A GENERAL REVIEW OF THE DEFORMATION MONITORING TECHNIQUES AND A CASE STUDY: ANALYSING DEFORMATIONS USING GPS/LEVELLING

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ABSTRACT:

Being sure is very important that the movements of an engineering structure, which serves the human life of today's modern world, are exhibiting safe behaviours. So, a lot of deformation monitoring studies for determining and analysing different kinds of engineering structures such as high-rise buildings, dams, bridges, viaducts, industrial complexes etc., are implemented. During these studies, the used measurement techniques and systems, which could be geodetic or non-geodetic, are determined considering the type of the structure of which deformations will be monitored, its environmental conditions and expected accuracy from the measurements. As related the used monitoring techniques, the deformation measurement equipments are varied. Also according to professions who use the deformation monitoring techniques, these techniques and instrumentation have traditionally been categorized in to two groups: *geodetic surveys*, which include conventional (terrestrial such as precise levelling measurements, angle and distance measurements etc.), photogrammetric (terrestrial, aerial and digital photogrammetry), satellite (such as Global Positioning System-GPS, InSAR), and some special techniques; *geotechnical/structural measurements*, using lasers, tiltmeters, strainmeters, extensometers, joint-meters, plumb lines, micrometers etc. In this paper, some of these deformation measurement techniques which are thought as more important and mostly used by the geodesy specialists will be reviewed. The importance and need of carrying out the deformation measurements periodically in engineering structures will be emphasised.

Besides, a case study that is about implementing the deformation analysis of a large viaduct using GPS and Precise Levelling measurements will be discussed here. As it is well known, engineering structures (such as in this viaduct) are subject of deformation due to factors such as changes of ground water level, tidal phenomena, tectonic phenomena etc. In this study, the design, execution and analysis of deformations in a high-way viaduct are going to be mentioned in detailed explanations as an example of implementing the two geodetic techniques in deformation monitoring of large engineering structures. During the study, the control network points were positioned with GPS measurement technique and height differences were supported with precise levelling measurements. As the result of measurement campaigns, the X, Y, Z cartesian coordinates and height differences were determined from the GPS measurements and precise levelling measurements respectively. Later on, deformation analysis using the height differences according to provided data from the GPS and the data from the precise levelling were carried out separately. Then, the 3D deformation analysis using the GPS measurements data was carried out too. Founded results will be given in the paper.

1. INTRODUCTION

Engineering structures (such as dams, bridges, viaducts, high rise buildings, etc.) are subject to deformation due to factors such as changes of ground water level, tidal phenomena, tectonic phenomena, etc. Monitoring and analyzing deformations of these structures constitutes a special branch of Geodesy Science. There are several techniques for measuring the deformations. These can be grouped mainly into two as geodetic and non-geodetic techniques.

Each main measurement technique has its own advantages and drawbacks. Geodetic techniques, through a network of points interconnected by angle and/or distance measurements, usually supply a sufficient redundancy of observations for the statistical evaluation of their quality and for a detection of errors. They give global information on the behaviour of the deformable structure while the non-geodetic techniques give localized and locally disturbed information without any check unless compared with some other independent measurements. On the other hand, the instruments, which are used in non-geodetic

measurements, are easier to adapt for automatic and continuous monitoring than conventional instruments of geodetic measurements. Geodetic techniques have traditionally been used mainly for determining the absolute displacements of selected points on the surface of the object with respect to some reference points that are assumed to be stable. Non-geodetic techniques have mainly been used for relative deformation measurements within the deformable object and its surroundings (Anonym, 2002).

The major motivation of this study is geodetic techniques in monitoring deformations of engineering structures and analyzing. In determining deformations according to geodetic techniques constitutes terrestrial measurement techniques or space based positioning techniques and/or combination of both techniques. Until the very beginning of 1980's, the deformations in engineering structures had been determined just using conventional measurement techniques. After that, by starting to use GPS measurement technique in geodetic and surveying applications, this very precise satellite based

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positioning technique has become to use in deformation measurements (Erol, 1999).

GPS technique has benefits of high accuracy and simultaneous 3-D positioning; however there are handicaps about vertical positioning using this technique. Because, the height component is the least accurately determined GPS coordinate, predominantly due to inherent geometric weakness and atmospheric errors (Featherstone et al., 1998; Çelik et al., 2001).

Therefore, using GPS measurement technique in deformation measurements with millimeter level accuracy requires some special precautions that increase the measurement accuracy in GPS observables via eliminating or reducing some error sources such as using forced centering equipments, applying special measuring techniques like rapid static method for short baselines or designing special equipments for precise antenna height readings (Erol and Ayan, 2003).

In some cases, even these special precautions might be insufficient to reach the necessary accuracy level; at that time to support GPS measurements with another measurement technique would be very useful as an improving solution.

In this study, 1D and 3D deformation analysis of a large viaduct using GPS and Precise Levelling measurements are implemented. The control network points were positioned with GPS measurement technique and height differences were supported with precise levelling measurements. As the result of measurement campaigns, the X, Y, Z cartesian coordinates and height differences were determined from the GPS measurements and precise levelling measurements respectively. Later on, deformation analysis using the height differences according to provided data from the GPS and the data from the precise levelling were carried out separately. Then, the 3D deformation analysis using the GPS measurements data was carried out too. Explanation on used analysis methods and founded results will be given in addition to general review of deformation analysis methods in the following sections.

2. OVERVIEW OF DEFORMATION MEASUREMENT TECHNIQUES

As it is mentioned in the introduction part, measurement techniques were divided mainly into two different groups as geodetic and non-geodetic techniques. These main techniques can also be divided sub-techniques. In the following, it can be found short descriptions of the used techniques in deformation measurements.

GPS; Global Positioning System offers advantages over conventional terrestrial methods. Intervisibility between stations is not strictly necessary, allowing greater flexibility in the selection of station locations than for terrestrial geodetic surveys. Measurements can be carried out during night or day, under varying weather conditions, which makes GPS measurements economical, especially when multiple receivers can be deployed on the structure during the survey. With the recent developed rapid static positioning techniques, the time for the measurements at each station is reduced to a few minutes (Anonym, 2002).

Photogrammetry; If an object is photographed from two or more survey points of known relative positions (known coordinates) with a known relative orientation of the camera(s), relative positions of any identifiable object points can be determined from the geometrical relationship between the intersecting optical rays which connect the image and object points. Aerial photogrammetry has been extensively used in determining ground movements in ground subsidence studies in mining areas, and terrestrial photogrammetry has been used in monitoring of engineering structures. The main advantages of using photogrammetry are the reduced time of field work; simultaneous three dimensional coordinates; and in principle an unlimited number of points can be monitored (Anonym, 2002).

Tilt and Inclination Measurements; The measurement of tilt is usually understood as the determination of a deviation from the horizontal plane, while inclination is interpreted as a deviation from the vertical. The same instrument that measures tilt at a point can be called either a tiltmeter or an inclinometer depending on the interpretation of the results (Anonym, 2002).

The some of new techniques for deformation monitoring can be listed as follows.

Insar; Elevations can be determined from Synthetic Aperture Radar (SAR) images by interferometric methods. This involves the use of two antennas, displaced either vertically or horizontally, installed on the same satellite or aircraft platform. One of the antennas transmits the signal, but both receive it, resulting in two images being created. The most accurate form of interferometric measurement is differential interferometry (InSAR), which involves the determination of elevation differences between two epochs of terrain measurement. In this case, the variations in the radar signal phases are determined between the two epochs, which reveal terrain surface deformations that may have occurred between the two occasions when the images were recorded. It is claimed that height differences as small as one centimeter can be detected by this method. Such a technique therefore has the potential of being a cost effective, near-continuous, remote method of measuring terrain subsidence due to mining, and ground movement due to land subsidence, earthquake or volcanic activity, etc. (http://www.gmat.unsw.edu.au/snap/work/insar.htm, accessed May 2004).

Pseudolite; It is well known that for GPS-based deformation monitoring systems, the accuracy, availability, reliability and integrity of the positioning solutions is heavily dependent on the number, and geometric distribution, of the satellites being tracked. However, in some situations, such as in urban canyons, monitoring in valleys and in deep open-cut mines, the number of visible satellites may not be sufficient to reliably determine precise coordinates. Furthermore, it is impossible to use GPS for indoor applications and due to limitations of the GPS satellite geometry; the accuracy of the height component is generally 2 or 3 times worse than the horizontal components. These factors make it difficult to address GPS deformation monitoring applications in areas where the number of visible satellites is limited or satellite geometry is poor, especially where real-time high accuracy height component monitoring is needed, as in such applications as ground subsidence or deformation monitoring of man-made structures. Therefore, in order to improve the performance of GPS-only deformation monitoring systems, the integration of GPS with other technologies needs to be investigated.

Pseudolites (pseudo-satellites), which are ground-based transmitters of GPS-like signals, can significantly enhance the

satellite geometry, and even replace the GPS satellite constellation in some circumstances (such as deformation monitoring indoors).

The geometry of the satellite constellation can be improved by the careful selection of the pseudolite location(s). In the case of GPS, the measurements with low elevation angles are usually rejected in order to avoid serious multipath, tropospheric delay and ionospheric bias. However, this is not necessary in the case of pseudolites. The quality of the measurements with less than half degree elevation angle (from the pseudolite transmitter to the GPS receivers) is still very high. Therefore, high quality pseudolite measurements with low elevation angles, when included in data processing, can be expected to significantly improve the ambiguity resolution performance and solution accuracy, especially in the height component. The availability is also increased because a pseudolite provides an additional ranging source to augment the GPS constellation (Dai et al., 2001).

Laser Scanning; Existing techniques (e.g., surveying, GPS) used to monitor large structures such as buildings, viaducts, dams and bridges, while very accurate, are greatly hindered by their low point density. Data acquisition time limits monitoring to only a few samples located at strategic points on the structure. Ground-based laser scanning is a new technology that allows rapid, remote measurement of millions of points, thus providing an unprecedented amount of spatial information. This in turn permits more accurate prediction of the forces acting on a structure. As an emerging technology though, several issues concerning instrument calibration, sensitivity analysis, data processing and data filtering techniques require investigation (http://www.cage.curtin.edu.au/~geogrp/projlaser.html, May 2004).

For any particular application of deformation measurements, the most appropriate technique (or combination of techniques), which are going to be used, are determined as related to type of the structure, required accuracy and also economical aspects.

3. NETWORK AND DATA

In this study, the deformations of the Karasu viaduct were investigated using GPS and precise levelling data. Karasu viaduct is 2160 m in length. As the longest viaduct of the Turkey, It is located in the west of Istanbul in one part of the European Transit Motorway. The first 1000 meter of this viaduct crosses over the Büyükçekmece Lake and the piers of the structure were constructed in to this lake (see Figure 1).

The viaduct consists of two separate tracks as northern and southern and was constructed on 110 piers (each track has 55 piers). There is 40-meter width between two piers and also one deformation point is constructed with in every 5 piers sequence.

The deformation measurements of Karasu involved four measurement campaigns. The first campaign was carried out in June 1996, the second in March 1997, the third in October 1997 and the last one in April 1998. These four campaigns include GPS measurements and precise levelling measurements. With the aim of investigating the deformations of this structure, before carrying out the measurement campaigns, a well designed local geodetic network had been established, and it was measured using GPS technique according to designed

session plan. Also, precise levelling measurement technique was applied between network points.

During GPS measurements, Trimble 4000 SSI and Leica System 300 dual frequencies receivers were used. Leveling measurements were carried out using Koni 007 precise level.

The network has 6 reference points, set around the viaduct and 24 deformation points, set on the building of the viaduct and they are established especially on the piers where expected to be most stable places on the structures (see Figure 1).
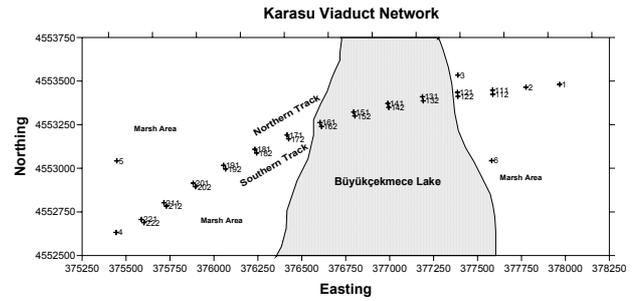
Figure 1: The configuration of geodetic network.

4. DEFORMATION ANALYSIS USING HEIGHT DIFFERENCES

With the aim of determining the deformations in engineering structures, landslide areas, crustal deformations etc., the geodetic networks are built. The observations are carried out in this geodetic network with certain intervals, and by this way stable network points and instable network points are verified. This provides to determine the changes on the observed structure or area (Demirel, 1987).

In general, the deformation analysis is evaluated in three steps in a geodetic network. In the first step, the measurements, which were carried out in t_1 and t_2 measurement epochs, are adjusted separately according to free adjustment method; outliers and systematic errors are detected and eliminated in this step. In the second step, global test procedure is carried out and by this test it is ensured that if the network point, which were assumed as stable, stayed really stable in the $\Delta t = t_2 - t_1$ time interval or not. In the global test, after the free adjustment calculations of the networks separately, the combined free adjustment is applied to both epoch measurements. (Ayan, 1982; Ayan et al., 1991).

After determining a group of stable points as the result of global test, following step of the analysis is the localizing of height changes. For doing this, T_H test values are calculated for the every network points, except the stable points, and they are compared with F critical value that is given in the Fisher distribution table (Erol and Ayan, 2003).

$$d = H_2 - H_1 \qquad S_0^2 = \frac{v_1^T P_1 v_1 + v_2^T P_2 v_2}{f_1 + f_2} \qquad f_0 = f_1 + f_2$$

$$T_H = \frac{d^T Q_{dd}^{-1} d}{r S_0^2} \qquad Q_{dd} = Q_{H_1 H_1} + Q_{H_2 H_2} \qquad r = f_G - f_0$$

If the $T_H > F_{r, f_0, 1-\alpha}$, it is said that the height of the point changed significantly. Otherwise, it is resulted that d height

difference is not a displacement but it is caused by the random measurement errors (Erol and Ayan, 2003).

The conventional deformation analysis method was applied to all three data sets (levelling derived height differences, GPS derived height differences and the combination of both) as is going to be mentioned below. And the results that obtained from evaluating these three data sets are shown in the graphics (see Figure 2, Figure 3 and Figure 4).

Stochastic information involves the number of instrument setup while evaluating levelling derived height differences in the deformation analysis algorithm.

On the other hand, in the algorithm of the second evaluation, the ellipsoidal height differences (dh_{ij}), which were derived from the GPS baseline solutions, are used as measurements,. Also, the stochastic information of measurements comes from the GPS baseline solutions.

In the third evaluation, height differences, derived from both measuring techniques, are used together and deformation analysis is applied according to results of evaluation of combined data groups. A very important point that has to be considered in the first step of this evaluation is that the both measurements groups derived from both techniques do not have the same accuracy naturally. And so, the stochastic information between these measurement groups (relative to each other) has to be derived. In this study, for computing the weights of both measurement groups, Helmert Variance Component Estimation (HVCE) Technique has been used.

In here, only the graphics, which shows the height differences of northern-track points, are given. Because of that the deformations in both tracks points have the same character, so it has been found sufficient to give results belongs to only one track of the viaduct.
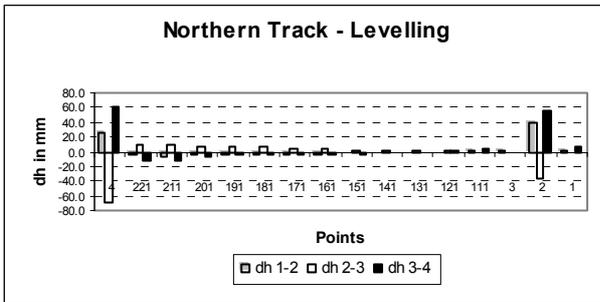
Figure 2: Levelling derived height differences between consecutive epochs for each point in the northern track
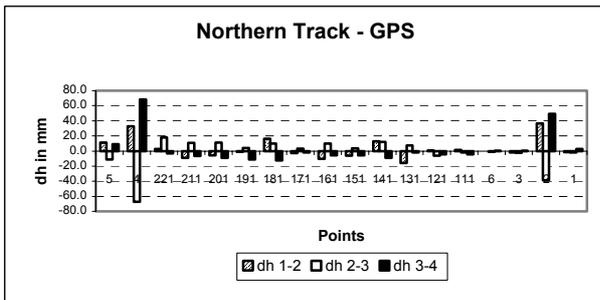
Figure 3: GPS derived height differences between consecutive epochs for each point in the northern track
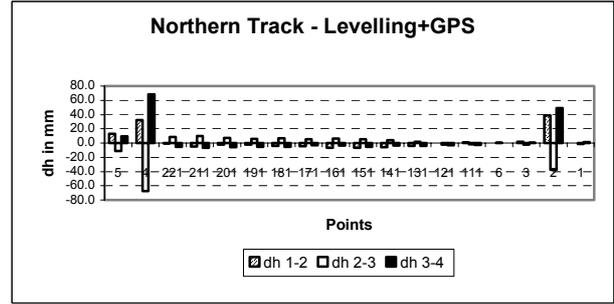
Figure 4: Combined derived height differences between consecutive epochs for each point in the northern track

5. 3D DEFORMATION ANALYSIS WITH S-TRANSFORMATION USING GPS DATA

It can be accomplished the datum consistency between different epochs by employing the S-Transformation. Also, the moving points are determined by applying this transformation consecutively. S-transformation is an operation that is used for transition from one datum to another datum without using a new adjustment computation. With another word, S-transformation is a transformation computation of the unknown parameters, which were determined in a datum, and their cofactor matrix, from the current datum to the new datum. The equations that imply the transition from the datum i to datum k are given below (Demirel, 1987; Başkaya, 1995).

$$\underline{x}^k = \underline{S}_k \, \underline{x}^i$$

$$\underline{Q}_{xx}^k = \underline{S}_k \underline{Q}_{xx}^i \underline{S}_k^T$$

$$\underline{S}_k = \underline{I} - \underline{G}\left(\underline{G}^T \underline{E}_k \underline{G}\right)^{-1} \underline{G}^T \underline{E}_k$$

5.1 Global Test Using S-Transformation

A control network is composed by the datum points and deformation points. By the help of the datum points, the control network, which were measured in t_i and t_j epochs, are transformed to the same datum.

While inspecting the significant movements of the points, continuous datum transformation is needed. Because of this, first of all, the networks, which are going to be compared each other, are adjusted in any datum such as using free adjustment.

In the result of free adjustment, the coordinates of the control network points, which were measured in any epoch, are divided into two groups as f (datum points) and n (deformation points).

Then, datum i and datum j were able to be transformed into the same datum k by the help of the datum points. As the result, the vectors of coordinate unknowns and also their cofactor matrix are founded for the datum points in the same k datum.

By the global test, it is determined if there is any significant movements in the datum points or not. In the result of global test, if it is decided that there is deformation significantly in one part of the datum points, determining the significant point movements using S-transformation (localization of the deformations) step is started. In this step, as is assumed that each of the datum points might change its position, for each

datum point, the group of datum points is divided in to two parts: the first part includes the datum points, which are assumed as stable and the second part includes the datum point, which is assumed as instable. And all the computation steps are repeated for each datum points. By this way, for all of the datum points, the probability of to be stabile or not has been tested. At the end, the exact datum points are derived.

5.2 Determining the Deformation Values

After determining the significant point movements, the block of datum points, which don't have any deformations, are to be determined. By the help of these datum points, both epochs are shifted the same datum and deformation values are computed as explained below.

The deformation vector for point P:

$$\underline{d} = \begin{bmatrix} x_k^j - x_k^i \\ y_k^j - y_k^i \\ z_k^j - z_k^i \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} \tag{20}$$

and the magnitude of the vector:

$$d = \sqrt{\underline{d}^T \underline{d}}$$

To determine the significance of these deformation vectors, which are computed according to above equations, the H_0 null hypothesis is carried out as given below.

$$H_0 : d = 0$$

And the test value:

$$T = \frac{\underline{d}^T Q_{dd}^{-1} \underline{d}}{3s_0^2}$$

While this test value is compared with critical value, $F_{3, f; 1-\alpha}$, and if $T \rangle F_{3, f; 1-\alpha}$, it is said that there is a significant 3D deformation in point P (Denli, 1998).

After GPS Data evaluation, 3D deformation analysis was carried out as it is mentioned. After the analyses, the founded horizontal displacements can be seen in Figure 5.
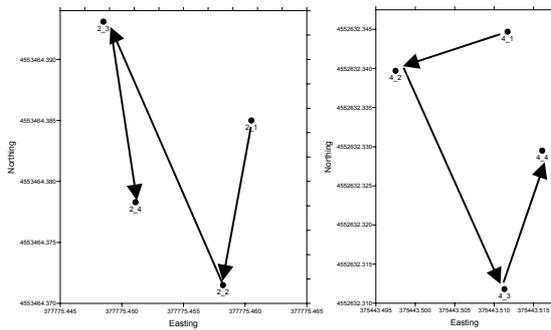
Figure 5: Horizontal displacements in points 2 and 4.

6. RESULTS and CONCLUSION

As is very well known that, the weakest component of a position obtained by GPS is the height component. This is mainly because of the weakness of geometric structure of GPS. Because of this, GPS technology in determining vertical deformations, it has to be supported by precise levelling measurements in vertical positioning.

In the first step of the process, the data, which were handled from both measurement methods, were processed for each epoch separately. By this way, the results, which had been derived from the independent solutions for each epoch, were compared. This was done to obtain an impression of the quality of the data, of possible inherent problems and to get first hints on instable points, thus, allow applying a suitable deformation analysis strategy. This operation covers the only height component. In the result of this comparison, the benefits of the precise levelling measurements have been seen. The precise levelling measurements have an effective role for checking the heights derived from GPS measurements and also for clarifying the antenna height problems, which could be occurred during the GPS measurements and effect the height component directly.

After these processes, deformation analyses were carried out by using height differences from levelling measurements, from GPS measurements and also by using combination of height differences from both GPS and levelling measurements respectively. In general meaning, the results were confirmed each other. In the results, it was surprisingly found that the maximum height changes were in point 2 and point 4 (as it is seen in the graphics), which were assumed as stabile at the beginning of the project and even though that their constructions are pillar. According to analysis of precise levelling data, on the contrary of the situation in point 2 and point 4, there weren't seen any changes in heights in the deformation points on the viaduct. On the other hand, according to the second evaluation (using GPS derived height differences) in some points on the viaduct, height changes were reported.

In the third evaluation, as a result of the Helmert Variance Component Estimation it was computed that the weight of height differences from levelling equals to 30 times of the weight of GPS derived height differences. This result was reached in the third iteration step.

In the graphics, height changes according to consecutive measurement campaigns are seen. As the graphics of the first, the second and the third evaluations are compared, it is not reliable enough to use GPS measurements without special precautions for the GPS error sources, such as multipath, atmospheric effects, antenna height problems etc., in vertical deformation analysis of engineering structures. According to investigation of the result of the second evaluation, it was seen changes in heights of some points on the viaduct. However, while the first and third evaluations are considered, it is understood that these changes, seemed to be deformations on the object point of the viaduct according to results of the second evaluation, were not significant and caused by the error sources in GPS measurements especially antenna height problems.

Thereafter, the 3D deformation analysis were carried out according to theoretical aspects that were explained in section 5 and the results of the priory adjustments and 1D deformation analysis aid to the decision phase of 3D deformation analysis, while the network points are being grouped as stable or instable. According to analyses results, horizontal displacements were investigated in point 2 and point 4 in the outside of the viaduct (see Figure 5).

However, at a first glance, it was surprising to found deformations in point 2 and point 4 that had been chosen as reference points, after geological and geophysical investigations, the origin of these results was captured. According to that the area is a marsh area that this characteristic might widen also underneath of these two reference points, 2 and 4. The uppermost soil layer in the region is not seemed to be stabile and the foundations of the constructions of the reference points are not founded as deep as the piers of the viaduct and so they are affected the environmental conditions easily. And these conditions supported the results of the analysis that in the object points on the viaduct, there were not exposed any significant movements. On the other hand, it is possible to mention about the correlation between vertical movements in two reference points, 2 and 4, and wet/ dry seasons, because the uplift and sinking movements of these reference points seems very synchronic as related to seasonal changes in amount of water (see Figures 2, 3 and 4).

The results of this study, experienced with measurements of Karasu viaduct, are thought to be very important remarks for deformation analysis studies using GPS measurements. As the first remark, GPS measurement technique can be used for determining deformations with some special precautions for eliminating GPS error sources. These include using forced centering mechanisms to avoid centering errors, using special equipments for precision antenna height readings, using special antenna types to avoid multipath effects etc. Also, during the data processes it is necessary to clear the cycle slips from the data, and to consider insufficiencies of the used tropospheric and ionospheric models.

However, even though these precautions, to provide better results in deformation analysis, GPS measurements have to be supported with Precise Levelling measurements.

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