

# PERFORMANCE OF KALMAN FILTER FOR GPS DATUM CONVERSION

B.L.Malleswari<sup>a,\*</sup>, M.Seetaa<sup>a</sup>, I.V.MuraliKrishna<sup>b</sup>, K.Lal Kishore<sup>b</sup>, Nagaratna.P.Hegde<sup>c</sup>

<sup>a</sup> Associate Professor, C.B.I.T, Gandipet, Hyderabad, Andhra Pradesh, India,

[blmalleswari@gmail.com](mailto:blmalleswari@gmail.com)

<sup>b</sup> Director, Registrar, JNTU, Kukatpalli, Hyderabad, Andhra Pradesh, India,

[ivm@ieee.org](mailto:ivm@ieee.org), 9848049624

<sup>c</sup> Associate Professor, Dept of CSE, V.C.E, Hyderabad, A P, India,

[nagaratnap@yahoo.com](mailto:nagaratnap@yahoo.com)

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## ABSTRACT:

This paper focuses on modeling of errors like ionospheric delays, atmospheric delays Tropospheric delays, Multipath effects and dilution of precision etc., affecting the GPS signals as they travel from satellite to user on Earth. As these errors degrade the accuracy of GPS position, an attempt is made to improve the accuracy in locating the GPS receiver by filtering the range measurements. This study emphasizes the coordinate conversion between World Geodetic System (WGS - 84) and Universal Transverse Mercator (UTM) using single frequency ML-250 hand held GPS receiver and smoothening of these coordinates using Kalman filter. A recursive filtering technique, Kalman filter is used for greater accuracy in estimating the position of user by considering the initial state of the system, statistics of system noise and measurement errors from sensor noise measurements. The results of proposed Kalman filter technique give better accuracy with more consistency and are found superior to the standard one.

## 1. Introduction

### 1.1 Global Positioning System:

Global Positioning System is a Satellite-based system that uses a constellation of 24 satellites to give an accurate position of user and GPS provides a global absolute positioning capability with respect to a consistent terrestrial reference frame and considered as an absolute global geodetic positioning system. GPS receivers have been developed to observe signals transmitted by the satellites and achieve sub-meter accuracy in point positioning and a few centimeters in relative positioning. The GPS satellites are positioned in such a way that at least five to eight satellites are accessible at any point on earth and at any time. GPS is based on a system of coordinates called the World Geodetic System 1984 (WGS-84 whose coordinates are the latitude, longitude, and height) (Kaplan.E.D, 1996) and (Parkinson, 1996).GPS data is observed in WGS 84 and Universal Transverse Mercator (UTM). The most common map projection and grid system used for land navigation is the Universal Transeverse Mercator (UTM)

system. A key advantage of the UTM projection is its preservation of the shape of the small areas on a map and its grid coordinates permit easy calculations using plane trigonometry (Langely B Richard, 2000). In UTM, the ellipsoid is portioned in to 60 zones with a width of  $6^{\circ}$  longitude each. A scale factor of 0.996 is applied to the central meridian (Leick, 1995). The scale factor is to avoid fairly large distortions in the outer areas of zone. There are several sources of error that degrade the GPS position from few meters to tens of meters (Pratap Misra, 2001). These error sources are Ionospheric, Atmospheric delays, Satellite and Receiver Clock Errors, Multipath, Dilution of Precision, Selective Availability (S/A) and Anti Spoofing (A-S) as described by Hoffmann - Wellenhof et al (1998). The errors could be transmitted via VHF/UHF links and the users can make use of the corrections to fix their positions more accurately. These errors can be reduced to arrive at a more accurate estimate of coordinates of user by means of a recursive algorithm-KALMAN FILTER. The emphasis is given on the above errors to analyze the Kalman filter (Grewel.M. S et al, 2001).

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\* Corresponding author. This is useful to know for communication with the appropriate person in cases with more than one author.

The GPS receiver is giving both WGS-84 data observed in latitude, longitude & altitude and UTM data observed in Northing & Easting (Langely.R.B, 2000). In the conversion process of WGS 84 to UTM, accuracy must be obtained without distortions. In the work attempted by Ravindhra et al (Feb, 2002), only the datum conversion from WGS- 84 to UTM and inaccuracy were discussed. The role of the noise in GPS is only at satellite and receiver segments. The modeling of the noise in satellites and receiver segments are discussed by Langely (March 1997).

## 2. The Kalman Filter:

A significant mathematical toolbox used for stochastic estimation from noisy sensor measurements is *Kalman filter*. Kalman filtering is based on linear mean square error filtering (estimation) and it is essentially a set of mathematical equations that implement a Predictor-corrector type estimator which is *optimal*. It minimizes the estimated *error covariance* —when some presumed conditions are met. For the given spectral characteristics of an additive combination of signal and noise, the linear operation on this input combination yields the best (meaning minimum square error) separation of the signal from the noise is to be known.

The specific feature of Kalman filter is about the description of its mathematical formulation in terms of state space analysis as per (Bozic, 2000) and its solution is computed recursively. As each update estimate is computed from the previous estimate and the input data, only previous estimate requires storage. The filter is a computational algorithm that processes measurements to deduce a minimum error estimate of the system by utilizing knowledge of the system, measurement dynamics, and statistics of the system, noises measurement errors and initial condition information. It is to improve the quality of datum conversion using this smoothening technique. It also reduces the error while converting from WGS-84 to UTM data. Here by employing this smoothening technique, Kalman filter puts up better WGS-84 to UTM conversion efficiency. The effects of ionospheric delays have already been discussed by Klobchar (May 1987). In the article [13], smoothening of WGS- 84 with the help of Kalman filter has been discussed. But in this proposed technique, the Kalman filter is used to smoothen the WGS coordinates. Since the time of its introduction, the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. This is likely due in large part to advances in digital computing, relative simplicity and robust nature of the filter itself. Rarely do the conditions

necessary for optimality actually exist, and yet the filter apparently works well for many applications in spite of this situation as emphasized by (Peter S Maybeck, 2001). Langely (October, 2000) discussed about the GPS receivers tracking 8 or more satellites simultaneously and determining the positions using all of the available observations with a least – squared – error algorithm.

## 3. Kalman Filter Algorithm

The Kalman filter addresses the general problem of trying to estimate the state  $X \in H^n$  of a discrete-time controlled process that is governed by the linear stochastic difference equation as in equation 1.

$$X_K = AX_{K-1} + BU_K + W_{K-1} \quad \dots \quad (1)$$

With a measurement  $Z_i \in H^M$  that is (as stated in equation 2).

$$Z_K = X_K + V_K \quad \dots \quad (2)$$

The random variables  $W_K$  and  $V_K$  represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions

$$P(W) = N(0, Q) \quad \dots \quad (3)$$

$$P(V) = N(0, R) \quad \dots \quad (4)$$

As per Peter S Maybeck (2001), the process noise covariance  $Q$  and measurement noise covariance  $R$  matrices as in equations 3 & 4 might change with each time step or measurement, however here we assume they are constant. The complete set of discrete Kalman filter equations and their solutions can be found in Gelb (1974).

The  $n \times n$  matrix  $A$  in the difference equation (1) relates the state at the previous time step  $K-1$  to the state at the current step  $K$ , in the absence of either a driving function or process noise. Note that in practice  $A$  might change with each time step, but here we assume it is constant. The  $n \times l$  matrix  $B$  relates the optional control input  $U \in H$  to the state  $x$ . The  $m \times n$  matrix  $H$  in the measurement equation (2) relates the state to the measurement  $Z_K$ . In practice  $H$  might change with each time step or measurement, but here we assume it is constant. The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update

equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback-- i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems.

#### 4. Filter Parameters and Tuning

In the actual implementation of the filter, the measurement noise covariance  $\mathbf{R}$  is usually measured prior to operation of the filter. Measuring the measurement error covariance  $\mathbf{R}$  is generally practical (possible) because we need to be able to measure the process anyway (while operating the filter) so we should generally be able to take some off-line sample measurements in order to determine the variance of the measurement noise. The determination of the process noise covariance  $\mathbf{Q}$  is generally more difficult as we typically do not have the ability to directly observe the process we are estimating. Sometimes a relatively simple (poor) process model can produce acceptable results if one "injects" enough uncertainty into the process via the selection of  $\mathbf{Q}$ . Certainly in this case one would hope that the process measurements are reliable. In either case, whether or not we have a rational basis for choosing the parameters, often times superior filter performance (statistically speaking) can be obtained by tuning the filter parameters  $\mathbf{Q}$  and  $\mathbf{R}$ . The tuning is usually performed off-line, frequently with the help of another (distinct) Kalman filter in a process generally referred to as system identification, which is clearly stated in (Bozic, 2000).

In closing we note that under conditions where  $\mathbf{Q}$  and  $\mathbf{R}$  are in fact constant, both the estimation error covariance  $\mathbf{P}_K$  and the Kalman gain  $\mathbf{K}_K$  will stabilize quickly and then remain constant. If this is the case, these parameters can be pre-computed by either running the filter off-line, or for example by determining the steady-state value of  $\mathbf{P}_K$ . It is frequently the case however that the measurement error (in particular) does not remain constant. For example, when sighting beacons in our optoelectronic tracker ceiling panels, the noise in measurements of nearby beacons will be smaller than that in far-away beacons. Also, the process noise  $\mathbf{Q}$  is sometimes changed dynamically during filter operation – becoming  $\mathbf{Q}_k$  in order to adjust to different dynamics. For example, in the case

of tracking the head of a user of a 3D virtual environment we might reduce the magnitude of  $\mathbf{Q}_k$  if the user seems to be moving slowly, and increase the magnitude if the dynamics start changing rapidly. In such cases  $\mathbf{Q}_k$  might be chosen to account for both uncertainties about the user's intentions and uncertainty in the model.

#### 5. Results and Discussions:

Using single frequency ML 250 GPS hand held receiver, the field data is collected at different locations in around Hussain Sagar, Hyderabad. The receiver is giving WGS – 84 data and UTM data (Nrx & Erx). The WGS – 84 coordinates are smoothed using Kalman filtering algorithm and then converted into UTM coordinates to test the accuracy and consistency. It is quite evident that, the converted values (Nprg and Eprg) without applying Kalman filter are giving poor resolution. And they are more inconsistent as the variance is more for the converted data. Then WGS 84 data is smoothed by Kalman filtering technique and then converted into UTM data which is giving a very small variance. So, lesser the variance more will be the consistency. Again the same GPS receiver's WGS 84 data is fed to Web Software “Coordinate. Transform” to validate UTM data by feeding longitude and latitude (Ns/w and Es/w). This converted data is also again analyzed and compared for all the sets. After comparing the variances of all the three (Ns/w, Nrx and Nprg (KF); Es/w, Eprg (KF) and Erx), it is found that the converted data, which is developed from Kalman filtering technique (Nprg(KF) and Eprg(KF)), is having better consistency. A comparative analysis is emphasized in the tables 1, 2 and 3. The accuracy of results obtained from program conversions has been validated. All the parameters are expressed in meters.

#### 6. Conclusions

A comparison of accuracy is suggesting, accuracy through Kalman filter application is certainly yielding better results. The present study & data analysis methodology showed that the variations in the signal related to WGS- 84 data can be smoothed using Kalman filter with in the range of studies made and the analysis is found to yield better accuracies. However the extensive application of the methodology for the data in bringing out the limitations of the smoothing of the signal, a statistical evaluation atleast in robust domain could throw some light on actual information content & loss of the information through Kalman filtering.

### Standard Deviation for set1 Table 1

N RX	N s/w	NPrg (kf)	E Rx	E s/w	EPrg (kf)
515.1	977	416	582.	681.9	296.1
635	.5152	.5819	316	64	57

### Standard Deviation for set2 Table 2

N RX	N s/w	NPrg (kf)	E Rx	E s/w	EPrg (kf)
811.3	1247.	559.520	199.0	204.0	143.11
243	6232	6	924	43	42

### Standard Deviation for set3 Table 3

N RX	N s/w	NPrg (kf)	E Rx	E S/W	EPrg (kf)
466.	720.4	270.82	521.3	697.5	345.287
5707	6406	07	621	67	5

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