

# BLOCK BUNDLE ADJUSTMENT FOR THEODOLITE STATIONS IN CONTROL NETWORKS - THE CASE OF THE GUGGENHEIM MUSUEM IN BILBAO

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Commission V , WG V/4

**KEY WORDS:** Control Network, Surveying, Bundle Adjustment, 3D

## ABSTRACT :

In surveying missions the volume and the weight of the surveying equipment are often a problem. In a previous work, the A. proposed what he called blind traverse for control network assessment in close-range photogrammetry. It is a procedure that forms the so-called topographic models, linking together a couple of Theodolite Stations by their observations to the common points, similarly to what happens in photogrammetry with the photogrammetric models. The theodolite relative orientation has three +one unknown parameters, estimated by means of co-planarity conditions, observing a minimum of three common points. When all the possible couples of TS are formed, they are oriented in an absolute reference system with a procedure similar to independent model adjustment in aerial triangulation (S-transformations). The advantage consists in the possibility to avoid to link together the TS with the reciprocal observations, allowing to reduce the amount of needed equipment by more than 50% in volume and weights (from three tripods to one, etc). The term “blind” refers to the null visibility among TS. In the present paper, the analogy between photogrammetry and surveying is extended to block bundle adjustment. The angular observations of a theodolite stations are similar to an image bundle. Compared to the photogrammetric bundle, the unknown parameters for any single TS, are reduced to four only, say the three co-ordinates of the TS and the bearing of zero reading of the horizontal circle. The fundamental equations are the horizontal direction and vertical one. Since the approximated values of the co-ordinates for the unknown points are needed, they are supplied by the previous stage by the blind traverse. The benefits are again the independence of the theodolite stations and in comparison with the blind traverse a more adequate stochastical model and a better accuracy. A computer adjustment programme has been written and tested. The A. called it 3Dom, meaning 3d adjustment and freedom in the lay out of the control traverse. As an example the control network of the Guggenheim Museum in Bilbao is shown. The adjustment takes place in two steps: computation of the approximate co-ordinates by co-planarity (the blind traverse), and final adjustment by 3Dom.

## 1. THE TOPOGRAPHIC MODEL BY CO-PLANARITY

In previous papers, the A. introduced the so-called “topographic models”, (Fangi G, 1999). They consist in the following: taking advantage of the analogy with the photogrammetric model, the surveying observations of two (or more) theodolite stations, are coupled together by means of the co-planarity condition to the common aimed points.

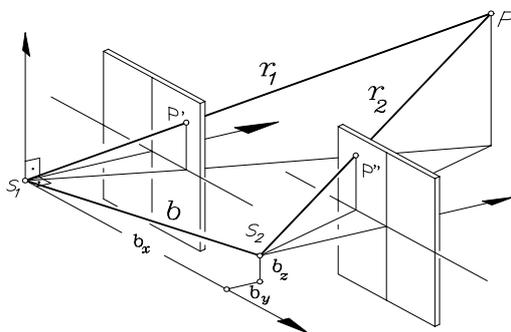


Figure 1 – The topographic model by intersection- The three coplanar vectors  $b$ ,  $r_1$  and  $r_2$ ..

When the topographic models are formed, they are oriented in the reference system, with a technique similar to the one of the aerial triangulation, say with the S-transformation. Obviously at least one distance is needed to correctly dimension the network. No approximate value is needed except the one for the zero bearing of the horizontal circle.

Therefore it is very advisable to roughly orientate the theodolite horizontal circle with the help of a magnetic compass. The benefit is the possibility to get rid of the reciprocal observations between adjacent stations, thus enabling a remarkable reduction of the needed equipment in terms of weight and volume.

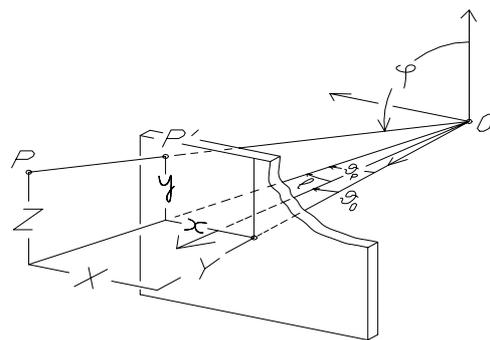


Figure 2 – Analogy theodolite-photogrammetric bundle:  $x$  and  $y$  image coordinates are equivalent to the angular measurements  $l$  and  $\phi$

Now the analogy between photogrammetry and surveying can be further extended to the block adjustment. A theodolite station can be compared to a photogrammetric bundle. In the three-dimensional adjustment, the orientation parameters per station are four compared to the six of the photogrammetric bundles. They are the three station co-ordinates  $X_0$ ,  $Y_0$ ,  $Z_0$  and the zero bearing  $\theta_0$  of the horizontal readings (figures 2,

3). The advantages are again the freedom in the selection of the theodolite stations, made independent from each other, as long as they observe a certain number of common points, like in photogrammetry no image is normally visible in any other image. The adjustment of a control network is similar then to a photogrammetric block bundle adjustment.

## 2 THE OBSERVATION EQUATIONS

Like in photogrammetry where the observations are the two image coordinates  $x$  and  $y$  per point, (that are indeed directions from the projection centre), the theodolite angular observations are the horizontal direction and the zenithal angle. In analogy with the co-linearity equations, we can set up two equations for the two direction, the horizontal one  $l$  and the vertical one  $\varphi$ . One can add, if it is the case, the equations of the measured distances. The surveyor is then free to place the theodolite where he likes. He needs only one tripod and the instrument only, and no more the complete traversing equipment. The survey can be greatly speed up. The disadvantage is that this procedure is more sensitive to gross errors that are more difficult to find and eliminate. In addition it can be difficult to estimate the approximated value of the co-ordinates for the unknown points. For this reason it is recommended to use the co-planarity to estimate the approximate coordinates of the unknown points. For any station it is very useful to measure the magnetic bearing by means of a compass, in order to get an approximate value of the bearing.

### 2.1 The Equation of the horizontal direction

The relationship that links the co-ordinates of the points A and B and the bearing  $\theta_{AB}$ , is:  
 $\theta_{AB} = \arctg((X_B - X_A)/(Y_B - Y_A))$ . (1)

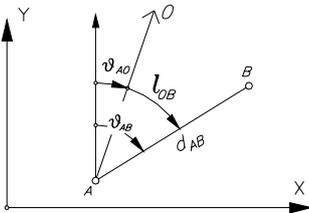


Figure 3 – Zero bearing  $\theta_{A0}$  and horizontal reading  $l_{OB}$

The discrepancy is the difference between the observed distance  $\theta_{AB}$  and its approximated value  $\theta^0_{AB}$ . Normally the bearing is not measured, since the direction of the  $Y$  axis is unknown; instead of it we measure the direction  $l_{OB}$  that is linked to the bearing by :  $l_{OB} + \theta_{A0} = \theta_{AB}$  (2)  
 Setting:  $X_A = X^0_A + dX_A, Y_A = Y^0_A + dY_A, Z_A = Z^0_A + dZ_A,$   
 $X_B = X^0_B + dX_B, Y_B = Y^0_B + dY_B, Z_B = Z^0_B + dZ_B,$  (3)  
 the last equation of the bearing becomes (after its linearisation about the approximate value  $X^0, Y^0, Z^0$ ) then the so-called equation of direction:

$$-\frac{Y_B^0 - Y_A^0}{d_{AB}^0} dX_A + \frac{X_B^0 - X_A^0}{d_{AB}^0} dY_A + \frac{Y_B^0 - Y_A^0}{d_{AB}^0} dX_B - \frac{X_B^0 - X_A^0}{d_{AB}^0} dY_B - g_{A0} = (l_{AB} - g^0_{AB}) + v$$
 (4)

To the traditional unknown co-ordinates another one is added, the so-called bearing of the origin  $\theta_{AB}$ .

### 2.2 The zenithal angle

The height  $Z_P$  of a point aimed from the station S of elevation  $Z_S$  is given by:

$$Z_P = Z_S + d_{SP} \text{ctg} \varphi_{SP} + \frac{1-k}{2R} d_{SP}^2$$
 (5)

deriving the zenithal angle  $\varphi_{SP}$ , we get:

$$\text{actg} \left[ \frac{\sqrt{(X_P - X_S)^2 + (Y_P - Y_S)^2}}{(Z_P - Z_S) - h_S + \Delta m_P} \left( 1 - \frac{Z_m}{R} \right) + \frac{1-k}{2R} \sqrt{(X_P - X_S)^2 + (Y_P - Y_S)^2} \right] = \varphi_{SP}$$
 (6)

with  $k$  refraction coefficient,  $Z_m = (Z_P + Z_S)/2$  mean height,  $R$  radius of the local sphere,  $h_S$  instrumental height in  $S$ ,  $\Delta m_P$  height of the target in  $P$

Its linearisation, neglecting the term  $Z_m/R$ , brings to:

$$\left[ \frac{(dZ_{SP}^0 - h_S + \Delta m_P)}{d_{SP}^0} + \frac{1-k}{2R} \right] \left[ \frac{X_P^0 - X_S^0}{d_{SP}^0} (dX_P - dX_S) + \frac{Y_P^0 - Y_S^0}{d_{SP}^0} (dY_P - dY_S) \right] + \frac{d_{SP}^0}{d_{SP}^0{}^2 + (dZ_{SP}^0 - h_S + \Delta m_P)^2} (dZ_P - dZ_S) = \varphi_{SP} - \varphi_{SP}^0$$
 (7)

with the position

$$\varphi_{SP}^0 = \text{actg} \left[ \frac{d_{SP}^0}{dZ_{SP}^0 - h_S + \Delta m_P} \right] + \frac{1-k}{2R} d_{SP}^0$$
 (8)

(Mussio, 1982). The unknowns are the variations of the coordinates  $dX, dY, dZ$  for the station S and for the aimed point P (6 in total). With this equation we take into account the existing correlation between the three coordinates. The correlations are strong, and then it is useful to consider them, in the case of local networks and of technical networks, where the visual directions are often very inclined. The estimate of the variance matrix of the unknowns is then more correct.

Provided that all the angular observations have the same accuracy, the weight of the observations will inversely be proportional to the square distance to the observed point.

### 2.3 The equation of the distance

Between two points S and P on the  $X, Y, Z$  plane, we can measure the slope distance  $d_{SP}$ . The equation of the distance, is then  $d_{SP} = ((X_P - X_S)^2 + (Y_P - Y_S)^2 + (Z_P - Z_S)^2)^{0.5}$  (9)

Its linearisation brings to:

$$\frac{X_P^0 - X_S^0}{d_{SP}^0} dX_S - \frac{Y_P^0 - Y_S^0}{d_{SP}^0} dY_S - \frac{Z_P^0 - Z_S^0}{d_{SP}^0} dZ_S + \frac{X_P^0 - X_S^0}{d_{SP}^0} dX_P + \frac{Y_P^0 - Y_S^0}{d_{SP}^0} dY_P + \frac{Z_P^0 - Z_S^0}{d_{SP}^0} dZ_P = (d_{SP} - d_{SP}^0) + v$$
 (10)

The normal system so formed will be singular since it is necessary to define a system.

To be able to determine the rank deficiency, it is necessary to keep in mind that in an altimetric problem it is equal to one, (one vertical translation), in a planimetric problem it is equal to three (two horizontal translations and one rotation in the  $x, y$  plane). In a spatial problem the rank deficiency is in general equal to six, however in this case for the nature of the observations it reduces again to four (three translations and a rotation in the plain  $xy$ ). Therefore a vertex and the direction to a second vertex have to be kept fixed.

### 3 THE APPROXIMATE CO-ORDINATES OF THE POINTS

The adjustment procedure requires the approximate value of the co-ordinates of the unknown points. They can be obtained in any possible way. One of the possible ways is then the procedure analogue to the aerial triangulation adjustment by independent models (Fangi, Thessalonic, 1999). Even with this computation the reciprocal visibility between the adjacent theodolite stations is not required. The computation takes place in two steps:

1. Formation of the models: By means of the co-planarity algorithm the three orientation parameters of one theodolite station with respect to a next one, are estimated. Then, by intersection of lines in space the relative co-ordinates of the intersected points are computed. Similarly to photogrammetry, the locus of the intersected points can be defined model, that, differently from the photogrammetric one, is already vertical, for the intrinsic nature of the geodetic measurements. There are two types of models: those formed by three vertices (three-section) and those formed by two models (bi-section). The computer programme tries first to form the trisected models, then the bisected models, comparing the observations of one station with all the other possible ones.

2. The final co-ordinates of the model points are worked out by means of an absolute orientation (four + one parameters). The ground co-ordinates of two points at least are then needed. When the models to be oriented are more than one, they can be linked with to procedure similar to the aerial triangulation with independent models. One model is equivalent to a traverse side. Compared to the triangulation in photogrammetry, the unknown parameters are less, all the equations are linear, and no iteration is needed.

No approximate value for any unknown parameter and coordinate is needed except the bearing of the zero reading for any theodolite station. It is the useful to get it with a magnetic compass.

### 4 THE COMPUTER PROGRAMME FOR ADJUSTMENT: 3Dom

A computer programme has been written called 3Dom (the name means three-dimensional adjustment, while the sound of the pronunciation is equal to freedom: the freedom to place the theodolite wherever it is useful and not where it is required to be visible from the other theodolite positions, freedom on the surveyor to travel with the least equipment). The programme is written in Fortran Power Station. It uses the Cholewky algorithm for triangularisation, solution of the normal system and inversion of the normal matrix. The design matrix is reduced, and coupled with a matrix of addresses (Mussio, 1984). In addition to the numerical report 3Dom has a graphical output, preparing a file with the extension .dxf. The ellipsoids of errors (Mussio, 1984) are drawn point-wise, along 20 meridians. The accuracies of the measuring instruments, theodolite for angles and distantimeter for distances, can be input.

### 5 THE CONTROL NETWORK OF THE GUGGENHEIM MUSEUM IN BILBAO

In the month of April 2004 a control network for the photogrammetric survey of the museum Guggenheim (figure 4) in Bilbao (Spain) has been set up. It is composed by

- 17 theodolite stations
- 751 points

We didn't make use of any target, choosing only natural points, taking advantage of the particular nature of the exterior panelling, composed by metallic plates of titanium, with very sharp edges (figure 5). The building is then an ideal environment to set up a very dense and accurate topographic network in short time.



Fig. 4 –The Guggenheim Museum in Bilbao.

From any traverse vertex the surroundings vertices have been observed with the help of a rod with a reflecting prism. A portable GPS Pathfinder Casio, with the accuracy of 10 m has been useful to get approximate positioning. In order to be able to observe the points of the top of the roof of the building, photographic images have been shot from the hill placed in the northern side of the museum. For the good orientation of the images taken with long focal length lenses, it is necessary to know the projection centre co-ordinates (Fangi, 1990, 1991). Therefore three theodolite stations have been placed rather far away from the museum, at a distance ranging from 500 m to 1.2 km. These stations have been linked to the net by resection in space. In this situation, with narrow angle resection, (figure 7), the geometrical configuration is poor and weak. Normally the determination leads to high values of variances of the co-ordinates.



Figure 5 – The exterior panelling in tiles of titanium helped in the selection of the observed point coincident with the vertices and with the edges of the buildings

Among the 751 observed points, some of them have been surveyed by irradiation (60) and mostly by intersection (691). We used a reflector-less theodolite. In spite of that, the remarkable distances did not allow the direct measure of the distance for mostly of the points, and we had to use the intersection to get the point coordinates.

#### 4.1 The Model Formation

The final adjustment with 3Dom has been preceded by the computation of the approximate co-ordinates with the model formation. The possible models were the combination of 17 coupled three by three  $(17/3)=680$  and the combination of 17 coupled two by two  $(17/2)=136$ , in total 816 possible topographic models could be formed; in reality only 27 of have been created by tri-section (three vertices), the remaining 29 by bi-section (two vertices). All of them have been oriented in the same reference system keeping fixed two vertices of the traverse. In this way the approximate co-ordinates of the points have been computed. In the following table the comparison between the co-ordinates by blind traverse and those got from the final 3Dom adjustment.

**Table 1** –Model Co-ordinates /Vs 3Dom co-ordinates (m)

- |                    |                 |                 |
|--------------------|-----------------|-----------------|
| 1) $s_x = .062$    | $s_y = .110$    | $s_z = .034$    |
| 2) $s_{x2} = .098$ | $s_{y2} = .140$ | $s_{z2} = .058$ |
| 3) $s_{xa} = .074$ | $s_{ya} = .129$ | $s_{za} = .092$ |
- 1)  $s_x$  = sample mean  
 2)  $s_{x2}$  = RMS value  
 3)  $s_{xa}$  = mean absolute values

In figure 6 the differences are shown, amplified 200 times. It is evident a systematic effect.

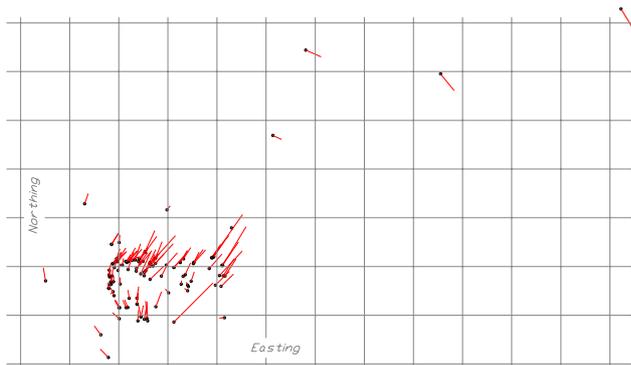


Figure 6. – The differences (amplified 200 times) between the model co-ordinates and 3Dom co-ordinates-

#### 4.2 The final Adjustments: 2+1 vs 3Dom

Two different types of adjustment have been performed, one with the traditional procedure where the planimetric computation takes place before the altimetric one iteratively. The used software was RETE (Fangi, 1996). The second adjustment by 3Dom software after removing all the reciprocal observations between the traverse vertices. 3D adjustment has the disadvantage to require more computer memory compared with 2+1 adjustment.

Due to the limited capacity of the computer Ram (512 Mbytes), all the points have been divided in three groups and the computation performed separately, taking some common points like the station points and those points with the maximum frequency. The features of the three adjustment blocks are shown in table 2

**Table 2** – The three groups of adjustment

	N points	Equations	Unknown	Redundancy
Group 1	128	812	404	408
Group 2	215	875	648	227
Group 3	278	1234	838	396

In table 3 the comparison of the sd. of the point co-ordinates by 2+1 adjustment with 3Dom adjustment. The low accuracy of the co-ordinates adjusted with traditional computation, can be explained with the fact that the theodolite stations are poorly linked together by reciprocal observations, not using the suitable equipment.

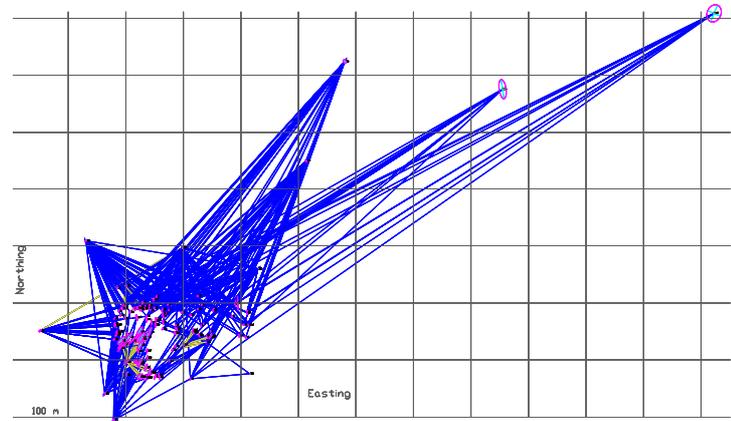


Figure 7 - The complete network

**Table 3** – Comparison RMS value for sd of all points

RMS value (mm)	$\sigma_x$	$\sigma_y$	$\sigma_z$
2+1 adjustment	34	24	14
3Dom	2	2	1

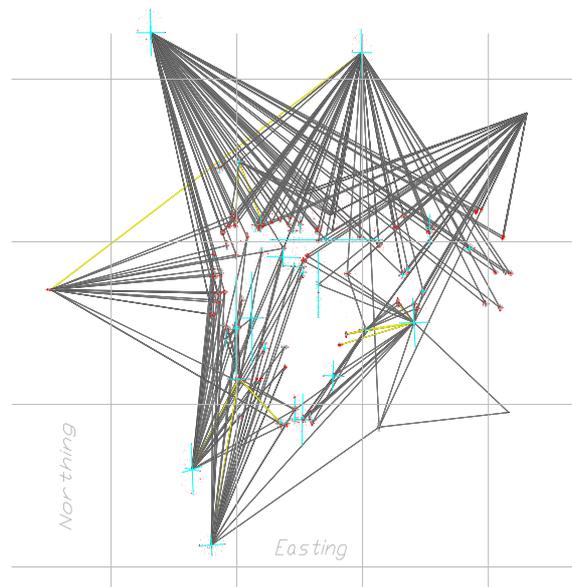


Figure 8. The central part of the control network of the Guggenheim museum in Bilbao with the ellipsoids of errors

The accuracy improves especially for narrow angle resection in space. In table 3 the sd. of the adjusted co-ordinates of far station point are shown.

Sd of the co-ordinates of the Far Stations – Comparison traditional 2+1 adjustment /vs. 3Dom adjustment (m)

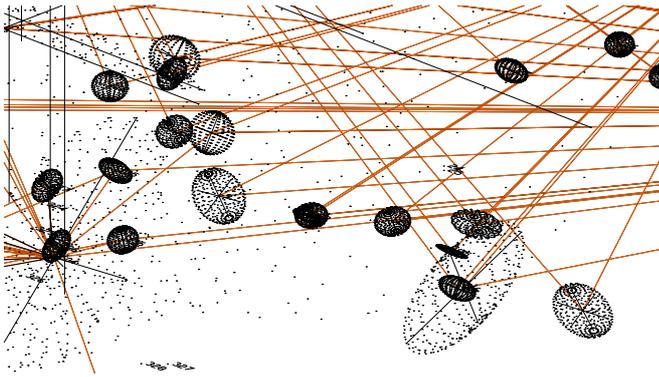


Figure 9 – A detail of the control network with the ellipsoids of errors



Figure 10 – The 756 points and 26 edges are visible, on the left the broken tower

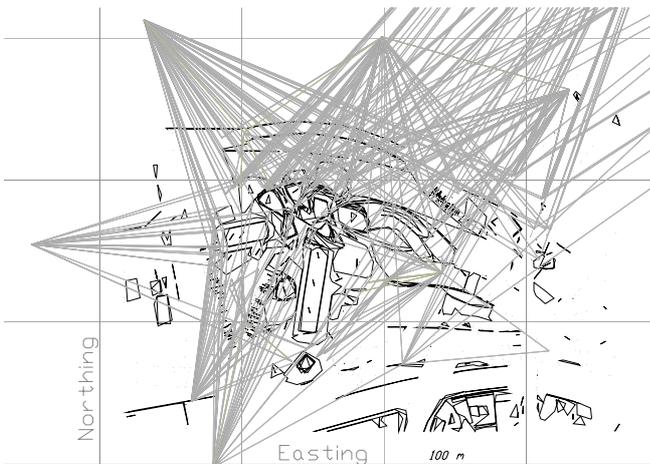


Figure 11 – The network superimposed to an existing map

Table 4-

Traditional adjustment	$\sigma_x$	$\sigma_y$	$\sigma_z$
14	$\pm 0.312$	$\pm 0.241$	$\pm 0.030$
15	$\pm 0.342$	$\pm 0.120$	$\pm 0.021$
16	$\pm 0.070$	$\pm 0.051$	$\pm 0.009$
3Dom Adjustment	$\sigma_x$	$\sigma_y$	$\sigma_z$
14	$\pm 0.014$	$\pm 0.009$	$\pm 0.016$
15	$\pm 0.018$	$\pm 0.012$	$\pm 0.035$
16	$\pm 0.004$	$\pm 0.003$	$\pm 0.001$

For resection in space the traditional adjustment gives the accuracy in the order on 0.1-0.3 m while the 3Dom adjustment brings the accuracy in the order of cm. The computation has been performed in a local reference system. After then the network has been adapted to the local datum.

## 5 CONCLUSIONS

The sd of the coordinates of the points are always inferior to 1 cm and in the 90% of the inferior cases to 0.5 cm. The  $s_0$  of the angular observations results 44 cc; in general the measure of the instrumental heights have strong influence on the genesis of the  $s_0$ . In the case of the 3dom adjustment with the independence of the theodolite stations, possible instrumental errors of measure of the height only have influence on the elevation of the occupied point and do not propagate.

The results are better in term of accuracy for the final adjusted coordinates compared to the traditional adjustment planimetry + altimetry. But the advantages are mainly in terms of operational conditions since there is the possibility do not to be obliged to inter-visibility between adjacent theodolite stations. The disadvantages are on the contrary the difficulty to estimate the approximate coordinates of the observed points. To find such coordinates the algorithm of co-planarity is suitable, avoiding again the constraint of the inter-visibility of the stations.

Finally the proposed procedure has an educational value showing the students the differences and the analogies between photogrammetry and surveying methods, algorithms and procedures.

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## Acknowledgements

The A. wants to thank the student Francesco Ieva for his help during the operations of survey.

Note: The present work has been financed by Cofin 2000

