

# ON IMPROVED GRAVITY MODELING SUPPORTING DIRECT GEOREFERENCING OF MULTISENSOR SYSTEMS

Dorota A. Grejner-Brzezinska<sup>a,\*</sup>, Yudan Yi<sup>a</sup>, Charles Toth<sup>b</sup>, Robert Anderson<sup>c</sup>, James Davenport<sup>c</sup>, Damian Kopcha<sup>c</sup> and Richard Salman<sup>c</sup>

<sup>a</sup>Department of Civil and Environmental Engineering and Geodetic Science, 470 Hitchcock Hall, Columbus, Ohio 43210, USA  
[dbrzezinska@osu.edu](mailto:dbrzezinska@osu.edu)

<sup>b</sup>Center for Mapping, The Ohio State University, USA

<sup>c</sup>National Geospatial-Intelligence Agency, Geospatial Sciences Division, St. Louis, Missouri, USA

## SS 3: Mobile Multi-Sensor Mapping Systems

**KEY WORDS:** GPS/INS, navigation, calibration, direct georeferencing.

### ABSTRACT:

Typically, inertial navigation systems assume the gravity field to be normal (ellipsoidal), meaning that the deflections of the vertical (DOV) are ignored in the gravity compensation procedure. This is one of the primary error sources in inertial navigation, especially detrimental in the stand-alone mode. Errors due to gravity field and system noise grow rather fast in the vertical channel, which normally requires some external aid (such as GPS or barometric altimeter), while the horizontal error growth is much slower and bound within the Schuler period. In this paper we present some preliminary results of testing GPS/INS navigation, supported by accurate external DOV information. The principal objective of our investigation is to determine to what extent accurate gravity data can improve georeferencing of airborne and land platforms, and how this accuracy propagates to a digital imaging sensor error model. The two primary interests addressed in this paper are: (1) the effect of accurate gravity information on the inertial sensor error estimation, and (2) the accuracy of stand-alone inertial navigation during a GPS outage with the DOV compensation. The high accuracy navigation grade LN 100 INS was tested in stand-alone mode and tightly integrated with dual frequency GPS data. The DOV compensation was performed using the unclassified 3D 2'x2' NGA (National Geospatial-Intelligence Agency) DOV grid, and tests comparing the navigation and calibration results with and without accurate gravity compensation under varying navigation conditions were analyzed. Due to the limited scope of this paper, only a sample of the airborne test results is presented, with a main focus on the land-based test results.

## 1. INTRODUCTION

Despite fundamental operational differences, GPS and inertial navigation systems (INS) are considered complementary positioning systems. GPS is, essentially, a geometry-based system, with the advantage of long-term position accuracy. Differential GPS, where systematic errors can be eliminated, can provide highly accurate cm-level position determination. Unlike GPS, an INS system is based on the laws of Newtonian physics and the initialization errors propagate throughout the trajectory. Although the long-term accuracy of a stand-alone INS cannot compare to that of GPS, its navigation solution is still necessary during the times of GPS signal loss. The GPS/INS systems based on high-quality inertial systems and supported by differential carrier phase GPS data can reach accuracies of a few centimeters per coordinate at the sensor's altitude (see, for example, Abdullah, 1997; Grejner-Brzezinska and Wang, 1998; El-Sheimy and Schwarz, 1999).

Typically, navigation algorithms consider the gravity field to be normal (ellipsoidal), meaning that the deflections of the vertical (DOV), defined as the difference between the actual gravity and the gravity model used (see Figure 1), are ignored in the gravity compensation procedure. This normally results in the inertial navigation error growth with time, especially detrimental for stand-alone INS operations. DOV is generally on the order of several arcseconds, except for in rugged terrain, and the global max/min values of 86/-113 arcsec ( $\xi$ , north DOV) and 108/-93 arcsec ( $\eta$ , east DOV) occur in the Himalayan region. In the US, particularly large DOVs occur in the Rocky Mountains and around trench regions (e.g., Hawaii). These are also the areas where the

DOVs change most rapidly. To limit the navigation error increase, some military systems incorporate active gravity field compensation, such as real-time DOV estimation from models. The horizontal error growth due to gravity field and system noise is much slower than in the vertical channel, and is bound within the 84.4-minute Schuler period. A typical horizontal error growth reaches 0.5-1.0 nm/hr for navigation-grade commercial systems. The vertical channel normally needs an external aid, such as GPS, to control its error growth.

### 1.1 DOV effects on inertial navigation

In this paper, the impact of the DOV compensation on sensor errors, position and attitude solutions is analyzed. A 3D 2' by 2' grid of NGA (National Geospatial-Intelligence Agency) DOV data was used in this study together with the WGS84 gravity model; DOVs were provided at eight altitudes: 0K, 10K, 15K, 20K, 30K, 50K, 70K, and 90K ft, and were interpolated for the sensor's altitude. The two main questions we attempt to answer are: (1) To what extent can accurate gravity information improve the accuracy of stand-alone inertial navigation during a GPS outage? (2) Can gravity compensation, combined with the INS static calibration technique (ZUPT, zero update point) provide better navigation accuracy during a GPS outage? We analyzed land-based and airborne test data representing different mission environments and dynamics. The results clearly indicate a positive effect of DOV compensation primarily on pitch and roll, but also on the horizontal coordinates. The details are discussed in section 2.

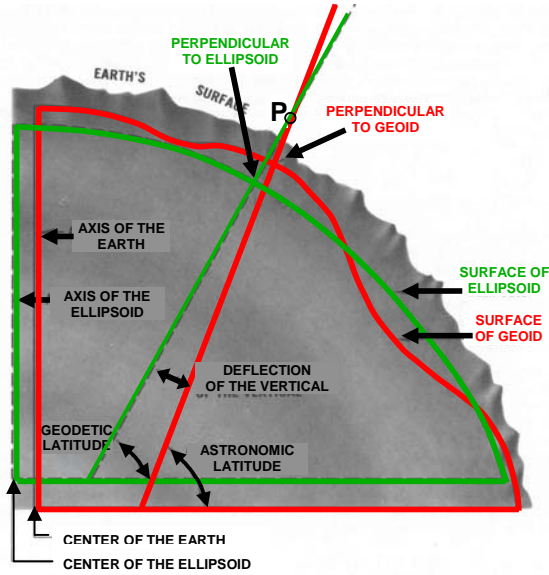


Figure 1. Deflection of the vertical.

Inertial navigation follows Newton's second law of motion defined in the inertial (nonrotating) frame (1):

$$\ddot{\bar{x}} = \bar{a} + \bar{g}(x) \quad (1)$$

$$\bar{g} = \bar{g}_m + \Delta\bar{g} \quad \text{and} \quad \Delta\bar{g} = \begin{bmatrix} -g_o\xi \\ -g_o\eta \\ \Delta g \end{bmatrix} \quad (2)$$

where  $\ddot{\bar{x}}$  = the total acceleration vector  
 $\bar{a}$  = the acceleration sensed by the accelerometer  
 $\bar{x}$  = the position vector  
 $\bar{g}(x)$  = the total gravitational acceleration vector  
 $\bar{g}_m$  = the gravity model  
 $\Delta\bar{g}$  = the difference between the actual gravity and the gravity model used  
 $g_o$  = the nominal value of gravity  
 $\xi$  and  $\eta$  = north and east DOVs, respectively  
 $\Delta g$  = the gravity disturbance, which corresponds to the gravity error  $\delta g$  (equation 3) in inertial navigation, if only the normal gravity term is used for gravity compensation.

Equation (3), expressing the navigation position errors to the first order due to errors in the system, is obtained by perturbing equation (1), i.e., by applying the differential operator,  $\delta$ . The solution of differential equation (3) provides expressions for the linearized error equations (Jekeli, 2001).

$$\delta\ddot{\bar{x}} = \frac{\delta g}{\delta x} \delta x + \delta g + \delta a \quad (3)$$

The primary observable provided by an accelerometer is the difference between kinematic inertial acceleration and mass gravitation; thus, errors in the observed accelerations are affected by errors in the gravity model used, translating to

the sensor positioning errors, as seen in equation (3). These, in turn, translate into errors in the coordinates of objects and points extracted from the directly oriented imagery, if a GPS/INS system is used to support a camera or a LiDAR (Light Detection and Ranging) system. Several models, ranging from normal gravity to high-order spherical harmonic expansion, can be used to approximate the Earth's gravity field. Historically, normal gravity has been sufficient for inertial navigation, as already mentioned. However, modern mapping systems based on high-accuracy GPS/INS may require better representation of the Earth's gravity in the navigation algorithm, especially during extended losses of GPS lock.

The total error dynamics equation in matrix form is as follows (Jekeli, 2001):

$$\frac{d}{dt} \varepsilon^n = F^n \varepsilon^n + G^n u \quad (4)$$

where superscript  $n$  denotes the navigation frame  
 $\varepsilon$  = vector of attitude, velocity and position errors  
 $u$  = vector of gyro, accelerometer and gravity errors, which can be estimated together in a GPS/INS filter (see, for example, Grejner-Brzezinska and Wang, 1998)  
 $F$  and  $G$  = free-inertial dynamics matrices of the system.

A detailed analysis of (4) reveals coupling among the unknowns that, in general, may complicate the estimation process (see, Jekeli, 2001). The errors in DOVs enter directly into the horizontal velocity errors in linear combination with the attitude errors. This, generally speaking, makes the parameter separation difficult in the estimation procedure (Grejner-Brzezinska and Wang, 1998). Thus, using DOVs in gravity compensation, which introduce less tilt error, leads to less coupling of the horizontal accelerations into the vertical axis. Therefore, it can be expected that (high-accuracy) DOV compensation should decrease not only the positioning error, but also improve the attitude determination.

## 2. PROCESSING STRATEGY AND TEST RESULTS

### 2.1 Test data and processing software

The GPS/INS system used in the analyses presented here is the OSU-developed AIMS<sup>TM</sup> system (see, for example, Grejner-Brzezinska and Wang 1998; Toth 1998). The positioning module of this system is based on a tight integration of dual frequency differential GPS carrier phases and raw velocity and angular rates provided by a medium-accuracy and high-reliability strapdown Litton LN-100 INS. LN-100 is based on Zero-lock<sup>TM</sup> Laser Gyro (ZLG<sup>TM</sup>) and A-4 accelerometer triad (0.8 nmi/h CEP, gyro bias – 0.003°/h, accelerometer bias – 25µg). The land based GPS/INS data used in this study were collected on January 31, 2001 near the OSU campus and the airborne data set was collected in Tucson, Arizona on May 6, 2002. The average DOVs along the land trajectory were about 6 arcsec ( $\eta$ ) and below 0.5 arcsec ( $\xi$ ); and 4–6 arcsec ( $\eta$ ) and 3–4 arcsec ( $\xi$ ) for the airborne test, with a sigma of 1 arcsec. Figure 2 illustrates  $\eta$  together with the corresponding positioning error for the land-based test.

## 2.2 Test solutions and their analyses

The following solutions were obtained and compared: (1) free INS navigation solution, (2) free INS solution supported by DOVs (3) GPS/INS solution (“true” reference), (4) free INS solution supported by DOVs and ZUPTs (land-based only), (5) GPS/INS solution supported by DOVs (gravity-enhanced reference solution).

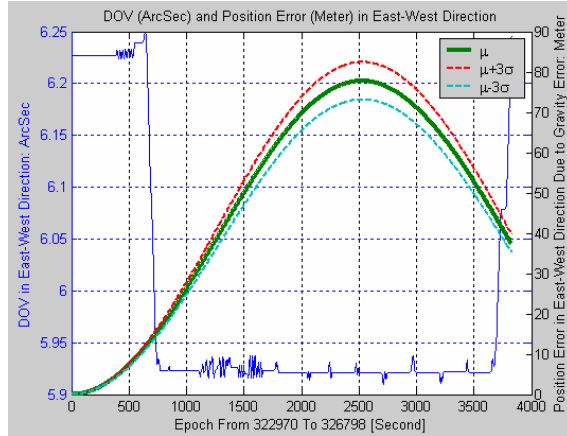


Figure 2. East-West DOV and corresponding position error (mean DOV ( $\mu$ ) was used) bounded by  $\pm 3\sigma$ .

## 2.3. DOV effects on gyro and accelerometer errors

Figures 3a and 3b illustrate the actual DOV signature along the land-based trajectory, and the gravity errors in N, E and D directions, estimated by the Kalman filter in the case when only normal gravity was used in the compensation procedure (solution 3). Figure 4 displays the difference between the true and the estimated DOVs, and the difference in the third component, the gravity disturbance, which is the actual difference between the two estimates (solution 3 and 5) since the gravity disturbance is not compensated for together with DOVs, for the airborne data set. For clarity, note that once DOVs are compensated for, they are fixed to a sigma of  $\pm 1$  arcsec in the GPS/INS filter, and only the gravity disturbance is estimated.

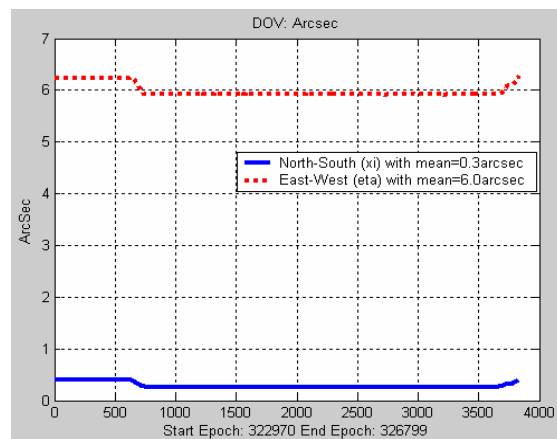


Figure 3a. DOV for land-based GPS/INS dataset.

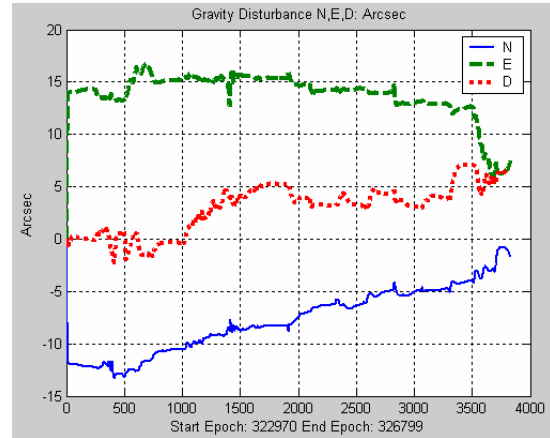


Figure 3b. DOVs and gravity disturbance (D) estimated by GPS/INS filter with no DOV compensation (solution 3); land-based data set.

Figures 3a-b and Table 1 show the difference between the estimated DOVs and the true DOVs at the level of  $\sim 8$  arcsec. Clearly, the estimated DOVs are substantially different from the true values, indicating that the estimates absorb some signal from other parameters in the state vector. This suggests that introducing DOVs directly may remove noticeable errors in tilt and possibly in gyro and accelerometer error estimates, as these components are correlated. Indeed, further analysis of the differences between the IMU errors obtained with the two solutions (3 and 5), with and without DOVs, indicates that direct DOV compensation affects the sensor error estimates. Figures 5-6, for example, illustrate the accelerometer bias and its RMS estimated by GPS/INS filter with no DOV compensation, and Figure 7 displays the difference between the accelerometer bias from Figure 5 and the one estimated by the filter where DOV compensation was included. Figure 8 illustrates the corresponding RMS improvement due to the DOV compensation.

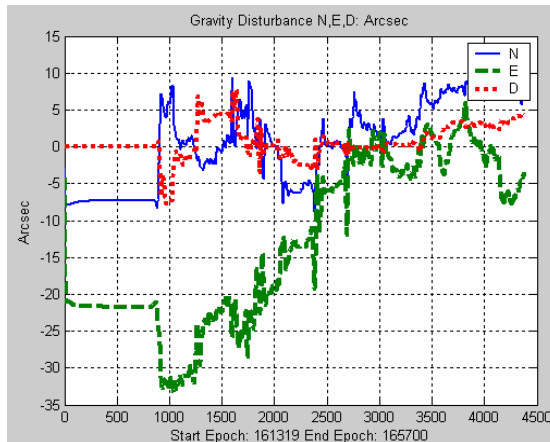


Figure 4. Difference between the true and the estimated (solution 3) DOVs; D is the difference between the gravity disturbance estimated by solutions (3) and (5); airborne data set.

Error Difference	Mean	Std	Units	Change wrt no DOV comp. (%)
AccBias X	-0.0	0.5	μg	0.0
AccBias Y	-0.0	0.4		0.1
AccBias Z	-1.1	0.4		34.4
AccSF X	15.8	5.0	ppm	111.3
AccSF Y	-0.4	2.1		3.2
AccSF Z	1.7	0.7		6.8
GyroBias X	0.6	2.0	arcsec	0.4
GyroBias Y	4.2	5.2	per hour	7.3
GyroBias Z	-12.0	6.0	hour	2.7
GravErr N	7.4	2.8	arcsec	103.8
GravErr E	-7.9	2.3		56.9
GravErr D	0.6	0.2		17.2
RMS Difference				
AccBias X	-0.2	0.1	μg	7.8
AccBias Y	-0.1	0.0		7.6
AccBias Z	-0.0	0.0		0.2
AccSF X	-1.4	0.3	ppm	6.2
AccSF Y	-0.8	0.2		3.5
AccSF Z	-0.3	0.0		1.3
GyroBias X	-0.4	0.4	arcsec	3.0
GyroBias Y	-0.5	0.5	per hour	3.6
GyroBias Z	-0.4	0.2	hour	1.2
GravErr N	-4.8	0.1	arcsec	100.0
GravErr E	-4.4	0.2		100.0
GravErr D	-0.1	0.0		3.3

Table 1. Error estimation difference between solutions (5) and (3) (the difference in DOVs is between the true DOVs and DOVs estimated in solution (3)).

#### 2.4. DOV effects on navigation solution

A comparison of the GPS/INS solution enhanced by the DOV data with the GPS/INS-only solution indicates that the effect of DOVs on position estimates is below a cm, while pitch and roll are improved (~4 arcsec RMS improvement) by applying DOV compensation. More visible effects of using DOVs can be observed by comparing the free-navigation INS solution with the INS solution supported by DOVs, as presented in Figures 9-10. Clearly, height and heading are not visibly affected by external DOV information. To fully assess the impact of DOV compensation on free-INS navigation, we compared solutions with different durations of GPS signal blockage (30 to 360 s) and different times of INS calibration with the GPS signal before loss of lock (~1300 s and ~460 s calibrations were considered). As illustrated in Table 2, a longer calibration period prior to a GPS gap may contribute to a slower error growth during the gap, especially for longer gaps; also, the effects of DOV compensation become more visible for longer GPS gaps (30 and 60-s gaps were tested but they show no visible impact on the results, and are not included in Table 2). For example, a 120 s gap with a 1300 s prior calibration results in cm-level improvement in horizontal coordinate RMS, while a gap of 360 s shows an ~21 cm improvement. Heading does not seem affected, and pitch and roll improvement is at the level of ~4 arcsec for all cases.

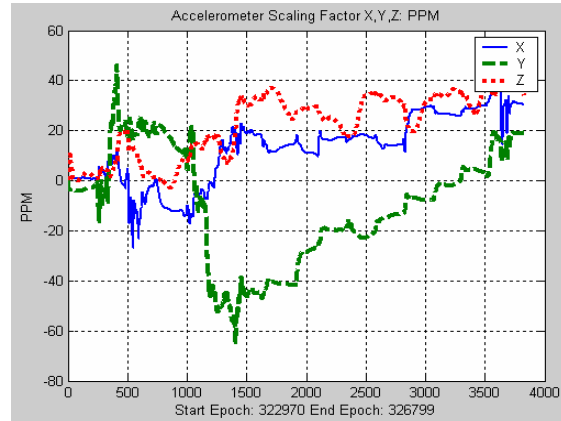


Figure 5. Accelerometer scale factor estimated by GPS/INS filter; no DOV compensation.

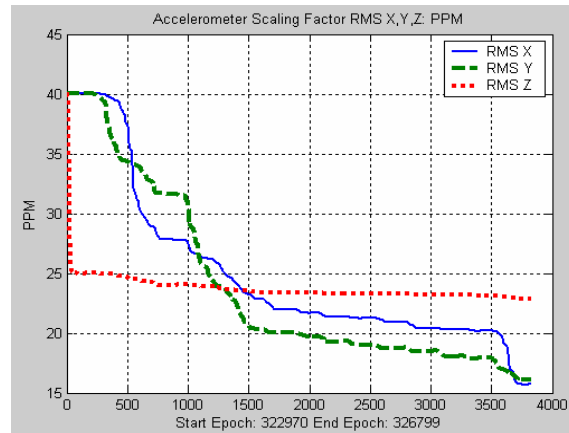


Figure 6. RMS for accelerometer scaling factor (SF) from Figure 5.

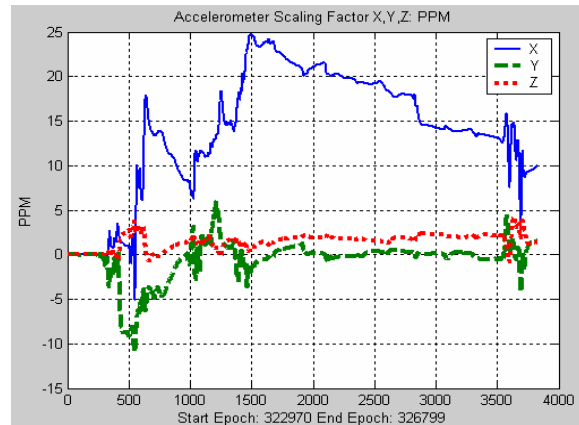


Figure 7. Difference in accelerometer SF between solutions with and without DOV compensation.

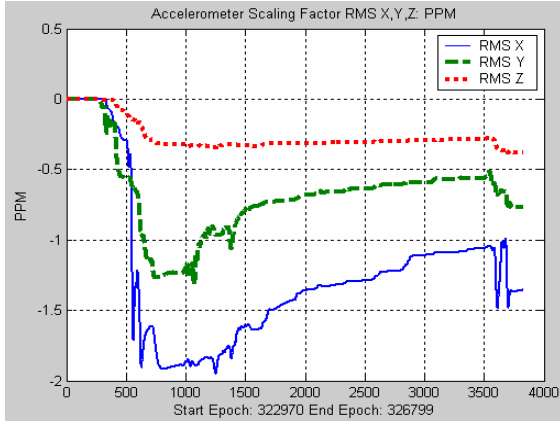


Figure 8. Difference in RMS of accelerometer SF corresponding to Figure 7; negative sign indicates improvement due to DOV compensation.

Further improvement in the INS/DOV solution can be achieved by applying periodic ZUPTs. ZUPT seems to have relatively more impact on the position coordinates, as compared to the attitude angles. The calibration performed during ZUPT also affects the INS/DOV trajectory portion that follows the ZUPT event, and thus, the difference between the two solutions still exists even though both solutions are based on INS/DOV only after the ZUPT event. For more details on ZUPT effects on INS navigation accuracy, see Grejner-Brzezinska et al. (2001). The effects of using DOVs in the navigation algorithm and performing ZUPTs to calibrate the (observable) errors can be clearly seen by comparing the reference GPS/INS/DOV solution under a favorable GPS constellation with the corresponding INS/DOV/ZUPT solution. The INS/DOV/ZUPT solution was calibrated by GPS prior to turning off the GPS signal; no ZUPTs were performed for the GPS/INS/DOV solution, even for the static portion of the trajectory. Our tests indicate that the free INS solution supported by DOVs and ZUPTs is capable of providing horizontal coordinates within an absolute difference of 1-3 cm (Figures 11-12), as compared to the reference “truth” (GPS/INS/DOV), while the attitude angles compare at a 1 arcsec level (Table 3). More details on the impact of the DOV compensation on the navigation solution can be found in Grejner-Brzezinska et al. (2003).

### 3. SUMMARY AND CONCLUSIONS

Based on the analyses summarized in this paper it can be concluded that the attitude components (primarily pitch and roll) are more affected by DOVs than the position coordinates, and the effect is more pronounced during the loss of GPS lock. The combined effects of DOVs and ZUPTs were analyzed for the land-based data set, indicating that while DOVs influence primarily the attitude, the ZUPTs have more impact on the position solution. It was also demonstrated that the use of DOVs and ZUPT calibration during the loss of GPS lock is capable of bringing the combined solution to accuracy comparable with the reference GPS/INS/DOV solution. The data sets used here were collected in test areas with relatively small DOVs; still their effect on the sensor errors and ultimately on the position and attitude solutions is visible. More tests are needed in areas with larger DOV magnitude and variation.

Component	Mean	Std	Max	Min	Units
120-second test duration, 1300 s prior calibration					
RMS N	-10	11	0	-39	mm
RMS E	-11	12	0	-41	mm
RMS Ht	0	0	0	0	mm
RMS Vn	0	0	0	-1	mm/s
RMS Ve	0	0	0	-1	mm/s
RMS Vd	0	0	0	0	mm/s
RMS Head.	0	0	0	0	arcsec
RMS Pitch	-4	0	-4	-4	arcsec
RMS Roll	-4	0	-4	-4	arcsec
360-second test duration, 1300 s prior calibration					
RMS N	-214	230	0	-793	mm
RMS E	-218	234	0	-806	mm
RMS Ht	0	0	0	-1	mm
RMS Vn	-2	2	0	-6	mm/s
RMS Ve	-2	2	0	-6	mm/s
RMS Vd	0	0	0	0	mm/s
RMS Head.	0	0	0	0	arcsec
RMS Pitch	-4	0	-3	-4	arcsec
RMS Roll	-4	0	-4	-4	arcsec

Table 2. Position and attitude accuracy improvement between the INS/DOV and INS-only solutions, summary of statistics; land-based test.

Difference	Mean	Std	Max	Min	Units
North	13	31	55	-75	mm
East	16	15	38	-13	mm
Height	-26	35	67	-101	mm
Heading	0	0	1	-1	arcsec
Pitch	0	0	1	0	arcsec
Roll	0	0	1	-1	arcsec

Table 3. Position and attitude difference between solutions (5) and (4); summary of statistics (land-based test); 85-s ZUPT.

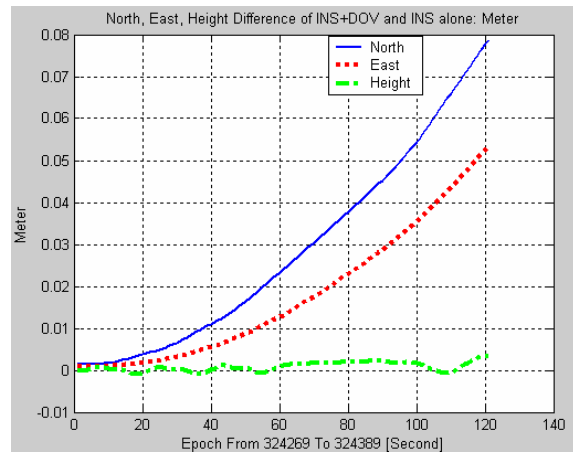


Figure 9. Coordinate difference between INS/DOV and INS-only solutions after ~1300 s of GPS-based calibration; land-based test.

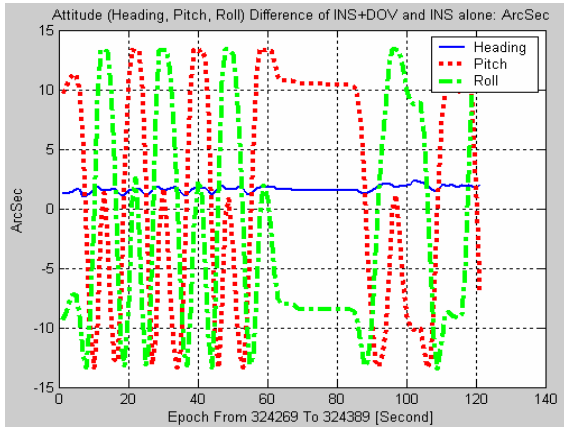


Figure 10. Attitude difference between INS/DOV and INS-only solutions after ~1300 s of GPS-based calibration; land-based test.

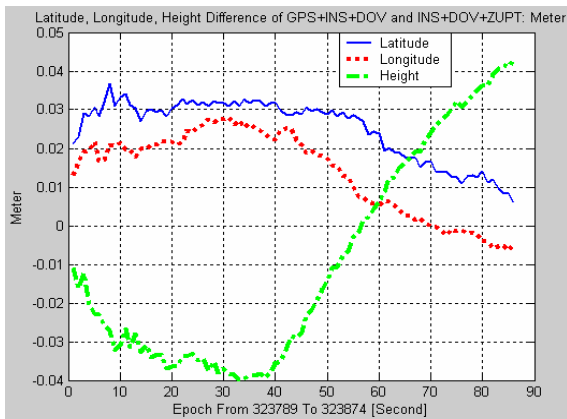


Figure 11. Coordinate difference between GPS/INS/DOV and INS/DOV/ZUPT solutions (85-s ZUPT event); land-based test.

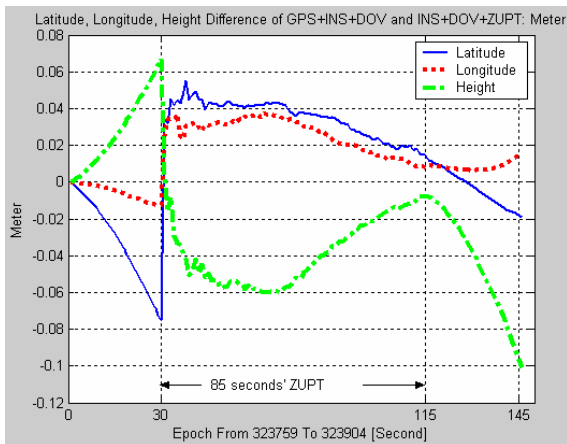


Figure 12. Coordinate difference between GPS/INS/DOV and INS/DOV/ZUPT solutions; land-based test; 30-s free INS navigation shown before and after the ZUPT event.

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