ACCURATE REGISTRATION OF ALS DATA WITHOUT CONTROL POINTS

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ABSTRACT:

This paper demonstrates the use of a matching algorithm to register an ALS data set to a reference surface composed of ground points obtained using kinematic stop-and-go GPS. The matching algorithm minimises the normal distances between the points of the ALS data set and the facetted reference surface. A particular feature of this experiment is to sample the reference surface in several high density patches, rather than spreading the sampling evenly over the surface. The resulting triangulated reference surface is composed of several patches of small triangles in an environment of extremely large triangles. The weighting technique adopted was based on interpolation errors, which are estimated using autocorrelation theory. A spatial covariogram is generated and the interpolation errors are determined using an exponential function fitted on the experimental covariogram. In practice, the accurate registration of ALS data is important, as it provides the foundation of DEMs used in sensitive areas such as flood studies. The method presented has proven to be a very successful tool and is an improvement on existing methods.

1. INTRODUCTION

The registration of data sets in the same coordinate system is essential for the fusion of data obtained from similar or different sources. The determination of the registration accuracy by commercial firms is often undertaken by comparing points of one surface to heights interpolated between points on the other surface. The last few years have seen research undertaken in the development of surface matching algorithms to automate the registration. High redundancy is achieved with these algorithms, as each point of one surface can potentially participate in the formation of a normal equation for a least squares adjustment. These algorithms should become an important tool for data fusion, especially with data acquisition techniques such as airborne laser scanning (ALS), which lack thematic information.

This paper presents a weighted least squares surface matching algorithm developed at the University of Newcastle, Australia. The weighting technique used in the algorithm permits a surface to be registered accurately to a reference surface with that reference surface covering 20% of the other surface only.

The accuracy of the registration is related to the density of the reference data (referred to as S_1 in this paper) and, to a lesser degree, to the roughness of the surface. A distinction is made between the precision of the matching, defined by the residuals of the least squares, and the accuracy of the registration, which compares the matched points of the second surface to their true position. The second surface is the data set which is transformed by the least squares matching algorithm in the coordinate system of S_1: it is referred to as S_2 in this paper.

The aim of this paper is to present the weighted least squares software and to demonstrate its performance through an application. The application involves registering a set S_2 of 27,000 points in the coordinate system of S_1, a set of 900 points. S_1 is a patchy set obtained with stop-and-go global positioning system method (GPS), while S_2 is a set of points filtered from a larger set sampled with an airborne laser scanner.

2. THE TOOLS OF THE EXPERIMENT

2.1 Least squares Matching Algorithm

2.1.1 Background: Surface matching, also referred to as registration without control points, describes an automated method used to find the parameters of a transformation which minimises the separation between two 2.5D surfaces. Early matching algorithms were presented by Rosenholm and Torlegård (1988) and Ebner and Strunz (1988). In both instances, better results were reported using a gridded DEM to orient photogrammetric data than with conventional methods. Similar methods are used in deformation studies and monitoring (Pilgrim, 1996). Registration methods using a control surface are justified particularly in areas where permanent control markers are not possible (abdominal deformation in pregnancy (Karras and Petsa, 1993)), unethical (dental erosion (Mitchell, 1995; Mitchell and Chadwick, 1998, 1999)) or impractical (coastal erosion (Buckley, 2003; Mills et al., 2003)). Maas (2002) developed a least squares matching (LSM) technique for laser scanner data strips adjustment: systematic deformations due to GPS/INS systems are corrected by solving for the three shift parameters T_X, T_Y and T_Z.

The algorithms mentioned minimise the difference in heights between the points of one surface to the facets of the other surface. The algorithm presented in this research minimises the normal distances between the points of S_2 to triangular plane patches of S_1. A similar algorithm was trialled on small artificial data sets (Schenk et al., 2000) in a comparison study of matching algorithms. Habib et al. (2001) report on such an algorithm where the parameters of the transformation are found with a system featuring a Hough transform.

2.1.2 The Matching Algorithm: Let the three rotation angles ω, ϕ and κ, the three translations T_X, T_Y and T_Z and the scaling factor s be the seven parameters of a conformal transformation which moves the set of points S_2 to a position S_2', which minimises the set of the normal distances from the points of S_2 to the facets of S_1. The normal distance from a point $I' \in S_2'$ to a plane defined by the three vertices $(P, Q, R) \subset S_1$ of the triangle enclosing the point I' is given by:

$$D = \frac{|a.x_i' + b.y_i' + c.z_i' - d|}{\sqrt{a^2 + b^2 + c^2}} \tag{1}$$

where a, b, c and d are functions of P, Q and R; $I'(x_i', y_i', z_i')$ is a function of ω, ϕ, κ, T_X, T_Y, T_Z, s and $I(x_i, y_i, z_i)$ with $I \in S_2$.

An approximation D^* of the normal distance D is obtained by linearising Equation 1 using a Taylor expansion and keeping only the first order derivatives :

$$\begin{aligned} D^* \quad \approx \quad & D_0 + \frac{\partial D}{\partial \omega}\Delta\omega + \frac{\partial D}{\partial \phi}\Delta\phi + \frac{\partial D}{\partial \kappa}\Delta\kappa \\ & + \frac{\partial D}{\partial T_X}\Delta T_X + \frac{\partial D}{\partial T_Y}\Delta T_Y + \frac{\partial D}{\partial T_Z}\Delta T_Z \\ & + \frac{\partial D}{\partial s}\Delta s \end{aligned} \tag{2}$$

where:
D_0 is the distance D evaluated at the initial value of the parameters of the transformation;
$\Delta\omega$, $\Delta\phi$, $\Delta\kappa$, ΔT_X, ΔT_Y, ΔT_Z and Δs are corrections to initial values of the parameters.

Let V be the difference between the approximated normal distance D^* and the exact distance D:

$$D = D^* - V$$

Equation 1 can thus be written as:

$$\begin{aligned} D \quad = \quad & D_0 + \frac{\partial D}{\partial \omega}\Delta\omega + \frac{\partial D}{\partial \phi}\Delta\phi + \frac{\partial D}{\partial \kappa}\Delta\kappa \\ & + \frac{\partial D}{\partial T_X}\Delta T_X + \frac{\partial D}{\partial T_Y}\Delta T_Y + \frac{\partial D}{\partial T_Z}\Delta T_Z \\ & + \frac{\partial D}{\partial s}\Delta s - V \end{aligned} \tag{3}$$

With the model requirement that $D = 0$ and rearranging Equation 3:

$$\begin{aligned} V \quad = \quad & D_0 + \frac{\partial D}{\partial \omega}\Delta\omega + \frac{\partial D}{\partial \phi}\Delta\phi + \frac{\partial D}{\partial \kappa}\Delta\kappa \\ & + \frac{\partial D}{\partial T_X}\Delta T_X + \frac{\partial D}{\partial T_Y}\Delta T_Y + \frac{\partial D}{\partial T_Z}\Delta T_Z \\ & + \frac{\partial D}{\partial s}\Delta s \end{aligned} \tag{4}$$

This calculation repeated over the n points of S_2 is arranged in matrix notation to give:

$$\mathbf{V} = \mathbf{L} + A\mathbf{X} \tag{5}$$

where:
V is an $n \times 1$ vector of residuals
L is a $n \times 1$ vector of “observables” D_0 (also called the “absolute term”)
A is an $n \times 7$ matrix of the first order derivatives (often called the “design” matrix)
X is a 7×1 vector of correction to the initial values of the parameters (to be estimated)

The conventional least squares solution for a system of weighted observations is given by:

$$\hat{\mathbf{X}} = -(A^T G^{-1} A)^{-1} A^T G^{-1} \mathbf{L} \tag{6}$$

where:
$\hat{\mathbf{X}}$ is the vector of least squares estimators
G is the variance/covariance matrix of the observations.

2.2 Weighting Techniques

The denser the data, the more faithful the triangulated surface is to the true surface. The use of weights in a least squares fit becomes essential when the reference data is irregularly distributed. Large triangles are formed in sparse data areas, and small weights have to be given to the corresponding normal equations to counterbalance the resulting large interpolation errors. The weighting technique adopted for this project is based on the interpolation errors σ^2, which are estimated from an exponential model fitted on the experimental covariogram. A comparison study of weighting techniques for least squares surface matching found that the best results in a sparse reference data environment were obtained with this method (Pâquet, 2003a).

This method involves the production of a covariogram or covariance function, from which the mean of the product of the heights can be evaluated for any given distance between the points. The covariances for a distance d are obtained using:

$$C(d) = \frac{\Sigma_{i=1}^{n}(z_i.z_{i+d})}{n} \tag{7}$$

The data point spacing is irregular and the distance and product of the heights are calculated for each pair of points in the data set. These results are then classed in bins, according to the range of the distance separating them. Finally the mean distance, mean product and standard deviation of the products are calculated in each bin. The covariogram is the plot of the value of the mean product versus the mean distances. The experimental covariogram fitted with an exponential model is shown in Figure 1. The weight of the normal equation is then computed as the inverse of the standard error of prediction $\sigma^2(O)$:

$$W = 1/\sigma^2(O) \tag{8}$$

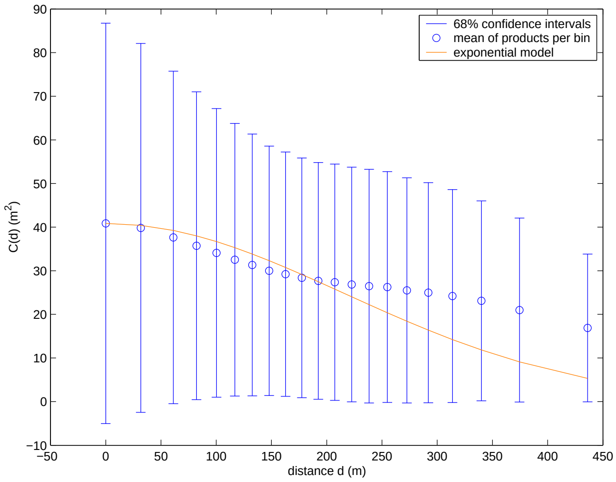


Figure 1: Covariogram Fitted with Exponential Model

The standard error of prediction is given by (Heiskanen and Moritz, 1967, p. 267):

$$\sigma^2(O) = C_0 - 2\Sigma_{i=1}^{n}\lambda_{oi}C_{oi} + \Sigma_{i=1}^{n}\Sigma_{k=1}^{n}\lambda_{oi}\lambda_{ok}C_{ik} \quad (9)$$

where the subscripts refer to horizontal distances between points (i.e.: ik is the distance between I and K), O is the point interpolated and $I = K$, with $i = k = \{1, 2, ..., n\}$, are points of known properties used to find the standard error of the interpolated property of O.

The method is adapted for the matching programme. The property of the variables is the height. The number of points used to estimate the error is limited to the three vertices of the enclosing triangle, and can be found analytically with, for a point O interpolated in an enclosing triangle of vertices P, Q and R:

$$\begin{aligned}\sigma^2(O) \quad = \quad & C_0 - 2(\lambda_1 C_{o-p} + \lambda_2 C_{o-q} + \lambda_3 C_{o-r}) \quad (10)\\ & + 2(\lambda_1\lambda_2 C_{p-q} + \lambda_1\lambda_3 C_{p-r} + \lambda_2\lambda_3 C_{r-q})\\ & + C_0(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)\end{aligned}$$

where the weights λ_i of the covariance factors are determined using:

$$\lambda_1 = \frac{x_o(y_r - y_q) + x_q(y_o - y_r) + x_r(y_q - y_o)}{x_p(y_r - y_q) + x_q(y_p - y_r) + x_r(y_q - y_p)} \quad (11a)$$

$$\lambda_2 = \frac{x_o(y_p - y_r) + x_p(y_r - y_o) + x_r(y_o - y_p)}{x_p(y_r - y_q) + x_q(y_p - y_r) + x_r(y_q - y_p)} \quad (11b)$$

$$\lambda_3 = \frac{x_o(y_q - y_p) + x_p(y_o - y_q) + x_q(y_p - y_o)}{x_p(y_r - y_q) + x_q(y_p - y_r) + x_r(y_q - y_p)} \quad (11c)$$

The covariance factors are estimated from the covariance function shown in Figure 1 for the distances shown in the subscripts. C_0 is the covariance for the nil distance, that is, the mean of the square of the products of the height of the points (the points are multiplied with themselves when the distance is nil). The exponential model (or Gaussian model) is given by (Mikhail, 1976, p. 405):

$$C(d) = C(0).e^{-k^2 d^2} \quad (12)$$

The value of $k = 0.0032728$ in the exponential model is determined by a least squares method. Note that the model does not fit the experimental data accurately for the longest distances (see Figure 1): however, out of the 1719 triangles generated by the Delaunay triangulation of S_1, only 6 had sides larger than 240m.

3. EXPERIMENT AND RESULTS

3.1 Aim

The aim of this experiment is to demonstrate the ability of the matching algorithm to register S_2 in the coordinate system of S_1. Given the characteristics of the data (§ 3.2), S_2 was first matched to S_1. The two sets were assumed then to be registered in the same coordinate system (§ 3.3). The experiment (i.e.: the testing of the algorithm) was undertaken: S_2 was transformed with known parameters, then matched with the algorithm. The ability of the algorithm to return S_2 back to its registered spatial position was measured by:

1. comparing the parameters of the initial transformation and the parameters of the matching transformation,
2. computing the mean of the absolute displacement of the coordinates of S_2.

3.2 Data Characteristics

The surface S_2 is a data set of 27,748 points. The set was extracted from a laser project covering the Greater Newcastle area. The accuracy reported for the Greater Newcastle project included a mean elevation difference of 0.1m based on direct observations of 12 test points. A comparison to 12 derived test points (interpolated from surface model) produced a standard deviation of 0.25m. Only height accuracy was estimated. The position of the set was fixed by GPS/INS with two survey control points, and one reference point situated at the aerodrome of departure of the plane used for the survey.

The reference set S_1 of 884 points was sampled with a GPS system using a stop-and-go kinematic method. Its accuracy varies from point to point but averages approximately 30mm both horizontally and vertically. The matched sets are shown in Figure 2.

3.3 Data Preparation

The experiment tests the ability of the matching programme to register a large dense ALS set using a sparse GPS set. The GPS set is made up of six clusters of dense data. The Delaunay triangulation of S_1 generates small triangles in the clusters, and large triangles which do not represent accurately the shape of the terrain between the clusters. After normalising the values of the data to minimise numerical errors (Pilgrim, 1991), S_2 is matched to S_1, resulting in a small adjustment. An adjustment can be expected to occur as the ALS set registration method is prone to planimetric

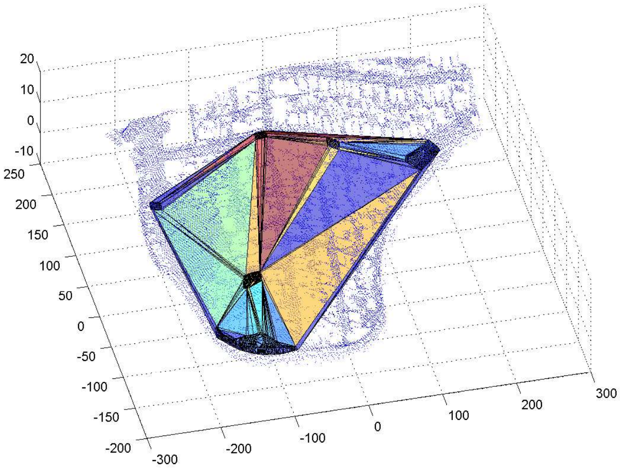


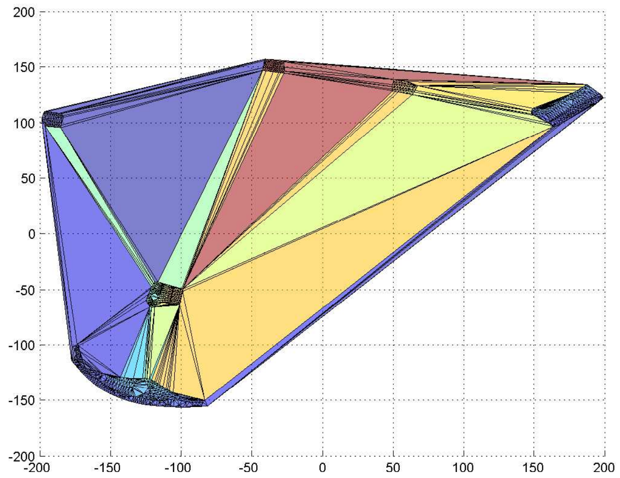
Figure 2: ALS and GPS Data Sets

errors (Huising and Gomes Pereira, 1998), and the set used was part of a much larger set adjusted globally. Moreover, the programme matches S_2 on the triangulated model S_1, not on the true surface. The accuracy of the matching to the true surface depends on the faithfulness of the model to the true surface and is a function of the density of the reference surface S_1 (Pâquet, 2003b). The parameters of the adjustment are:

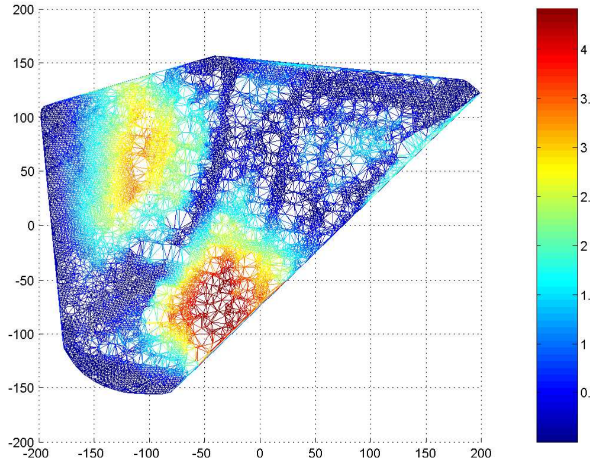
$$\begin{array}{llcr}
\text{Rotation Angles (degrees):} & \omega & = & 0.0159 \\
 & \phi & = & 0.0072 \\
 & \kappa & = & -0.0650 \\
\text{Translation (m):} & T_X & = & 0.0773 \\
 & T_Y & = & -0.0031 \\
 & T_Z & = & -0.0496 \\
\text{Scaling:} & s & = & 0.9999
\end{array}$$

It is assumed that the position of S_2 after the preparation adjustment accurately describes the true surface. No checks are available to determine this accuracy. The sole measure of precision is that provided by the least squares method. The separation between the two surfaces after iteration termination can only be assessed with the residuals of the least squares adjustment.

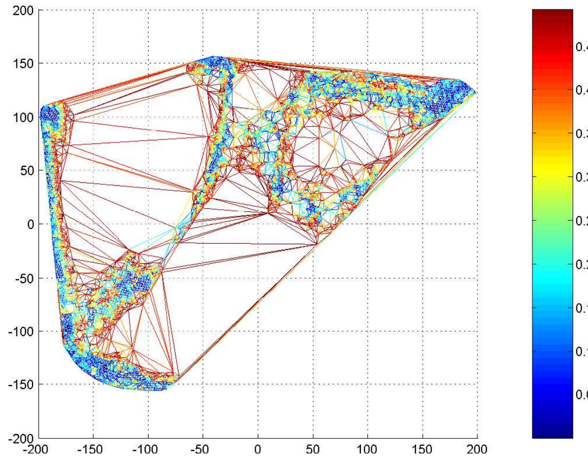
The residuals of the least squares are not directly indicative of the accuracy of the registration: some of the residuals occur in the large patches formed by the triangulation between the six clusters of GPS data. The total number of triangles in S_1 is 1,719. The number of triangles with an area under 6m^2 is 1,569. Their combined surface area is $28{,}637\text{m}^2$, which represents 38.37% of the total triangulated area of $74{,}638\text{m}^2$. The total number of residuals obtained in the matching is 14,649 (out of 27,748 points: see explanation below). Assuming that the smallest residuals occur within the smallest triangles, the matrix of residuals is sorted by size. The 5,620 smallest residuals (or 38.37% of the matrix of residuals) have values ranging from 0.000038m to 0.4918m. The mean and standard deviation are respectively 0.2123m and 0.1420m. By contrast, the largest residual in the matrix of residuals has a value of 4.4123m, and the mean and standard deviation are 1.1466m and 1.0356m respectively. This assumption is validated in Figure 3(c), which shows residuals of value smaller than 0.1m produced within the six clusters.



(a) Reference Surface S_1



(b) All Residuals



(c) Smaller Residuals

Figure 3: Configuration and Magnitude of Residuals

S_2 has a total area of approximately $149{,}740\text{m}^2$. The six clusters of S_1 represent an area of approximately $28{,}637\text{m}^2$ or 19.1% of the ALS surface. Of the 27,748 points of S_2, only 14,649 were included in the last iteration of the programme to form the normal equations of the least squares. The others either did not find correspondence or were eliminated as outliers. Experience shows that better matching results are obtained with the points of S_1 sampled in the periphery of S_2, but this characteristic was not shown in this experiment. S_2 is now assumed to be registered in S_1 coordinate system. The two sets are now ready for the experiment.

3.4 Measure of the Accuracy of the Matching

After the adjustment, the programme is tested by transforming S_2 with known parameters. This transformation is referred to as the initial transformation. S_2 is then matched to S_1. The process is undertaken with different values for the parameters of the initial transformation. A measure of the accuracy of the matching (i.e.: the ability of the programme to return S_2 to its original spatial position) is demonstrated by comparing the parameters of the matching transformation which undo the initial transformation (i.e: the inverse operation).

The matching accuracy is not easily assessed by interpreting the simultaneous effect of the differences between the three rotations, the three translations and the scaling of both initial and matching transformations. An additional measure of the accuracy is therefore given by the mean and standard deviation of the mismatches of the points of S_2. To obtain this mismatch, the coordinates of S_2 in its original position are compared to the coordinates of S_2 after the initial and matching transformation. The absolute mismatch between each point is obtained with the distance formula. The mean $\overline{m}$ of the mismatches of the 27,749 points of S_2 is computed with the standard deviation σ_m.

3.5 Results

Four trials were undertaken for the experiment. The results of the trials are shown in Tables 1, 2, 3 and 4. The Tables show the parameters of the initial transformations and the parameters of the matching transformation. The absolute values of the differences between the parameters are shown in the last column. The last three rows of the Tables indicate the number of iterations undertaken by the matching programme, the mean of the mismatches $\overline{m}$ and the standard deviation σ_m.

From the four trials, it can be observed that the largest rotational error was $\kappa = 0.0255^o$ in trial 2, while the largest translation error was $T_X = 0.0798m$ which occurred in trial 4. The largest mismatch occurs in trial 4 where S_2 was shifted by an initial transformation including a translation of 2m.

4. CONCLUSIONS

This article reports on a weighted least squares matching algorithm developed at the University of Newcastle, Australia. The algorithm measures the separation between the surfaces along the normal distances between the points of one surface to the plane patches of the other. The weighting technique of the least squares involves the production of a spatial covariogram. The weights are computed as the inverse of the interpolation errors estimated from an exponential function fitted on the experimental covariogram.

The experiment presented in this article demonstrates the ability of the programme to register an ALS data set of 27,749 points in the coordinate system of a data set of 884 points obtained with a stop-and-go GPS method. The reference set is sparsely sampled, and contains 6 clusters densely sampled. The weighting technique of the algorithm efficiently allocates large weights to points interpolated close to the vertices of the patches generated by the Delaunay triangulation, where the interpolation error is minimal. Likewise, small weights are allocated to the normal equations of the least squares which involve points far away from vertices where large interpolation errors occur. Four trials involving different sets of parameters showed that the programme is able to register the ALS surface with mean errors approximating 100mm in the worst of the four cases. The weighted least squares matching program can be used to orient data sets which may include buildings and protruding features. The reference surface is sampled between the protrusions (i.e.: in the streets and parks) using a precise and dense GPS method. The sampling is ideally patchy - the size of the patches is as important as the density of the patches to the final accuracy of the matching. The programme is thus a useful tool for data reconstruction.

Table 1: Matching Results

	initial	results	$\mid\Delta\mid$
ω	0	-0.0019	0.0019
ϕ	0	-0.0013	0.0013
κ	0	-0.0005	0.0005
T_X	-1	0.9629	0.0371
T_Y	-1	0.9897	0.0103
T_Z	-10	9.9875	0.0125
s	1	1.0002	0.0002
Iterations	10		
$\overline{m}$	0.0479		
σ_m	0.0178		

(ω, ϕ and κ in decimal degrees)
(T_X T_Y, T_Z, $\overline{m}$ and σ_m in metres)

Table 2: Matching Results (cont.)

	initial	results	$\mid\Delta\mid$
ω	0	-0.0029	0.0029
ϕ	0	-0.0019	0.0019
κ	0	0.0255	0.0255
T_X	1	-1.0122	0.0122
T_Y	1	-1.0618	0.0618
T_Z	10	-10.0452	0.0452
s	1	1.0004	0.0004
Iterations	5		
$\overline{m}$	0.0980		
σ_m	0.0426		

(ω, ϕ and κ in decimal degrees)
(T_X T_Y, T_Z, $\overline{m}$ and σ_m in metres)

Table 3: Matching Results (cont.)

	initial	results	$\mid\Delta\mid$
ω	0	-0.0036	0.0036
ϕ	0	-0.0023	0.0023
κ	-1	0.9825	0.0175
T_X	0	0.0696	0.0696
T_Y	0	-0.0343	0.0343
T_Z	0	-0.0105	0.0105
s	1	1.0000	0.0000
Iterations	5		
$\overline{m}$	0.0720		
σ_m	0.0321		

(ω, ϕ and κ in decimal degrees)
(T_X T_Y, T_Z, $\overline{m}$ and σ_m in metres)

Table 4: Matching Results (cont.)

	initial	results	$\mid\Delta\mid$
ω	0	-0.0003	0.0003
ϕ	0	-0.0026	0.0026
κ	0	0.0167	0.0167
T_X	-2	2.0798	0.0798
T_Y	0	0.0114	0.0114
T_Z	0	-0.0024	0.0024
s	1	0.9999	0.0001
Iterations	9		
$\overline{m}$	0.1096		
σ_m	0.0290		

(ω, ϕ and κ in decimal degrees)
(T_X T_Y, T_Z, $\overline{m}$ and σ_m in metres)

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