MULTIVARIATE MATHEMATICAL MORPHOLOGY BASED ON PRINCIPAL COMPONENT ANALYSIS: INITIAL RESULTS IN BUILDING EXTRACTION

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ABSTRACT:

Today, colour or multichannel satellite and aerial images are increasingly becoming available due to the commercial availability of multispectral digital sensors and pansharpening function of the commercial remote sensing software tools. Comparing to their monochromic counterparts, colour image data can offer not only more useful information about landscape but also the correlations among channels. Recently, multivariate mathematical morphology has received increased attention due to its rigorous mathematical theory and its powerful utility in multichannel image analysis. In this paper, a new morphological method for multichannel remotely sensed image processing is presented and analyzed. The proposed method utilizes a multivariate ordering principle based on principal component analysis. To define the colour morphology the colour vectors are ordered by using the first principal component analysis. On the basis of this ordering, new infimum and supremum are defined. Using the new infimum and supremum, the fundamental erosion and dilation operations are defined. Two series of experiments have been prepared to test the performance of the proposed method by using Ikonos and QuickBird pansharpened images and colour aerial images acquired over a built-up area.

1. INTRODUCTION

As a methodology analyzing spatial structures in remotely sensed image data, mathematical morphology has become more and more popular in the image processing community not only due to its rigorous mathematical theory but also its powerful utility in image analysis. Generally speaking, mathematical morphology uses the morphological operations to analyze and recognize geometrical properties and structure of objects in images. So far, mathematical morphology has been developed as a complete and efficient tool for analyzing the spatial organization in binary and grayscale images (Serra, 1982). It is categorized into binary morphology and grayscale morphology.

Initially, mathematical morphology was proposed by Matheron (1975) for investigating the geometry of the objects of a binary image in his classical book on random set. In the grayscale case, complete lattices are used as the mathematical basis for the grayscale morphology. The basic idea behind the grayscale morphology is on the assumption that the set of all possible images forms a complete lattice. Based on this assumption, the set of all operators mapping one grayscale image into another also constitutes a complete lattice.

Both from a practical and theoretical point of view, colour mathematical morphology can be of great interest. First, colour is known to play a significant role in human visual perception and is becoming more and more relevant to computer vision as colour sensors become more widely available. It is well known that in an image a great deal of extra information may be contained in the colour, and this extra information can then be used to simplify image analysis, e.g., object identification and feature extraction based on colour. So it is necessary to develop a new effective technique to analyze colour images. Secondly, since binary mathematical morphology and grayscale mathematical morphology are intended to analyze binary images and gray-scale images, respectively, it would be interesting, from a pure theoretical point of view, to extend morphological theory to process colour images.

Although some techniques developed for grayscale mathematical morphology can be extended to colour images by applying the operators to each channel of a colour image separately, for example, the most straightforward scheme for the extension is to treat a colour image as an independent monochrome image and the grayscale morphological operator is directly applied to each colour component separately. Unfortunately, this procedure has some drawbacks, e.g., producing new colours that are not contained in the original image and may lead missing of the correlations between components (Astola et al., 1990; Goutsias et al., 1995).

The extension of concepts from grayscale morphology to colour morphology raises some important problem (Louverdis, 2002; Vardavoulia, 2002). First, an appropriate colours ordering must be found to define colours morphological operations that will retain the basic properties of their grayscale counterparts. Secondly, a colour space that determines the way in which colours are represented must be chosen. Third, an infimum and a supremum operator in the selected colour space should be defined well. It would be perfect for the two operators to be vector preserving, so that they do not introduce new colours that do not exist in the original image.

In this paper, a new reduced ordering based on fuzzy first principal component in RGB colour space is proposed. On the basis of the vectors ordering, new infimum and supremum operators that are both vector preserving are defined. Then, colour morphology, which takes into consideration the vector nature of colours, is introduced. Using new infimum and supremum operators, the basic morphological operations in RGB colour space: erosion, dilation, opening, and closing, are defined. Last, as an example, the proposed colour morphology

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is used to colour edge detection and building roof extraction from remotely sensed images.

The paper is organized as follows. Section 2 is devoted to a series of background notions in vector ordering, multivariate data analysis, and colour morphology. In Section 3, a new reduced ordering based on ordinal first principal component analysis is introduced. The basic morphological operations such as dilation, erosion, closing, and opening based on the new vector ordering are proposed in Section 4. Section 5 is devoted to the applications of the proposed morphological operators to colour edge detection and building roof extraction. In Section 6, preliminary results of building extraction from pansharpened Ikonos and QuickBird and colour aerial imagery are given followed by discussion and outlook in Section 7.

2. BACKGROUND

2.1 Ordering Vector

A set of multivariate data consisting of n m-dimension random vectors can be modeled as an $n \times m$ data matrix **X**. The rows of the matrix **X** will be written with $\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots \boldsymbol{X}_n$ corresponding to n observations. The columns of the matrix **X** will be written with $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots \boldsymbol{x}_m$ corresponding to p variables. The element locating at the row i and the column j in the matrix **X** is x_{ij} representing jth variable on the ith observation, i.e.,

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \\ \vdots \\ \boldsymbol{X}_n \end{bmatrix} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m] = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \quad (1)$$

where, $\boldsymbol{X}_i = [x_{i1}, x_{i2}, \cdots x_{im}], (i = 1, 2, \cdots n)$ (2)

and $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots x_{nj}], (j = 1, 2, \cdots m)$ (3)

The aim of ordering multivariate data $\boldsymbol{X}$ is to arrange them to the form $\mathbf{X}_i \prec \mathbf{X}_j \prec \cdots \prec \mathbf{X}_k$ according to each variable $\mathbf{x}_j$, j = 1, 2, …, m, where the symbol $\prec$ means less preferred to and the subscripts $i, j, \ldots, k$ range over mutually exclusive and exhaustive subsets of integers 1,2, …, n.

Unfortunately, ordering multivariate data are not straightforward, because there is not the notion of the natural ordering in a vector field as in the one-dimensional case. Although there is no unambiguous form of multivariate data ordering scheme, much work has still been done to order the data. Barnett (1976) proposed the so-called sub-ordering principles to rule the ordering. The sub-ordering principles are classified in four groups: (1) marginal ordering (M-ordering), in which multivariate data is ordered along each one of its m-dimensions independently; (2) condition ordering (C-ordering), in which the multivariate vectors are ordered conditionally on one of components. Thus, one of the components is tanked and other components of each vector are listed according to the position of their ranked component; (3) partial ordering (P-ordering), in which multivariate data is to partition the vectors into groups, such that the groups can be distinguished with respect to order, rank, or extremeness (Titterington, 1978); and (4) reduced ordering (R-ordering), it reduces vectors to a scalar value according to a measure criterion. Mardia (1976) further developed the sub-classification of reduced ordering: distance ordering and projection ordering. The distance ordering refers to the use of any specific measures of distance, and the projection ordering considers ordering the sample by using the first principal component (PC1) or higher.

2.2 First Principal Component Analysis

An obvious extension of the univariate notion of mean and variance leads to the following definitions (Mardia et al., 1979).The mean of jth variable, $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots, x_{nj}]^T$, is defined as

$$\bar{x}_j = \frac{1}{n}\sum_{r=1}^{n} x_{rj} \quad (4)$$

The variance of the jth variable, $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots, x_{nj}]^T$, is defined as

$$s_{jj} = \frac{1}{n}\sum_{r=1}^{n}(x_{rj} - \bar{x}_j)^2 = s_j^{\ 2} \quad j = 1, 2, \ldots, m \quad (5)$$

The covariance between the ith variable, $\boldsymbol{x}_i = [x_{1i}, x_{2i}, \cdots, x_{ni}]^T$, and jth variables, $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots, x_{nj}]^T$, is defined as

$$s_{ij} = \frac{1}{n}\sum_{r=1}^{n}(x_{ri} - \bar{x}_i)(x_{rj} - \bar{x}_j) \quad i, j = 1, 2, \ldots, m \quad (6)$$

The vector of means, or mean of the matrix **X,** is

$$\bar{\boldsymbol{X}} = [\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_m] = \frac{1}{n}\sum_{r=1}^{n} \boldsymbol{X}'_r = \frac{1}{n}\boldsymbol{X}'\boldsymbol{1} \quad (7)$$

where $\boldsymbol{1}$ is a column vector of n one, i.e., $\boldsymbol{1} = [1, 1, \cdots, 1]^T$. Also the variance-covariance matrix **S** of the matrix **X** is

$$\boldsymbol{S} = [s_{ij}] = \frac{1}{n}\sum_{r=1}^{n}(\boldsymbol{X}_r - \bar{\boldsymbol{X}})(\boldsymbol{X}_r - \bar{\boldsymbol{X}})' = \frac{1}{n}\boldsymbol{X}'\boldsymbol{H}\boldsymbol{X} \quad (8)$$

where $\boldsymbol{H} = \boldsymbol{I} - \frac{1}{N}\boldsymbol{1}\boldsymbol{1}'$ denotes the centering matrix and **I** denotes identity.

It is obvious that **S** is symmetric and position semi-definite. By the spectral decomposition theorem, the variance-covariance matrix $\boldsymbol{S}$ may be written in the form

$$\boldsymbol{S} = \boldsymbol{G}\boldsymbol{L}\boldsymbol{G}' \quad (9)$$

where **L** is a diagonal matrix of the eigenvalues of **S**, that is

$$\boldsymbol{L} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda_m \end{bmatrix} \quad (10)$$

where $\lambda_1 \geq \lambda_2 \geq, \cdots, \geq \lambda_m \geq 0$, **G** is an orthogonal matrix, its column vectors $\mathbf{g}_i$ (i = 1, 2, …, m) is the standardized eigenvectors corresponding to the eigenvalues λ_i (i = 1, 2, …, m) of **S**, i.e.,

$$\boldsymbol{G} = [\boldsymbol{g}_1, \boldsymbol{g}_2, \cdots, \boldsymbol{g}_m] \quad (11)$$

According to the eigenvectors, PC1 is defined as

$$\boldsymbol{y}_1 = (\boldsymbol{X} - \boldsymbol{1}\bar{\boldsymbol{X}}')\boldsymbol{g}_1 \quad (12)$$

The eigenvalue λ_1 of the sample variance-covariance matrix **S** can be written as

$$\lambda_1 = \sum_{j=1}^{P} s_{jj} r^2(\boldsymbol{x}_j, \boldsymbol{y}_1) \tag{13}$$

where $r^2(\boldsymbol{x}_j, \boldsymbol{y}_1)$ is the sample correlation coefficient between $\mathbf{x}_j$ and $\mathbf{y}_1$. Let $\boldsymbol{a} = (a_1, a_2, \cdots a_m)^T$ be a standardized vector, i.e., $\mathbf{a'a} = 1$. Then **Xa** givens *n* observations on a new variable defined as a weighted sum of the columns of **X**. The sample variance of this new variable is $\mathbf{a}'\mathbf{Sa}$.

Theorem 1 No standardized linear combination (SLC) of $\mathbf{X}_i$ (i = 1, 2, …, n) has a variance larger than λ_1, the variance of PC1 (Mardia et al., 1979).

From Theorem 1, not surprisingly, the standardized linear combination with the largest variance is PC1.

2.3 Colour Morphology

Similar to the extension from binary morphology to grayscale morphology, the extension of concepts of grayscale morphology to colour image processing also need some rules to organize colour values. Today, complete lattices are considered as the right mathematical framework for colour morphology, not only because the framework will retain the basic properties of its grayscale counterparts, but also it could be used in applications similar to those of the corresponding grayscale operations. An inherent difficulty in the framework is that there is not an obvious and unambiguous method of fully ordering colours (vectors). So far, there is not a unified colour morphology theory, due to the variety of ordering schemes and colour spaces that can be used. Different approaches to colour morphologies can be classified by following the classification of ordering schemes stated in Section 2.2: marginal morphologies (Talbot et al., 1998), partial morphology (Vardavoulia et al., 2002), and reduced morphologies (Comer and Del, 1998).

3. ORDERING VECTORS

Although principal component analysis is an important and essential technique for data reduction and has been widely used in remote sensing image processing, when the variables are ambiguous, it makes no sense to estimate PC1 as the linear combination of variables with standardized weights having maximal, because the linear combination of ambiguous variables is not well defined. However, according to Theorem 1 in Section 2.2, Eq. (13) provides an alternative way to define a sample PC1, ordinal PC1 (Korhonen and Siljamaki, 1998).

Definition 1 Vector $\boldsymbol{y} \in R^n$ is PC1 of the centered variables $\mathbf{Hx}_j$, j = 1, 2, …, m, if $\boldsymbol{y}$ is a solution vector of the following:

$$\max_{\boldsymbol{y}}\{\lambda = \sum_{j=1}^{m} s_{jj} r^2(\boldsymbol{y}, \boldsymbol{x}_j) \big| \boldsymbol{y}'\boldsymbol{y} = 1\} \tag{14}$$

From the Definition 1 on PC1, we can extend the definition of the PC1 to cases in which some restrictions are imposed on the vector **y** and variance s_{jj}.

Definition 2 The ordinal PC1 based on fuzzy pair wise comparison matrix $\boldsymbol{y} \in Z^n$ is a vector, $y_i \in \{1, 2, \cdots, n\}$, i = 1, 2, …, n, and $y_i \neq y_k, \forall i \neq k$, which determines a rank order for observations such that the sum of products on squares of some correlation coefficients between the variables $\mathbf{x}_j$, j = 1, 2, …, m, and **y** with the variance of variable $\mathbf{x}_j$, j = 1, 2, …, m is maximal. That is

$$\lambda = \sum_{j=1}^{m} s_{jj} r^2(\boldsymbol{x}_j, \boldsymbol{y}) \tag{15}$$

where $r(\boldsymbol{x}_j, \boldsymbol{y})$ is a correlation coefficients between the variables $\mathbf{x}_j$, j = 1, 2, …, m, and **y**, and s_{jj} is the variance of variable $\mathbf{x}_j$, j = 1, 2, …, m.

Definition 3 Let variable $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots, x_{nj}]^T$, $\forall x_{ij} \in Z^+, \forall x_{ij} \leq X_j$. The $n \times n$ matrix $\mathbf{M}(\mathbf{x}_j) = [\mu_{hk}(\mathbf{x}_j)]$ is a fuzzy pair wise comparison matrix describing the fuzzy rank order of observation according to variable $\mathbf{x}_j$ if

$$\mu_{hk}(\boldsymbol{x}_j) = \frac{1}{X_j}(x_{kj} - x_{hj}), \quad \forall h, k = 1, 2, \cdots, n \tag{16}$$

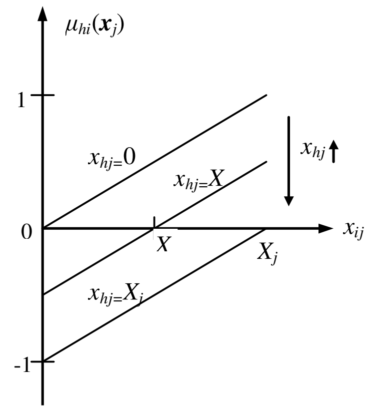

Fig.1 The membership function for fuzzy pair wise comparison matrix.

As defined in Definition 3, the μ_{hk} denotes the membership grades representing the comparison relationship between x_{hj} and x_{kj} according to the rank C. The value of the membership grades calculated by Eq. (16) is in [-1, 1], which shows not only the degree of the relationship between x_{hj} and x_{kj}, but also the fact that these two elements are positively or negatively related. We have illustrated the membership function of Eq. (16) in Figure 1.

To find the ordinal PC1 defined in Definition 2, it is necessary to define a rank correlation coefficient. Following the original idea of Daniels (1946), there are two ways to compute the rank correlation coefficients, Spearman's and Kendall's rank correlation coefficients (Kendall, 1962), as follows.

$$r_K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \frac{\boldsymbol{f}_i'\boldsymbol{f}_j}{\sqrt{\boldsymbol{f}_i'\boldsymbol{f}_i}\sqrt{\boldsymbol{f}_j'\boldsymbol{f}_j}} \tag{17}$$

where r_K is called Kendall's rank correlation coefficient, and $\boldsymbol{f}_i$ is calculated by

$$\boldsymbol{f}_j = [\mu_{11}, \cdots \mu_{1n}, \cdots\cdots, \mu_{n1}, \cdots, \mu_{nn}]^T \tag{18}$$

The variance of variable $\boldsymbol{x}_j = [x_{1j}, x_{2j}, \cdots, x_{nj}]^T$, which is used to measure the degree of dispersion for the variable, is computed by

$$s_{jj} = \frac{1}{N}\sum_{i=1}^{N}(x_{ij} - x_{Rj})^2 \tag{19}$$

where x_{Rj} is a referent element for x_{ij}. All referent elements consist to a referent variable for the variable $\mathbf{x}_j$.

λ in Eq. (13) corresponding to Kendall's rank correlation coefficient can be written as.

$$\lambda = \sum_{j=1}^{m} s_{jj} r_K^{\,2}(\boldsymbol{x}_j, \boldsymbol{y}) = \sum_{j=1}^{m} s_{jj} \frac{(\boldsymbol{g}_{x_j}{}'\boldsymbol{g}_y)}{(\boldsymbol{g}_{x_j}{}'\boldsymbol{g}_{x_j})(\boldsymbol{g}_y{}'\boldsymbol{g}_y)} = \sum_{j=1}^{m} s_{jj} \frac{(\boldsymbol{x}_j{}'\boldsymbol{y} - C_j)^2}{A_j B} \tag{20}$$

where $A_j = \boldsymbol{x}_j{}'\boldsymbol{x}_j - \frac{(\boldsymbol{x}_j{}'\boldsymbol{1})^2}{n}$, $B = \sum_{i=1}^{n} i^2 - \frac{n(n+1)^2}{n}$,

$C_j = \frac{(\boldsymbol{x}_j{}'\boldsymbol{1})(n+1)}{2}$.

To find a numerical solution to the ordinal principal component, a mathematical optimization model should be constructed. We formulate the optimal problem as an integer programming (IP) model:

Given

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \\ \vdots \\ \boldsymbol{X}_n \end{bmatrix} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m] = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \tag{21}$$

Max

$$\sum_{j=1}^{P} s_{jj} \frac{(\boldsymbol{x}_j{}'\boldsymbol{y} - C_j)^2}{A_j B} \tag{22}$$

Subject to

$$\boldsymbol{y} = [y_1, y_2, \cdots, y_n]^T, y_i \in \{1, 2, \cdots, n\} \, and \, y_i \neq y_j, \forall i \neq j, i, j = 1, 2, \cdots, n \tag{23}$$

The combination optimization problem stated above can be solved by using a branch and bound approach (Winston, 1991) that is a basic technique for solving integer and discrete programming problems. The method is based on the observation that the enumeration of integer solutions has a tree structure. The main idea behind the branch and bound method is to avoid growing the whole tree as much as possible, because the entire tree is just too big in most real problems. Instead branch and bound grows the tree in stages, and grows only the most promising nodes at any stage. It determines which node is the most promising one by estimating a bound on the best value of the objective function that can be obtained by growing that node to later stages.

In this paper, we model our integer programming problem above as a variable-integer assignment problem. We have n integers, 1 thought n, to assign to n variables, y_i thought y_n. Each variable can take exactly one integer, and all integers must have an assigned variable. The object is to maximum the total profit described by Eq. (22). In general, when there are n variables and n integers there are n possible assignments. Figure 2 shows an example of structure tree for a three-variable-integer assignment problem.

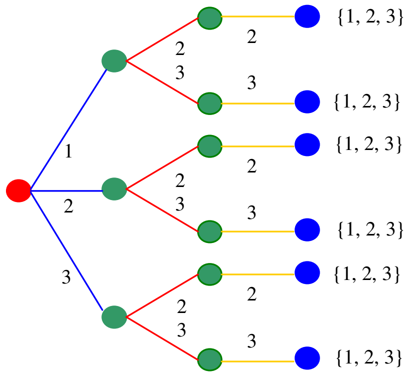

Fig. 2. The structure tree for the 3 variable-integer assignment problem.

: Root Node = All Solutions.

: Bud Node = Partial Assignments.

: Leaf Node = Complete Assignments.

— : y_1; — : y_2; — : y_3.

The formal branch and bound formation follows.

- ***Root node in the branch and bound tree***: all solutions.
- ***Bud node***: a partial assignment of integer to variable. For example, a partial assignment {1, ?, ?} represents the assignment of integer 1 to variable y_1.
- ***Leaf node:*** a complete assignment of integers to variables, e.g., a complete assignment {1, 2, 3} represents the assignment of integer 1 to y_1, 2 to y_2, and 3 to y_3.
- ***Objective function***: for each leaf node, its objection function can be computed by Eq. (22).
- ***Bounding function:*** for each bud node, we first create a pseudo-leaf by combining the partial assignment for this bud node and the maximum unassigned integer. For example, for the bud node {1, ?, ?}, its pseudo-leaf node is {1,3, 3}. Then we use the pseudo-leaf and Eq. (22) to compute the bounding function for the bud node.
- ***Bud node selection policy:*** global best value of the bounding function.
- ***Variable selection policy:*** choose the next variable in the nature order y_1 to y_N.

Terminating rule: when the best solution objective function value is better than or equal to the bounding function value associated with all of the bud nodes.

4. COLOUR MORPHOLOGY

4.1 Definitions of Infimum and Supremum

To extend the vector ordering approach to colour imagery, it is necessary to define the colours as vectors. In this paper, a colour C in RGB colour space is represented as a vector $\mathbf{X}_c = [R_c, G_c, B_c]$, where R_c, G_c, B_c denote the red, green, and blue components of the colour C, respectively. Therefore, a colour image can be viewed as a vector field. Given a colour image C and a pixel p in C in which the colour is $\mathbf{X}_p = [R_p, G_p, B_p]$. Let W be the $N = n \times n$ window consisted of the neighbourhood of the pixel p. All N colours in W can be written as a ($N \times 3$) data matrix $\mathbf{X}$ including N observations and three variables, that is,

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_N \end{bmatrix} = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3] = \begin{bmatrix} R_1 & G_1 & B_1 \\ R_2 & G_2 & B_2 \\ \vdots & \vdots & \\ R_N & G_N & B_N \end{bmatrix} \quad (24)$$

where $\mathbf{X}_i = [R_i \ \ G_i \ \ B_i]$, $i = 1,2, \ldots, N$,

$$\begin{cases} \boldsymbol{x}_1 = [R_1, \ R_2, \cdots, R_N]^{\mathrm{T}} \\ \boldsymbol{x}_2 = [G_1, G_2, \ldots, G_N]^{\mathrm{T}} \\ \boldsymbol{x}_3 = [B_1, \ B_2, \ldots, B_N]^{\mathrm{T}} \end{cases} .$$

In order to define the fundamental colour morphological operators, new infimum and supremum should be defined first. Using the vector ordering approach described in the previous section, we define the infimum operator $\wedge$ in $\mathbf{X}$ as follows:

$$\wedge \mathbf{X} = \wedge\{\mathbf{X}_1, \mathbf{X}_2, \cdots \mathbf{X}_N\} = \min\{\mathbf{X}_1, \mathbf{X}_2, \cdots \mathbf{X}_N\} \quad (25)$$

In a similar way, we define the supremum operator $\vee$ in $\mathbf{X}$ as follows:

$$\vee \mathbf{X} = \vee\{\mathbf{X}_1, \mathbf{X}_2, \cdots \mathbf{X}_N\} = \max\{\mathbf{X}_1, \mathbf{X}_2, \cdots \mathbf{X}_N\} \quad (26)$$

The application of these two operators to a particular window W results in only one output vector that is included in the input window W. Consequently, it is obvious that the proposed operators are colour preserving since no new colour (vector), which is not present in the window, is generated.

4.2 Definitions of Colour Morphological Operators

As stated in Section 4.1, we model a colour image as a vector field. For the definition of morphological operators in the vector field we need to point out the structuring element *G*. In this paper, we define a small single colour image as the structuring element for our colour morphological operators in which its size is the $m \times n$ window *W* related to a pixel *p* and its colour is either the colour of the pixel *p* or another colour of interest given by the user. We use the colour as the referent vector to calculate the variance s_{jj}.

Definition 4 Let *C* is a colour image, *W* is a window corresponding to pixel *p* having data matrix **X**, and *G* is a structuring element with the same size as *W* and the same colour as *p*. The colour dilation δ_C^G, erosion ε_C^G, closing χ_C^G, and opening o_C^G of *C* by *G* is the colour image given by:

$$\delta_C^G(C) = \{\vee \boldsymbol{X}, \forall p \in C\} \quad (27)$$

$$\varepsilon_C^G(C) = \{\wedge \boldsymbol{X}, \forall p \in C\} \quad (28)$$

$$\chi_C^G(C) = \varepsilon_C^G(\delta_C^G(C)) \quad (29)$$

$$o_C^G(C) = \delta_C^G(\varepsilon_C^G(C)) \quad (30)$$

From the Definition 4, the colour dilation of colour image *C* by the structuring element *G* keeps the steps as follows:

a. First, we construct a window *W* at pixel *p* consisting of the neighbourhood of *p* as the structuring element.
b. Then, we order all observations in data matrix X corresponding to W using the vector ordering approach.
c. Step (b) results in a rank order of colours in W. we find the infimum (supremum) in these colours.
d. The colour of the dilation (erosion) at the pixel p is the infimum chosen in above step.

5. COLOUR EDGE DETECTION

Edge detection is one of the basic techniques for many image processing tasks, such as image segmentation, image compression, feature extraction and so on. Various edge detection techniques have been proposed (Pratt, 1991). Generally, edges are defined as a discontinuity in some image attributes, for example, the brightness for grayscale images. For colour images, the situation is different. Several definitions of colour edges have been proposed (Pratt, 1991). First, a colour edge can be said to exist if and only if the luminance field contains an edge. This definition ignores discontinuities in hue and saturation that occur in regions of constant luminance. The second way to define a colour edge is to check if an edge exists in any of its constituent primary components. The third definition is based on forming the sum of gradients of the primary values or some linear or nonlinear colour component. A colour edge is said to exist if the gradient exceeds a threshold. Grayscale erosion and dilation have been successfully applied to extract the edges in grayscale imagery based on the subtraction of images (Dougherty, 1991; Lee et al., 1987). Unfortunately, these algorithms cannot be applied directly to colour imagery by means of colour erosion and dilation, since it does not make sense to subtract arithmetically two colours in RGB colour space.

In this section, a new algorithm to detect colour edge is introduced by using proposed colour morphological operators. According to the definitions of colour edges given by Pratt (1991), we define a loose colour edge is defined in the context of vector field. Colour edges are defined as any significant discontinuity in the vector field representing the colour image. Based on these definitions, we further define a basic colour edge detector as follows.

Definition 5 Let *C* is a colour image, the *W* is a window corresponding to pixel *p* having data matrix **X**, and *G* is a structuring element with the same size as *W* and the same colour as *p*. The colour edge detector $\varepsilon\delta_C^G$ of is the grayscale image given by:

$$\varepsilon\delta_C^G(C) = \left\| \delta_C^G(C) - \varepsilon_C^G(C) \right\| \quad (31)$$

where $\|\cdot\|$ represents an appropriate vector norm. It is worth making some remarks about the meaning of this colour edge detector, which point out that, in a uniform area of the image *C*, where all colours will be close to each other; the output of the detector will be small. However, its response on an edge will be large since dilation of image *C* will be created from the colours on the one side of the edge, which have 'large' vectors while erosion of image C will be created from another side with 'small' vectors.

6. EXPERIMENTAL RESULTS

To test the feasibility and the performance of the developed methodology, experiments have been conducted using real data (1) pansharpened 61cm resolution QuickBird satellite imagery, (2) pansharpened 1m resolution Ikonos satellite imagery, and (3) cm-level colour aerial images have been conducted. Three of the tested images are shown in Fig. 3. Each colour image has 24 bits per pixel and 150×150 pixels in size.

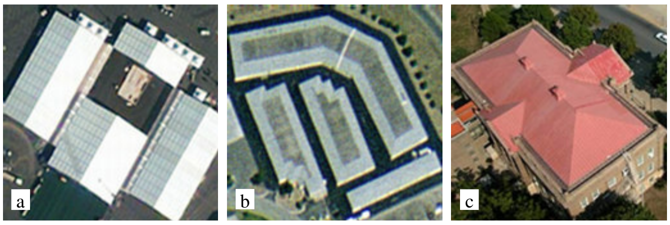

Fig. 3 Original images with 150×150 pixels. (a) QuickBird, (b) Ikonos, (c) aerial.

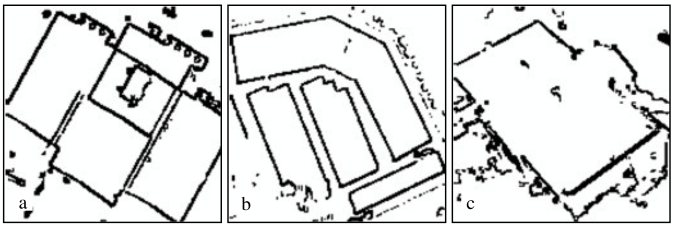

Fig. 6 Edge mapper. (a) QuickBird, (b) Ikonos, (c) aerial imagery.

The overall characteristics of the colour morphological operations are similar to that in grayscale case, i.e., the colour dilation eliminates 'dark' details, where the vectors have smaller rank than its surroundings, enhance 'light' details, where the vectors have larger rank than its surroundings, reduces 'dark' objects and enlarges 'light' objects; the colour erosion eliminates 'light' details, enhances 'dark' details, reduces 'light' objects, and enlarges 'dark' objects; the colour closing typically eliminates 'dark' details, and the colour opening eliminates 'light' objects. The results of the proposed colour edge detector using simply Euclidean distance as the vector norm are shown in Fig. 4. All building roofs in the three scenes have been extracted.

7. DISCUSSION AND OUTLOOK

A novel approach to colour mathematical morphology based on principal component analysis has been presented. The general design of colour mathematical morphology is conceptually sound and the algorithms were tested with phansharpened 1m Ikonos and 61 cm QuickBird satellite images and colour aerial imagery acquired over a built-up area in Toronto, Ontario. The proposed method extended the greyscale morphology to the colour morphology and provided its promising performance in colour image processing and feature extraction. Preliminary investigations suggest that building extraction can be automatically performed by using the developed colour edge detector. However, we didn't report those results because of space limitations. Detailed discussion is being presented in another publication. In future, the main focus will be on the roof extraction and aim at a global robust adjustment including the regularities of roof structure. Automatic procedures may fail in recovering the correct information due to the complexity of the task. Therefore, interactive tools for editing the extracted results are necessary. Future work will also include handling of road networks using the proposed method within an ongoing project on automated manmade object extraction from high-resolution colour satellite imagery. A strategy that integrates multiple cues including colour and attributed edges in a GIS environment will be invested and tested.

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