

# Mathematical Modeling of Ikonos Geo Image

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### ABSTRACT:

This paper presented a rigorous mathematical model based on orbital parameters for correction of the geo-rectified images such as Ikonos Geo, where the geometry of image at the time of imaging is lost and minimum information regarding the satellite movement in its orbit is available. Based on this model, the geo-rectified image is converted back to its geometry at the time of imaging at first. Then the Orbital Parameter Model is implemented to the resulted image to extract high accurate 3D spatial information from the image. The developed model has been tested on geo-rectified images such as SPOT L1B, and IKONOS Geo images taken over two test areas in Iran. This is followed by the results of various geometric accuracy tests carried out using different parameters and combination of control and check points. Then the results of this model are compared with those obtained through a generalized model based on Rational Function Model (RFM). The results show that, the orbital parameter model can achieve sub-pixel accuracy with little number of Ground Control Points (GCPs), where RFM gives the results with an accuracy about pixel, but with more number of well-defined and well-distributed GCPs.

### 1. INTRODUCTION

After launch and deployment of commercial high resolution satellites in recent years the issue of extracting precise 3D spatial information and large scale topographic mapping are highly considered by photogrammetry community. However, since in some cases raw images and ephemeris data are not released by their vendors, then the image geometry at the time of imaging is lost and it is difficult to reconstruct its geometry using rigorous models. Rigorous mathematical models based on collinearity equations need ancillary data for precise geometric correction of space images. These models can achieve high accuracy with little number of Ground Control Points (GCPs). However, lack of ephemeris data makes many investigators to use non-rigorous (generalized) models such as Rational Function Model (RFM). Although these mathematical models (RFM) can be computed without any knowledge about the sensor model but need so many well distributed GCPs.

In this paper a modified orbital parameter model is proposed for geometric correction of geo-rectified images. This model first converts the geo-rectified image back to its raw image format and then approximates the ancillary data from celestial mechanics and metadata to be used in orbital parameter model. The developed model has been tested on geo-rectified images such as SPOT Level 1B and Ikonos Geo images taken over two test areas in IRAN. This is followed by the results of various geometric accuracy tests carried out using different parameters and various combinations of control and check points. For the Ikonos Geo and SPOT Level 1B, the sub-pixel accuracy was achieved. This model is then compared with an iterative forward RFM where different terms (first, second or third order) and various state of denominator (same or different denominator) with different combination of control and check points are investigated in RFM.

### 2. Test Area

One SPOT-4 Level 1B stereo-pair, covering a part of Zanjan Province in the west part of Iran, was acquired on 14 May and 21 June 2000, for the purpose of this research project. The cross-track angles for the left and right images of SPOT stereo pair are +24 and -26.4 degrees respectively. For this project, 37 well distributed Points were established using differential GPS techniques. The accuracy of these points is estimated to be less than 1m.

An Ikonos Geo Panchromatic image was also employed covering an area of 11\*15 km<sup>2</sup> of central Hamedan city in the west of Iran. It was acquired on 7 October 2000 with an off-nadir angle of 20.4° and 47.4° sun elevation. The elevation within the IKONOS test area ranged from 1700m to 1900m. The ground control points and check points were extracted from 1:1000 scale digital maps produced by National Cartographic Center (NCC) using 1:4000 scale aerial photographs. The image coordinates of control and check points were monoscopically measured using Geomatica software package. For this image, 72 Points were selected from the test image. The points are distinct features such as buildings, and pool corners, wall and road junctions.

### 3. Mathematical Models

In this paper a rigorous orbital parameter model as well as an iterative forward rational function model (in different state of denominator) applied on the images. A brief description of these models are given in the followings.

#### 3.1 Orbital Parameter Model

An orbital parameter model can be applied to the pushbroom

images in order to determine their exterior orientation parameters. An orbital resection method has been developed to model continuous changing of position and attitude of the sensors by finding the orbital parameters of the satellite during the period of its exposure of the image. A bundle adjustment has been developed to determine these parameters using GCPs. This program has been tested already for SPOT Level 1A and 1B stereo pairs (Valadan and Petrie, 1998), MOMS-02 stereo images (Valadan, 1997), IRS-1C stereo pair (Valadan and Foumani, 1999), and Ikonos image (Valadan and Sadeghian, 2003).

The well known collinearity equation relates the points in the CT object coordinate system to the corresponding points in the image coordinate system. The relationship between these two coordinate systems is based on three rotations using combinations of the Keplerian elements mentioned above but computed with respect to the CT and CI systems, plus three rotations,  $\omega$ ,  $\varphi$ ,  $\kappa$ , for the additional undefined rotations of the satellite at the time of imaging. The off-nadir viewing angles of the linear array sensor must also be included as angle  $\alpha$  and  $\beta$ . the following equations "Eq. (4)" will then result:

$$\begin{pmatrix} x_i - x_0 \\ y_i - y_0 \\ -c \end{pmatrix} = S R \begin{pmatrix} X_i^g - X_0 \\ Y_i^g - Y_0 \\ Z_i^g - Z_0 \end{pmatrix}_{CT} \quad (4)$$

where

$$R = R_1(\alpha)R_2(\beta)R_3(\kappa)R_2(\varphi)R_1(\omega)R_2((f + \omega_p) - \frac{\pi}{2})R_1(\frac{\pi}{2} - i)R_3(\Omega - \pi) \quad (5)$$

- $\alpha$  : is the cross-track viewing angle,
- $\beta$  : is the along-track viewing angle,
- $\kappa, \varphi, \omega$  : are additional undefined rotations of the spacecraft at the time of imaging,
- $f, i, \omega_p, \Omega$  : are the true anomaly, orbital inclination, argument of perigee, and right ascension of the ascending node respectively,
- $x_i, y_i$  : are the image coordinates of the image point  $i$ ,
- $x_0, y_0$  : are the image coordinates of the principal point,
- $X_i^g, Y_i^g, Z_i^g$  : are the coordinates of the image point  $I$  in the Conventional Terrestrial (CT) coordinate system,
- $X_0, Y_0, Z_0$  : are the coordinates of the position of the sensor's perspective centre in CT coordinate system,
- $c$  : is the principal distance,
- $R_j$  : defines the rotation around the  $j$  axis, where  $j = 1, 2, \text{ or } 3$ .

Because of the dynamic geometry of linear array systems, the positional and attitude parameters of a linear array sensor are treated as being time dependent. The only available measures of time are the satellite's along-track coordinates. Thus the major components of the dynamic motion, the movement of the satellite in orbit and the earth rotation are modeled as linear angular changes of  $f$  and  $\Omega$  with respect to time, defined as  $f_i$  and  $\Omega_i$ :

$$\begin{aligned} f_i &= f_0 + f_1 x \\ \Omega_i &= \Omega_0 + \Omega_1 x \end{aligned} \quad (6)$$

where

$f_i$  and  $\Omega_i$  : are the true anomaly and right ascension of the ascending node of each line  $i$  respectively;  
 $f_i$  and  $\Omega_i$  : are the true anomaly and right ascension of the ascending node with respect to a reference line, for example the centre line of the scene; and  
 $f_1$  and  $\Omega_1$  : are the first values for the rates of changes of  $f_i$  and  $\Omega_i$

During the orientation of a pushbroom image, nine parameters of the orientation ( $f_0, \Omega_0, \alpha, i, f_1, \Omega_1, \omega_0, \varphi_0, \kappa_0$ ) find the position in space of the satellite and its sensor system and its crude attitude. Considering the attitude of a scan line as a reference, the attitude parameters  $\omega, \varphi, \kappa$ , and of the other lines can therefore be modeled by a simple polynomial based on the along-track ( $x$ ) image coordinates as "Eq. (7)":

$$\begin{cases} \omega = \omega_0 + \omega_1 x + \omega_2 x^2 \\ \varphi = \varphi_0 + \varphi_1 x + \varphi_2 x^2 \\ \kappa = \kappa_0 + \kappa_1 x + \kappa_2 x^2 \end{cases} \quad (7)$$

In the case of geo-rectified images the model first converts back the image to its geometry at the time of imaging. Then the Orbital Parameter Model is implemented to the resulted image to extract high accurate 3D spatial information from the image. A comparison of the raw and Geo image shows one major difference in terms of geometry, the number of pixels per line are different in raw and Geo images, because the rectified image is resample to unique space but in raw image because of off-nadir view angle this spaces are not same. To solve this problem, two coefficients computed to enable the final image to have the same size in two directions as a raw image. These coefficients are used to procedure the pixel and line coordinates of each point. However, additional displacements were introduced into the Geo imagery by the original corrections for each curvature/panoramic distortion. These displacements occur predominantly in the cross-track ( $y$ ) direction and, since they are approximately symmetrical about the image center line, parameters adjusting the attitude as a function of the cross-track image coordinates should give a good correction for these displacements by replacing the terms in equation (3) by a term for this purpose, which leads to the following (see also Valadan and Petrie, 1997) "Eq. (8)":

$$\begin{cases} \omega = \omega_0 + \omega_1 x + \omega_2 y^2 \\ \varphi = \varphi_0 + \varphi_1 x + \varphi_2 y^2 \\ \kappa = \kappa_0 + \kappa_1 x + \kappa_2 y^2 \end{cases} \quad (8)$$

### 3.2 Rational Function Model

Rational function model determines the image coordinates from the ratio of two polynomials of object coordinates as (Tao and Hu, 2000; OGC, 1999) "Eq. (1) and Eq. (2)":

$$x = \frac{P1(X, Y, Z)}{P2(X, Y, Z)} = \frac{\sum_{i=0}^{m1} \sum_{j=0}^{m2} \sum_{k=0}^{m3} a_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1} \sum_{j=0}^{n2} \sum_{k=0}^{n3} b_{ijk} X^i Y^j Z^k} \quad (1)$$

$$y = \frac{P3(X, Y, Z)}{P4(X, Y, Z)} = \frac{\sum_{i=0}^{m1} \sum_{j=0}^{m2} \sum_{k=0}^{m3} c_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1} \sum_{j=0}^{n2} \sum_{k=0}^{n3} d_{ijk} X^i Y^j Z^k}$$

where

$$P1(X, Y, Z) = a_0 + a_1 X + a_2 Y + a_3 Z + a_4 XY + a_5 XZ + a_6 YZ + a_7 X^2 + \dots + a_n Z^n \quad (2)$$

where (x,y) are normalized image coordinates and (X,Y,Z) are normalized object coordinates.

The normalization equation ("Eq. (3)") can be describe as (OGC, 1999):

$$x_n = \frac{x - x_o}{x_s} \quad (3)$$

$$y_n = \frac{y - y_o}{y_s}$$

Where  $x, y$  are image coordinates,  
 $x_o, y_o$  are shift value for image coordinates,  
 $x_s, y_s$  are scale values for image coordinates,  
 $x_n, y_n$  are normalized image coordinates.

The same equations can be used for object coordinates normalization. The process of normalization in both image and object coordinates is very important in RFM. This causes less computational input error and numeric stability, leading to better accuracy. RFM can be solved on two methods: Terrain-Independent (use the satellite parameters and sensor model with no GCPs) and Terrain-Dependent (that use only GCPs to compute the unknown parameters).

Since, for Geo-rectified images (such as IKONOS Geo) precise ancillary data is not available, therefore, in this paper the Terrain-Dependent forward RFM is investigated. In first step the unknown parameters are calculated iteratively using sufficient well distributed GCPs (Tao and Hu, 2001) and in the second step object coordinates are calculated using intersection method, where the approximate values of object coordinates of Independent Check Points (ICPs) determined from DLT equations. Different tests have been carried out in both cases (using similar and different denominators) where different combination of GCPs and ICPs have been used. Since the RFM is very sensitive to GCPs, different distribution were investigated.

#### 4. Geometric Accuracy Tests Using Orbital Parameter Model and RFM

The residual errors ( $\Delta X, \Delta Y, \Delta Z$ ) for 20 ground control points as well as 17 independent check points after running the bundle adjustment program based on orbital parameter model for SPOT L1B stereo pair are given as RMSE values and are summarized in Table 1. As can be seen from this Table, good results have been achieved using this stereo image. The graphical analyses of these results using vector plots of the errors occurring at each individual GCP and independent check point show that the residual errors are random and free from systematic effects.

The residual errors in terms of  $\Delta X, \Delta Y, \Delta Z$  in WGS 84 coordinates at the 35 GCPs and 37 ICPs after implementing orbital parameter model for Ikonos Geo image are summarized in Table 2. As can be seen again from this Table, good results have been achieved using this stereo image. The vector plots of the errors occurring at each individual GCP and independent check point show that the residual errors are random and free from systematic effects.

To have a better and more practical view on the results, the residual errors in UTM coordinates are given in Table 3.

The results of RFM, where the dominators are considered to be similar, on SPOT L1B and IKONOS Geo images are shown in Tables 4 and 5. Table 4 presents the  $\Delta X, \Delta Y, \Delta Z$  residuals in WGS 1984 coordinates for SPOT L1B image over the Zanjan project area when 20 Ground Control Points (GCPs) and 17 Independent Check Points (ICPs) have been used. In this table for example (+XY) means that  $P1(X, Y, Z) = a_0 + a_1 X + a_2 Y + a_3 Z + a_4 XY$ .

Table 5 shows the  $\Delta X, \Delta Y, \Delta Z$  residuals in WGS 1984 coordinates for Ikonos Geo image over the Hamedan project area when 35 Ground Control Points (GCPs) and 37 Independent Check Points (ICPs) have been used.

The results of RFM, where the dominators are considered to be different, on SPOT L1B and IKONOS Geo images are shown in Tables 6 and 7. Table 6 presents the  $\Delta X, \Delta Y, \Delta Z$  residuals in WGS 1984 coordinates for SPOT L1B image over the Zanjan project area when 20 Ground Control Points (GCPs) and 17 Independent Check Points (ICPs) have been used.

Table 7 shows the  $\Delta X, \Delta Y, \Delta Z$  residuals in WGS 1984 coordinates for Ikonos Geo image over the Hamedan project area when 35 Ground Control Points (GCPs) and 37 Independent Check Points (ICPs) have been used.

Comparing the results given in Tables 4 and 5 with the results given in Tables 6 and 7 shows that in RFM, when using different denominators, better accuracy can be achieved comparing to the case of using similar denominators.

Comparing the results given in Tables 6 and 7 with the results given in Table 1 and 3 shows that the residual errors obtained from the orbital parameter model are better than the results obtained from the rational function model. This shows the efficiency of the orbital parameter model proposed in this paper.

Table 1.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for SPOT L1B image over the Zanjan project area (20 GCPs and 17 ICPs) using orbital parameter model

Method	GCPs (meter)			ICPs (meter)		
	$\Delta X$	$\Delta Y$	$\Delta XY$	$\Delta X$	$\Delta Y$	$\Delta XY$
15 parameter for Geo image corrected	3.63	3.56	5.07	6.91	5.29	5.92

Table 2.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for Ikonos Geo image over the Hamedan project area (35 GCPs and 37 ICPs) using orbital parameter model

Method	GCPs (meter)			ICPs (meter)		
	$\Delta X$	$\Delta Y$	$\Delta XY$	$\Delta X$	$\Delta Y$	$\Delta XY$
15 parameter for Geo image corrected	0.73	0.71	1.01	0.73	0.83	1.10

Table 3. RMSE in UTM coordinate system for Ikonos Geo image over the Hamedan project area (35 GCPs and 37 ICPs) using orbital parameter model

Method	GCPs (meter)			ICPs (meter)		
	$\Delta E$	$\Delta N$	$\Delta PI$	$\Delta E$	$\Delta N$	$\Delta PI$
15 parameter for Geo image corrected	0.80	0.30	0.85	0.80	0.40	0.89

Table 4.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for SPOT L1B image over the Zanjan project area (20 GCPs and 17 ICPs) using RFM with the same denominator (P2=P4)

	Rmse of GCP Right Image (in pixel)			Rmse of GCP Left Image (in pixel)			Rmse of ICP Right Image (in pixel)			Rmse of ICP Left Image (in pixel)			RMSE OF OBJECT POINTS (in meter)			
	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta X$	$\Delta Y$	$\Delta Z$	$\Delta XY$
XY	1.04	0.83	1.33	1.03	0.69	1.25	1.34	0.73	1.53	1.09	1.32	1.72	14.59	11.95	11.30	18.85
+XZ	0.76	0.88	1.16	0.69	0.76	1.03	1.22	0.77	1.44	1.13	1.38	1.79	13.43	10.14	9.25	16.83
+YZ	0.51	0.66	0.84	0.46	0.43	0.63	1.20	0.74	1.41	1.03	1.12	1.52	10.62	10.55	9.53	14.96
+X <sup>2</sup>	0.58	0.67	0.89	0.38	0.44	0.58	1.13	0.89	1.44	1.06	1.08	1.51	12.25	10.06	9.79	15.85
+Y <sup>2</sup>	1.39	2.58	2.93	0.28	0.46	0.54	1.21	1.18	1.69	1.04	1.06	1.48	13.27	11.28	9.49	17.42
+Z <sup>2</sup>	0.67	0.81	1.05	0.25	0.41	0.48	2.06	0.90	2.25	1.08	1.08	1.53	13.38	12.56	9.41	18.36
+XYZ	0.16	0.22	0.27	0.26	0.33	0.42	1.31	1.12	1.73	1.05	1.18	1.58	11.49	12.75	9.77	17.16
+X <sup>3</sup>	0.10	0.23	0.25	0.14	0.36	0.39	1.33	0.93	1.62	9.16	3.44	9.78	12.55	12.36	10.64	17.61

Table 5.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for Ikonos Geo image over the Hamedan project area (35 GCPs and 37 ICPs) using RFM with the same denominator (P2=P4)

35 GCP 37 ICP	Rmse of GCP (in pixel)			Rmse of ICP (in pixel)			Rmse of ICP (in meter)		
	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta X$	$\Delta Y$	$\Delta XY$
XY	0.53	0.41	0.67	0.91	0.54	1.06	0.91	0.54	1.06
+XZ	0.52	0.39	0.65	0.93	0.51	1.06	0.93	0.51	1.06
+YZ	0.51	0.38	0.64	0.92	0.51	1.05	0.92	0.51	1.05
+X <sup>2</sup>	0.50	0.37	0.63	0.93	0.53	1.07	0.93	0.53	1.07
+Y <sup>2</sup>	0.50	0.38	0.60	0.93	0.53	1.08	0.93	0.53	1.08
+Z <sup>2</sup>	0.47	0.36	0.59	0.94	0.54	1.08	0.94	0.54	1.08
+XYZ	0.49	0.38	0.62	1.00	0.54	1.14	1.00	0.54	1.14

Table 6.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for SPOT L1B image over the Zanjan project area (20 GCPs and 17 ICPs) using RFM with different denominator (P2#P4)

20 GCP 17 ICP	Rmse of GCP Right Image (in pixel)			Rmse of GCP Left Image (in pixel)			Rmse of ICP Right Image (in pixel)			Rmse of ICP Left Image (in pixel)			RMSE OF OBJECT POINTS (in meter)			
	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta X$	$\Delta Y$	$\Delta Z$	$\Delta XY$
XY	0.56	0.65	0.86	0.44	0.59	0.74	1.18	0.68	1.36	0.99	1.12	1.49	10.65	10.14	9.49	14.70
+XZ	0.53	0.63	0.82	0.39	0.45	0.59	1.14	0.73	1.36	0.92	1.11	1.44	10.65	9.75	9.17	14.44
+YZ	0.32	0.56	0.65	0.35	0.39	0.53	1.23	1.02	1.60	0.94	1.13	1.47	11.49	10.57	10.87	15.62
+X <sup>2</sup>	0.21	0.53	0.57	0.26	0.35	0.44	1.42	1.01	1.74	1.09	2.45	2.69	14.19	12.83	13.35	19.12
+Y <sup>2</sup>	0.09	0.16	0.19	0.11	0.26	0.28	3.77	1.14	3.94	2.31	1.58	2.79	14.98	13.65	13.71	20.26
+Z <sup>2</sup>	0.03	0.07	0.08	0.02	0.23	0.23	2.97	2.68	4.00	1.64	1.31	2.10	38.55	16.97	22.97	42.12

Table 7.  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  residuals in WGS 1984 coordinates for Ikonos Geo image over the Hamedan project area (35 GCPs and 37 ICPs) using RFM with different denominator (P2#P4)

35 GCP 37 ICP	Rmse of GCP (in pixel)			Rmse of ICP (in pixel)			Rmse of ICP (in meter)		
	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta x$	$\Delta y$	$\Delta xy$	$\Delta X$	$\Delta Y$	$\Delta X$ $\Delta Y$
XY	0.50	0.38	0.63	0.91	0.49	1.04	0.92	0.49	1.04
+XZ	0.49	0.38	0.62	0.92	0.49	1.05	0.92	0.49	1.05
+YZ	0.57	0.37	0.68	0.99	0.48	1.10	0.99	0.48	1.10
+X <sup>2</sup>	0.97	0.36	1.03	1.05	0.50	1.16	1.05	0.50	1.16
+Y <sup>2</sup>	0.49	0.34	0.60	0.96	0.50	1.08	0.96	0.50	1.08
+Z <sup>2</sup>	0.84	0.35	0.91	6.02	0.52	6.05	8.26	0.52	8.28
+XYZ	2.23	0.86	2.39	10.64	8.02	13.33	3.80	2.18	4.38

## 5. Conclusions

This paper proposed a developed mathematical model based on Orbital parameter model as well as RFM based on non-rigorous model, where, tested on SPOT L1B AND Ikonos Geo images over two test area in Iran. As the above results show, orbital parameter model can achieve sub-pixel accuracy while, RFM give an accuracy about pixel.

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