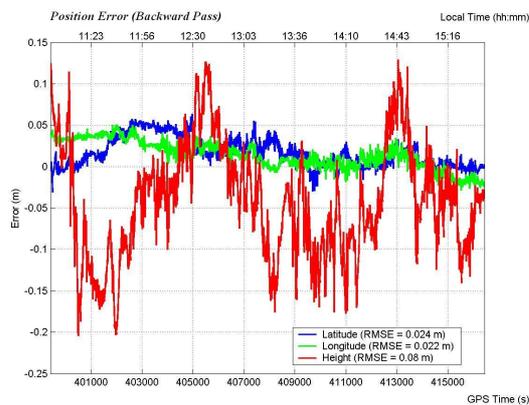


Figure 5: Position Errors between P<sup>3</sup> and JPL's GIPSY/OASIS II Solution

Table 1: Position Error Statistics (cm): P<sup>3</sup> vs JPL's GIPSY/OASIS II

	Latitude	Longitude	Height
Mean	1.7	1.4	-4.3
Std. Dev.	1.7	1.7	6.8
RMSE	2.4	2.2	8.0

#### 4.2 Aircraft – High Dynamics (~800 km/h)

The second data set was flown on September 21, 2003, with a flight duration of approximately 3.75 h. Maximum aircraft speed was up to 810 km/h. Final orbit and clock products from the International GPS Service (IGS) were used, with resolutions of 15 minutes and 5 minutes, and stated accuracies of 5 cm and 0.1 ns, respectively [IGS Data & Products, 2003].

The number of satellites and PDOP, aircraft altitude, and the forward and backward estimations of the zenith tropospheric delay and receiver clock offset are shown in Figures 6, 7, 8 and 9, respectively.

The results obtained using P<sup>3</sup> software were then compared to the data supplier's own in-house multi-reference station DGPS solution and also with JPL's GIPSY/OASIS II solution. The comparison with the in-house DGPS solution is shown in Figure 10, and with JPL's solution in Figure 11.

Given in Table 2 are the position error statistics for the comparison between P<sup>3</sup> and the in-house DGPS solution while

given in Table 3 are the position error statistics for the comparison between P<sup>3</sup> and JPL's GIPSY/OASIS II solution.

There appears to be a relatively strong correlation between the height error and the receiver clock offset when comparing the P<sup>3</sup> solution to the in-house DGPS solution, as shown in Figure 10. Furthermore, a large offset appears for the height component. Both these effects are significantly reduced in Figure 11 when the two PPP solutions are compared, suggesting that the DGPS solution is less accurate compared to the two PPP solutions. This is also reflected in the statistics shown in Tables 2 and 3. The increased height error in the latter one-third of the data set is probably due to the higher PDOP, which is shown in Figure 6.

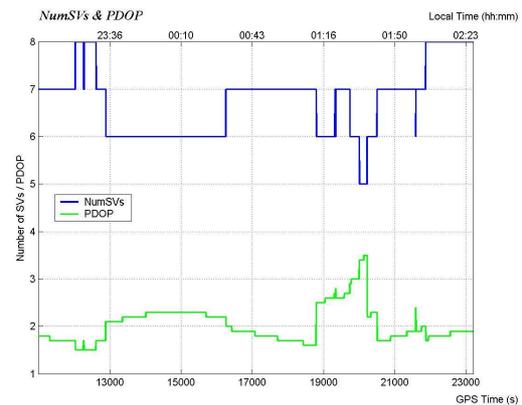


Figure 6: Number of Satellites and PDOP

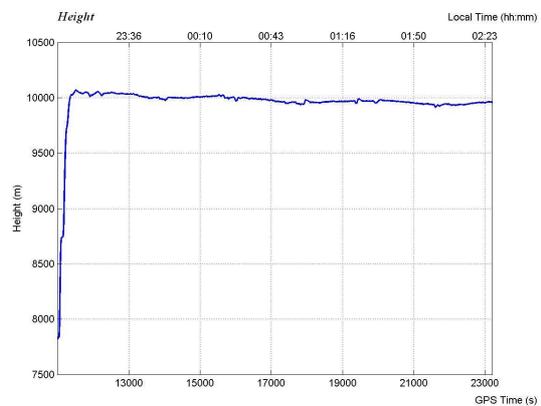


Figure 7: Altitude

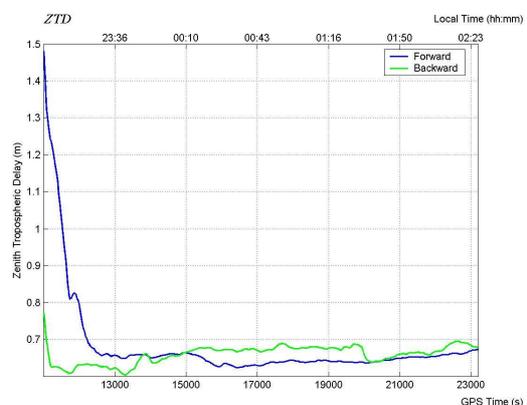


Figure 8: Zenith Tropospheric Delay

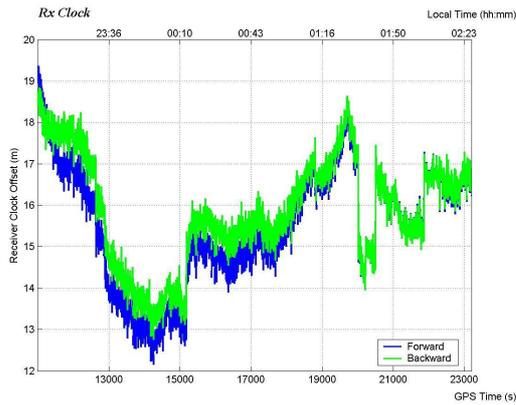


Figure 9: Receiver Clock Error

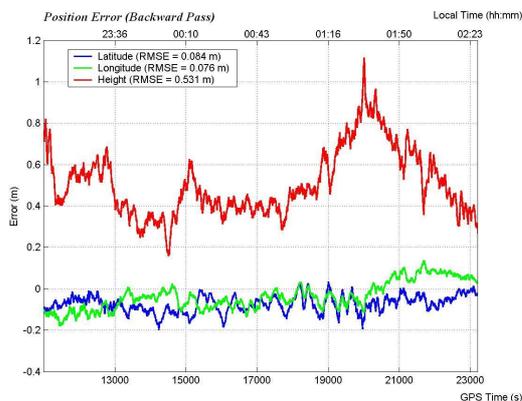


Figure 10: Position Errors between P<sup>3</sup> and the Data Supplier's In-House DGPS Solution

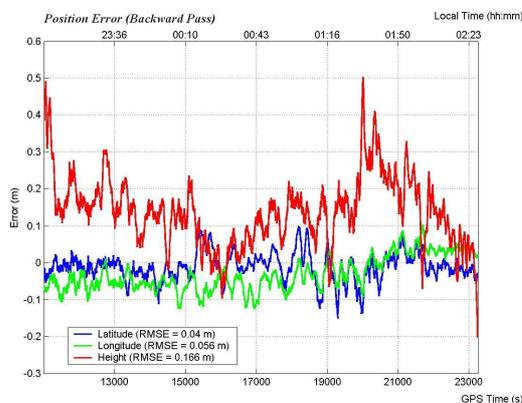


Figure 11: Position Errors between P<sup>3</sup> and JPL's GIPSY OASIS II Solution

Table 2: Position Error Statistics (cm): P<sup>3</sup> vs DGPS

	Latitude	Longitude	Height
Mean	-7.4	-3.6	50.7
Std. Dev.	4.0	6.7	15.9
RMSE	8.4	7.6	53.1

Table 3: Position Error Statistics (cm): P<sup>3</sup> vs JPL's GIPSY/OASIS II

	Latitude	Longitude	Height
Mean	-1.2	-3.1	13.7
Std. Dev.	3.8	4.6	9.4
RMSE	4.0	5.6	16.6

## 5. CONCLUSIONS

The purpose of this paper was to present a method that has been developed at the University of Calgary for high precision kinematic positioning using a single dual-frequency GPS receiver. P<sup>3</sup>, the software developed at the University of Calgary, was also described. Since the PPP approach does not require the deployment of base stations, errors associated with reference station coordinates as well as error de-correlation with increasing rover-reference receiver distance do not apply. Aside from globally consistent accuracy (rivaling DGPS accuracy in many instances), the PPP approach offers a significant cost saving since base stations do not need to be deployed.

Two airborne kinematic data sets have been analyzed and compared with other available solutions. It was found that the P<sup>3</sup> solution agrees well with the GIPSY/OASIS II solution provided by JPL, with differences at the centimetre level in the horizontal, and up to a couple of decimetres in the vertical. About half of the height discrepancy between these two solutions can be attributed to different precise clock product resolutions – 5 minutes for P<sup>3</sup>, and 1 second for JPL.

The second data set presented in this paper showed that the PPP approach would be more accurate than the DGPS approach in some instances especially over long baselines since it does not depend on error de-correlation between the rover and reference receivers or the coordinates of the reference receivers. As such, it is capable of providing a globally consistent solution with reduced logistical complexity in the field.

## ACKNOWLEDGEMENTS

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