THE SOLID IMAGE: AN EASY AND COMPLETE WAY TO DESCRIBE 3D OBJECTS

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ABSTRACT:
A digital photo image can be considered, with a good approximation, as a central perspective of the acquired object. If the internal and external orientation parameters of the camera are known, it is possible to establish the direction in the space of each point of the object which is represented by a pixel in the image.

Using a dense digital surface model (DDSM) of the acquired object each pixel (and therefore each direction in the space) can be associated to the value of distance between the centre of perspective and the point of the object that is represented by the pixel itself. In this way each pixel can be made to the 3D position of the corresponding point of the object in an absolute reference system.

The DDSM can be derived from an existing map or by using surveying instruments and procedures. A series of new instruments, based on laser technology, have recently been introduced onto the market. These instruments offer the possibility of obtaining a DDSM in a quick and cheap way.

The integration of the DDSM geometric data with the image radiometric data allows a new concept of the Solid Image to be obtained.

Solid images can be used in architectural, archaeological and land surveying applications to determine, in real time, the position of any point in a 3D reference system, using a common PC or to carry out correct three-dimensional measurements (lines, areas, volumes, angles, …), by just selecting some points on the image.

In addition, with the Solid Image even unskilled users can easily plot profiles, sections and plans using simple drawing functions, generate stereo models and realistic 3D models.

The developed software, which is able to generate and manage the solid image, is described and some examples are shown.

1. THE SOLID IMAGE

1.1 Definition of a Solid Image

Any image can, with a good approximation, be considered a central prospective of the photographed object.

If the orientation and calibration parameters of a digital image are known, it is possible to establish a direction in space in the x, y and z reference system, for each pixel of the image. (see figure 1).

If the coordinates of an image point \(i\) are measured, the horizontal and vertical angles \(\theta\) and \(\alpha\) can easily be determined as:

\[
\begin{align*}
\theta &= \arctan \frac{\xi_i}{c} \\
\alpha &= \arctan \frac{\eta_i}{\sqrt{c^2 + \zeta_i^2}}
\end{align*}
\] (1)

Where: \(\theta\) = the horizontal angle
\(\alpha\) = the vertical angle
\(c\) = the principal distance
\(\xi_i, \eta_i\) = the coordinates of the image point \(i\)

One image is not sufficient to determine the 3D position of an object point. If a single image is used it is possible, at the most, to establish its direction in the space. In order to reconstruct objects in three dimensions, it is necessary to use at least 2 images according to the classical principles of photogrammetry.

A valid alternative to the well known photogrammetric restitution techniques is offered by the knowledge of the distance \(d\) between the centre of perspective of an image and the object itself for each direction in the space. In this way it
is possible to establish the position of any object point O in the \( \xi, \eta \) and \( \zeta \) system, using simple geometric equations (see figure 2).

\[
\begin{align*}
\xi &= \rho \cos \theta \\
\eta &= \rho \sin \theta \\
\zeta &= z - \frac{c}{\tan \theta}
\end{align*}
\]

Where \( \theta, \alpha \) are determined as previously shown (1).

In order to determine the previously defined distances for each pixel of the digital image, a dense digital surface model (DDSM) can be used. These 3D models are today easily obtained by laser scanners.

The traditional RGB radiometric data of the image, together with a “distance matrix” derived from a laser scanner data set, lead to a new concept: the “Solid Image”:

![Figure 3. The structure of the Solid Image](image)

A digital image is usually made up of three matrices where the fundamental RGB (Red, Green and Blue) radiometric components are stored.

The distance values corresponding to each pixel are stored in a supplementary matrix D that has the same size as the RGB matrices, in terms of lines and columns.

A Solid Image is therefore made up of 4 matrices: R, G, B and D (see Fig. 3).

In order to obtain the distance values that have to be stored in the new matrix, the laser scanner and the photograph should be made from two taking points close to each other, in order to reduce the shaded areas, of which it is not possible to determine the 3D position.

In addition, it is possible to add to the previously described Solid Image another set of information. Recent laser scanners are also able to measure the reflectivity value for each point of the “cloud” (in terms of % of reflected laser beam).

This set of information is not yet frequently used in laser data treatment commercial software, but it should not be neglected when dealing with research. The reflectivity value is related to the type of material the object is made up of, an aspect that could be of fundamental importance in the analysis and development of 3D models.

These reflectivity values are known, as for the distances. This set of information can be interpolated and stored in a fifth “reflectivity matrix” A.

A Solid Image obtained using such modern laser scanners can therefore be made up of 5 matrices: R, G, B, D and A.

### 1.2 Calibration of the image

In order to fill the D and A matrices with correct distance and reflectivity values, it is first of all necessary to determine the image orientation (3 rotations + 3 translations) and the image calibration values (lens distortion, principal distance and coordinates of the principal point).

This can be achieved by measuring the image coordinates for a sufficient number of control points. The procedure can be made fully automatic if a set of high reflectivity markers is used to determine the control point positions both on the image and inside the DDSM [Bornaz, Rinaudo, Lingua 2002]. In this case, orientation and calibration parameters of the image are given in the laser scanner reference system.

### 1.3 Projection of the point cloud

In order to interpolate the D and A matrices, the first step consists in projecting each point of the DDSM onto the image. This operation consists in calculating, for each point of the DDSM, the corresponding image coordinates \( \xi, \eta \).

The mathematical model used to project the point cloud is that of the collinearity equations (in which the radial distortion components are added).

\[
\begin{align*}
\xi &= \xi_0 + \Delta \xi \\
\eta &= \eta_0 + \Delta \eta \\
\end{align*}
\]

Where: \( \xi_0, \eta_0 \) are the elements of the rotation matrix \( \xi, \eta \) are the image coordinates of the point \( X, Y, Z \) = the object coordinates of the projection centre \( c \) = the focal length \( \Delta x, \Delta y \) = the distortion components.

The distortion components are modelled by:

\[
\begin{align*}
\Delta \xi &= (\xi - \xi_0) \left( k_1 \cdot \rho^3 + k_2 \cdot \rho + k_3 \cdot \rho^3 \right) \\
\Delta \eta &= (\eta - \eta_0) \left( k_1 \cdot \rho^3 + k_2 \cdot \rho + k_3 \cdot \rho^3 \right)
\end{align*}
\]

Where: \( \rho \) = the distance of the pixels from the image centre \( k_1, k_2, k_3 \) = the radial distortion coefficients.

Other types of distortion have not been considered.

### 1.4 Interpolation of the distance and reflectivity matrices

The density of the pixels in the digital image is usually much higher than the density of the acquired point cloud. For this reason, when the point cloud is projected onto the digital image, the distance matrix is not completely filled: the distance values are only associated to some pixels (this
quantity depends on the density of the points acquired with
the laser scanner and on the image resolution in terms of dpi).
In order to fill the distance matrix, it is necessary to integrate
the missing values with an interpolation process. The
“average weighted method” has been used. The four nearest
pixels, of which the value of distance is known, are
considered. The interpolation of the distance values is carried
out using the formula:

\[ \delta_{ij} = \frac{d_1 \cdot \delta_1 + d_2 \cdot \delta_2 + d_3 \cdot \delta_3 + d_4 \cdot \delta_4}{d_1 + d_2 + d_3 + d_4} \]  

(5)

Where \( i, j \) = the indexes of the current pixel
\( \delta_{i...4} \) = the distances between the centre of the
perspective and each of the 4 object points
\( d_{1...4} \) = the distances, on the image, between
the \( i,j \) pixel and the pixels used for the
interpolation.

The same process is used for the interpolation of the
reflectivity matrix \( A \).

1.5 Validation of the procedure

A test area was used to calibrate and validate the proposed
method. It has been built in the photogrammetry laboratory at
the Politecnico di Torino. A total of 40 high reflectivity
markers, whose positions were accurately determined using
classical geodetic techniques, were realised in the laboratory.
A laser scanning was carried out, using a RIEGL LMS-Z420
laser scanner, whose accuracy is ±5 mm for the
coordinates. Simultaneously a series of digital images were
acquired, with a Nikon D1X mounted with a 28 mm lens.
After having carried out the survey, the orientation and
 calibration parameters of a digital image were calculated and
a Solid Image of the same image was created.
The image size is 1312 pixels in height and 2000 pixels in
width, the resolution is 2100 dpi, corresponding to a pixel
size of 12.1 μm.
The obtained results are:

Orientation parameters of the digital image

\[
\begin{align*}
X_0 &= -0.071 \ m & \omega &= -85.505 \ gon \\
Y_0 &= -0.038 \ m & \phi &= -298.742 \ gon \\
Z_0 &= 0.253 \ m & \kappa &= 114.379 \ gon
\end{align*}
\]

Calibration parameters (see (3) and (4))

\[
\begin{align*}
\xi_0 &= -0.160 \ Mm \\
\eta_0 &= 0.011 \ Mm \\
\xi &= 28.49 \ Mm
\end{align*}
\]

\[
\begin{align*}
k_1 &= 1,70376 \ E-06 \\
k_2 &= 3,98296 \ E-09 \\
k_3 &= 2,11153 \ E-12
\end{align*}
\]

First analysis was performed to investigate the accuracy of the
solid image.
All the possible distances between the markers placed on the
object were calculated and compared with those obtained
from the Solid Image.
The following results were obtained:

Mean and maximum differences

\[
\begin{align*}
\text{Minimum value} &= -8 \ mm \\
\text{Maximum value} &= 10 \ mm \\
\text{Mean value} &= 1.2 \ mm
\end{align*}
\]

2. SOFTWARE FOR THE CREATION AND MANAGEMENT OF A
SOLID IMAGE

2.1 The LSR 2004 software

A specific software package (LSR 2004) has been developed
by the authors in Visual Basic 6.0 to create and manage the
Solid Image and some of its first applications.
LSR 2004 (Laser Scanner Registration 2004) is a software
made up for terrestrial laser scanner data processing and the
creation and management of the Solid Image.

LSR 2004 allows:

- dense point clouds and high resolution digital images to be
  imported and exported;
- point clouds to be filtered to remove outliers and gross
  errors;
- the position of the high reflectivity points (markers)
  placed on the acquired object to be determined
  automatically;
- adjacent scans to be recorded and geo-referenced
  automatically using reference markers;
- triangulation to be carried out in the case of different
  scans;
- images to be calibrated;
- the Solid Image and its applications (3D colour model
  and artificial stereoscopic pair) to be created and
  managed;
- direct measurements to be carried out on the Solid
  Image.

LSR 2004 allows a Solid Image to be created through:

- an automatic calibration procedure of the digital image
- projection of the point cloud and
- interpolation of the D and A matrices.

It allows the operator to visualise the RGB image or each of
the two D or A matrices, codified in colour scales.

Figure 4 The LSR 2004 software – Solid Image of the
Tetrarchi statue in Venice

The resulting Solid Image is a high-resolution digital image
integrated with the 3D information of the photographed
object. The spatial position of the image points in terms of
X,Y,Z coordinates can be seen in real time. This is obtained
by simply moving the PC cursor onto the Solid Image: the
software reads the orientation and calibration parameters of
the image and the values contained in the R, G, B, D and A
matrices of each pixel of the image on which the cursor is
positioned and, using these values, it calculates the X, Y and
Z coordinates of the point. These values are made visible to
the operator, in real time, in a specific visualisation window
(see figure 4).

2.2 Calculation of a mean plane

LSR 2004 allows a set of measurements to be directly carried
out on the Solid Image. One of these measurements is the
computation of the mean plane of a set of points. This kind
of measurement could be useful, for example, for geologists,
who often have to determine the position of planes on the
external face of a rock mass, in order to understand the
internal fracture situation. It is in fact possible to
automatically determine the barycentre of the points and the
dip and dip direction values of the plane. The mean plane can
be calculated, using LSR 2004, in two different ways: by
choosing a set of points on the image or by defining an area
containing the points from which the mean plane can be
calculated (see figure 5).

3. Example of a Solid Image

3.1 Example of a Solid Image

The complete procedure for the creation of a Solid Image has
been performed on different objects acquired with laser
scanner devices. One of these objects is the front entrance of
the San Marco Basilica in Venice.

3.2 The survey

A complete survey of the front entrance has been performed
in order to obtain its Solid Image (topographic +
photogrammetric + laser scanner). A Riegl LMS-Z420 laser
scanner with an accuracy of ± 5 mm was used for the DDSM
acquisition. The minimum angular step is 10 mgon.
The laser scanner acquisition was carried out from a mean
distance of 20 m and with angular steps of 20 mgon, in about
1 hour 30 minutes.
In addition to the laser survey, a photographic survey of the
statue was performed using a semimetric Rollei 6008 camera
with a 28 mm lens.
A classical topographic survey was also carried out.

**Calibration of the image**

The image calibration was carried out as the first step. The estimation was performed using a bundle approach, using a software package made by our research group. In addition, the radial distortion coefficients have been estimated.

**Creation of a Solid Image**

When the calibration and orientation parameters of the digital image were known, the Solid Image creation was carried out. This procedure is completely automatic.

![Figure 8. The 3D model acquired with a laser scanner](image)

At the end of the process, it is possible to visualise the Solid Image, the D or the A matrices, which are codified in colour scale and to evaluate the 3D position of the points in real time. To do this, the cursor is simply moved by the operator onto the image.

![Figure 9. The distance matrix. The colour scale represents the different distances](image)

**Measuring the geometric elements**

It is easy to directly measure distances, angles and mean planes on the Solid Image, by selecting some points on the image (see fig. 10 and 11).

![Figure 10. Measuring the distance between two points](image)

![Figure 11. Measuring the mean plane](image)

**3.6 The artificial stereoscopic pair**

Another product that can be obtained from the Solid Image is an artificial stereo pair. This is obtained by defining a taking base that can be chosen by the operator.

A software developed by the authors based on the central perspective allows the original image to be projected onto two new images that form a stereoscopic pair. The interior orientation parameters of the two new images are the same as the original image while the external orientation parameters are slightly varied in order to realise a “normal taking”, with the base imposed by the operator and the axes of the virtual cameras perpendicular to this base and parallel to each other.
Some pixels do not have a radiometric value in the new images. This lack of information is due to prospective reasons: if the centre of the projection changes, some areas that were previously hidden in the original image could become visible in one of the two new images, but no information is available on these areas. In this case, the "holes" have to be represented as lack of information, for example in white, as there is no reason to carry out an interpolation process on these areas.

4. CONCLUSIONS

The Solid Image opens the way to a series of new products in the architectonic field and in the surveying of portions of the territory. A Solid Image easily allows:

- Determination, in real time, of the 3D position of any point in the image, using a simple PC;
- correct 3D measurements (lines, areas, volumes, angles, etc.) by only selecting some points of the image;
- extraction of profiles, sections, and planes using simple drawing functions on the image, even by inexperienced users;
- in general, the high quality colour images to be integrated with the 3D geometry of each pixel of the photograph.

A series of new applications of the Solid Image can be foreseen for various application fields.

5. BIBLIOGRAPHY

- L. Bornaz, A. Lingua, F. Rinaudo. High resolution digital image orientation using laser scanner data - Proceeding of the CIPA WG6 International Workshop on “Scanning for Cultural Heritage Recording” (pp. 52-57), Corfu

6. ACKNOWLEDGEMENTS

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