A PRELIMINARY STUDY ON SPATIAL SAMPLING FOR TOPOGRAPHIC DATA

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ABSTRACT:

In this paper, an effective spatial sampling method is preliminary explored to assess the quality of the topographic data. The most novel idea in this study is we think the error as noise in the signal and we hope to direct spatial sampling through realizing this idea. First, we perform a prior partition of the original topographic data on the basis of the different error levels after a rough classification. Second, errors are supposed to obey the normal distribution, so we can use different Gaussian filters to remove the errors in each region. Then error-free topographic data after filtering can be seen as the true data. Finally, on the basis of this result, we can obtain the difference between the original topographic data and the error-free topographic data and then configure an error surface. The extreme points on the error surface are seen as the representative of the sampling sets. Finding the extreme points on the error surface is equivalent to selecting the least amount of sample points; meanwhile, these sample points can reflect the true error of the topographic data. By means of the proposed method, we hope to find the most optimum selection of spatial sample points from feature points, which can depict an approximate distribution of the errors, as well as reflect the entire quality of the topographic map.

1. INTRODUCTION

What is the quality level of the produced mps? Can the maps satisfy the quality required by users? Such questions are raised after the maps are produced. Consequently, to know the quality of the maps is necessary and significant. However, there are many maps after an arbitrary project; we have no idea how to inspect the quality of every map. Usually, sampling is a common and effective way to solve such problems. However, the question of how to develop the sampling scheme and implement the sampling method on the topographic data has always been an important research topic.

The estimation and assessment of the quality of the topographic maps has been researched before and several standards have been established and distributed. Early in 1947, the National Map Accuracy Standards (NMAS) was worked out for the well-defined points (U.S. Bureau of the Budget, 1947). In 1990 and in 1998 respectively, ASPRS Accuracy Standards (ASPRS Specifications and Standards Committee, 1990) and the National Standard for Spatial Data Accuracy (NSSDA) (Federal Geographic Data Committee, 1998) from the Federal Geographic Data Committee (FGDC) were established and distributed. The significance of these standards is to lay out a regulation for surveyors and inspectors to understand the quality information of the data. However, all these standards have no customized sampling scheme for users, so it depends on users to develop a concrete sampling scheme themselves. Due to the discrepancy of sampling accuracy, it is acceptable for standards to only provide a general idea, such as using the RMSE value to measure the positional accuracy of the map.

In practice, the standards for accuracy estimation of the maps are applied widely around the world and improved upon based on the practical situation. In China, topographic data producers will check the quality of the maps in three steps, namely inter-team checking, cross-team checking and quality inspection station accepting, namely two grades checking and one grade verifying. In principle, all the points should be re-surveyed during inter-team and cross-team checking. Actually, sampling is implemented throughout these three steps. Commonly, Simple random sampling or Judge-based sampling is widely used to select the sample points. However, a single simple random sampling scheme can not select the sample points accurately as it is not based on the error distribution, and a single judge-based sampling scheme may result in points located in easy-going place will be surveyed and sampled carefully, while the points located in difficult to reach areas will be neglected and misestimated, and this can not reflect the true appearance and accuracy of the map.

Consequently, the simple random sampling and judge-based sampling supplement each other, and the sample points are superfluous in order to approach the true accuracy level of the map. We can use such methods to reflect the quality of the maps to a certain extent. However, such sampling schemes have not considered the error distribution, so in terms of experience, when the inspectors locate the sample area and decide on the sample size, they usually will consider a compromise between accuracy and the cost. To ensure the sampling accuracy, they always select superfluous sample points. In another words, the sample cost has to give way to the sample accuracy. Consequently, to balance the accuracy and the cost, especially to reflect the true error of the maps, we will try to find a new and effective sample scheme to assess the quality of the topographic data.

In this paper, a novel spatial sampling scheme is described. We look at the error of the topographic map as the noise in the signal processing field. Based on the assumption that the error is supposed to obey the normal distribution, the proposed method aims at finding an error surface and locating the sample points on the surface. In section 2, we reviewed several spatial sampling methods currently in use. The proposed spatial sampling method is introduced in section 3. The detail description about the selection of sample points and its distribution are depicted in section 4 and section 5. Conclusions are drawn in section 6.

2. BRIEF REVIEW AND ANALYSIS OF SPATIAL SAMPLING FOR TOPOGRAPHIC DATA

On one hand, spatial sampling on the remote sensed data and GIS raster data has been discussed for a long time. On the basis of a literature review, we can see that there are already some mature technologies and methodologies in use, and a systematic effort has been made to investigate various aspects of accuracy in spatial databases (Bogaert et al, 1999; Muller et al. 1999; Zhu et al, 2005; Banjevic et al, 2002, Wiens, 2005; Royle, 2002). On the other hand, research on spatial sampling is also evolved in the GIS vector data, including the modelling research on the point, line and polygon features. In addition, some software applications for analysing, evaluating and even visualizing the quality of spatial data have been developed (Devillers and Jeansoulin, 2006).

To design a spatial sampling scheme is mainly to decide the distribution and the size of sample points. Traditionally, sample points correspond with an actual feature point on a topographic map. Well-defined or apparent feature points are always the candidates for sample points. For lines and areas, the situation is more complex. Similarly, the points that define those lines and areas are always looked upon as the sample points.

Another important part of spatial sampling design is to decide on the sampling methods. There are many available methods, such as simple random sampling, stratified sampling, systematic sampling etc., as well as the combination of these single sampling methods. The sample method is dependent on the error source and distribution of the map to some extent.

The positional accuracy is generally the most important parameter used to estimate the quality of the topographic data. Furthermore, the RMSE value is the most popular index of the error measurement. The error assessment on the sample points is also measured with other parameters and used to describe the thematic accuracy, completeness accuracy, logical consistency accuracy and even the temporal accuracy of digital topographic maps.

Error models for point, line and polygon have been developed on the theoretical and/or experimental basis. Prior research, including Bolstad et al. (1990), Jennifer and Walsby (1995), pointed out that the error obeys normal distribution and gave the point error model. Linear error models were also obtained based on the strict statistical theory (Shi, 1994, 1997, 1998; Shi and Liu, 2000). Meanwhile, the error propagation model has been researched widely to estimate the error induced by different GIS operations (Shi et al, 2003, 2004). Further research on the error models and error propagation in GIS operation can not only help measure the amount of the error, but also help lay out the distribution of sampling points.

Besides the research on the error model, many spatial sampling schemes have been designed to select an optimal set of sample points providing information for parameter estimation and spatial inter/extrapolation on the basis of an appropriate statistical model (Angulo et al, 2001, Angulo, 2005). Spatial simulated annealing (SSA) was introduced to optimize spatial environmental sampling schemes, which are optimized at two levels and take into account sampling constraints and preliminary observations (Van Groenigen et al. 1998).

In this study, we hope to analyze the accuracy of the topographic data quantitatively; in a further step, we even hope to explore a spatial sampling scheme, using the least sample points while depicting the accurate error of the digital topographic maps. To obtain this result, we will try to develop a novel spatial sampling scheme on the basis of signal process theory.

3. SPATIAL SAMPLING SCHEME

3.1 Statistical Distribution of Error

There are three kinds of common error, namely random error, system error and rough error, which exist during the data capture process. Many methods and assessing rules have been researched to find the description of the error and its distribution, such as the normal distribution referred in section 2, L^P norm distribution (Meng et al, 1998; Zhou, 2003) and so on. To a certain extent, the error obeys normal distribution is the widely accepted conclusion. The French mathematician De Moivere and The Germanic mathematician Gauss have produced landmark work on error theory and point out the normal distribution of the random error. Especially, Gauss contributed the basis for the distribution pattern of random error and brought forward the error processing method (Bolstad et al. 1990, Goodchild 1991).

However, there are many other kinds of description for error distribution. In this study, we focus on finding a good sampling scheme, and not researching the error distribution itself. We need to find how to perform the spatial sampling assuming the error obeys the normal distribution..

Suppose we have a set of surveyed data $(a_1, a_2 ... a_n)$ and its corresponding error $(\varepsilon_1, \varepsilon_2 ... \varepsilon_n)$ obeys the normal distribution. On the basis of such known information, we try to perform the proposed spatial sampling scheme, as well as assess the error and its spatial distribution.

3.2 Overall Flowchart of the Proposed Sampling Method

In this study, the error is regarded as the noise in the signal viewpoint according to the discussions in section 3.1. Our purpose is to bring forward a reasonable spatial sampling scheme, which can perfectly reflect the spatial distribution of error and select sample points as little as possible. To solve this problem, we have two principle steps to take, namely fitting the estimation of the topographic data and seeking a description of the error surface. The flowchart of this study is listed in figure 1.

In the first part of this study, we try to obtain an approximate expression of the real topographic data. First, we perform a rough partition of the sample area on the basis of the production method of the topographic map. For example, we can divide the sample area into several sub-areas, and each sub-area will be tackled by the same algorithm individually as the equivalent level of error. Second, supposing the error obeys normal distribution, we use multi-scale filters to remove the different magnitude of error from the measuring data for each region. After removing the error, we anticipate the reserved data is only a representation of the real topographic data. So if we can extract the extreme points of the filter data, we can fit a function for the real topographic data. Finally, we put all the fitted functions from each sample area together to estimate the true assessment of the total topographic data.

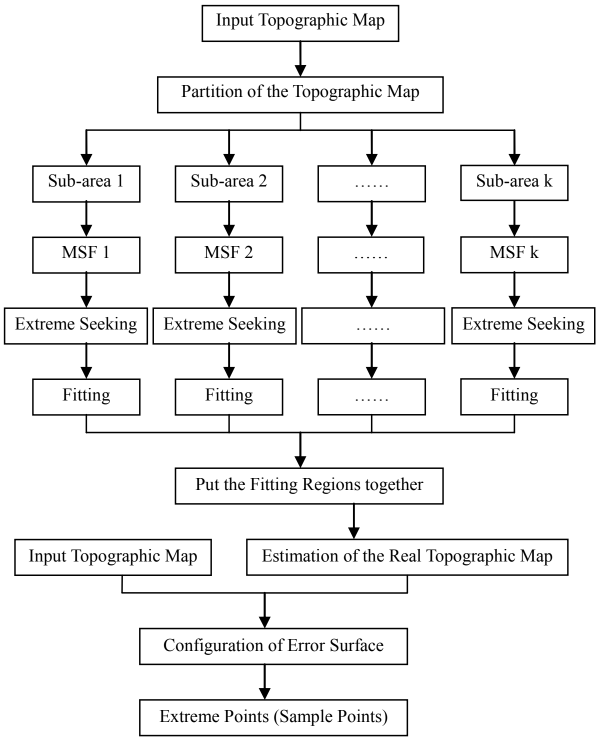

Figure 1. Flowchart for Spatial Sampling Scheme

MSF (Multi-scale Filter) involved in Figure 1 is assigned with different parameters. Filter in signal processing field is always used to filter the noise; here it is used to filter the error.

After filtering the error of the topographic map, we obtain an approximation of the real topographic data, and then combine it with the known measured data we already have. Obviously, the difference between the measured data and the approximation of the measured data should represent the error. Consequently, we can configure the error surface and further work out the extreme points. Finally, we can find the appropriate amount of sample points in an optimal location for this spatial sampling scheme. These key components of the overall sampling flowchart including partition of the topographic map, multi-scale filtering technique, extreme seeking, fitting technique, estimation of the real topographic data, configuration of error surface and determination of sample points will be discussed in detail in the following subsections.

4. PARTITION AND MULTI-SCALE

4.1 Prior Partition of the Sample Area

Different data capturing means can bring different errors to the topographic data; it can even have different effects on the same regions in the same topographic map. Usually, the means of field survey, aerial photogrammetry and paper map digitization are always used to produce the digital topographic map. Meanwhile, the errors of a topographic map will differ with the change of data producers. In addition, the different degrees of complexity of the terrain, and geographic features captured with the same means and even the identical producers, result in different errors of the topographic map.

Consequently, there are a lot of situations resulting in different errors of a topographic map; fortunately, we can obtain prior knowledge about the error distribution. Although we can not know the exact error distribution and the value of error, it's possible for us to roughly distinguish which part of a map has high error, which part has medium errors and which part has low errors. For example, in aerial photogrammetry, the fringes of some features are easy to distinguish and the accuracy of the stabbing points is comparatively higher and vice versa. Another example, in field surveys, the higher accuracy of traverse control survey, the lower error the features surveyed around the traverse.

On the basis of such prior knowledge, we can plan a rough partition of the topographic data to be evaluated. In this study, we conduct the research on a map obtained from field survey as an example. As the accuracy of all features vary with the accuracy of a traverse and the degree of difficulty of the feature points to be surveyed, we divide the map extent into two regions with different level of errors (Figure 2).

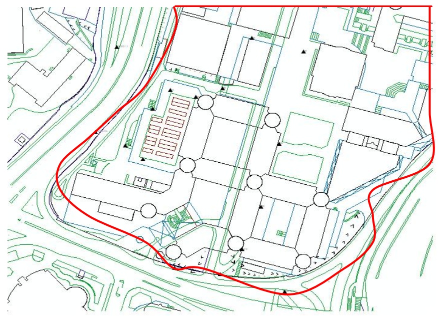
Figure 2. Prior Partition for a Sampling Extent

Figure 2 is the topographic data around the Hong Kong Polytechnic University, divided by a red line. PolyU buildings inside the red line are deemed to have a similar error, and so do roads outside the red line. The reason for such prior partition is that we have laid out two traverses in each partition respectively.

4.2 Multi-scale Filter on the Survey Data

Based on the accuracy of traverse, we have divided the sample area into several sub-areas. Although we have no idea about the true value of the error, we can roughly distinguish the distribution of error and then take further multi-scale filters on such measuring data. In this study, we will use the method of error removing based on the Gauss filter. We think of the measuring data with error as the signal with noise.

We need to seek the locations of the extreme points on the error surface, which are also contained in the real topographic data. The purpose of this is to fit more accurate topographic data. However, in practice, the real extreme point is covered by the error. So we want to use a particular filtering method to reveal the extreme point. Unfortunately, the real extreme point may be deduced by the filter to a certain extend, if the scale of the filter is too large. If the scale of the filter is too small, then the noise may not be removed completely. In fact, this is a classical problem of noise removing. In this section, we adopt a multi-scale filtering method to resolve this problem. We can describe the measuring data as a continuous curve in black and the error in orange.

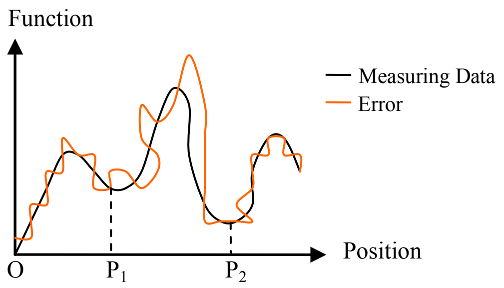

Figure 3. Error Simulation on Measuring Data

In figure 3, we find the error between the interval [P_1, P_2] is less than that outside this interval. Actually, such a condition is accordant with the case described in section 4.1. As the rough error distribution is known, we can process the error apart. Here the Gaussian filter is used to remove the noise. In the spatial domain, the isotropic Gaussian can be expressed as follows:

$$G(x,y)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{1}$$

Here, σ is used to control the filter degree based on the rough error partition.

Considering that in different sub-regions, the noise is also different. Therefore, it is not reasonable to use the Gaussian filter with a fixed scale (that is the value of σ is given and fixed) to filter the whole regions. A natural idea is to use different scale filters for different sub-regions, which is the mulit-scale filtering technique and is designed as follows:

Step1: the original topographic data is divided into K different sub-regions and denoted by $r_i^0\,(i=1,\dots,K)$. Their corresponding rough errors are denoted by $E_i\,(i=1,\dots,K)$, and suppose $E_1>E_2>\dots>E_K$ without loss of generality.

Step2: Given the filter scales in Equation (1) for the above sub-regions $\sigma_i^0\,(i=1,\dots,K)$, and let $\sigma_1^0>\sigma_2^0>\dots>\sigma_K^0$

Step3: The sub-regions are filtered by using the filters given in Equation (1) with the scales given in step 2 and denotes the filtered by $r_i^1\,(i=1,\dots,K)$.

Step4: set another groups of scales as $\sigma_i^1=\frac{\sigma_i^0}{2}(i=1,2,\dots,K)$, the same procedure as Step 3 is performed , and the corresponding results are denoted by $r_i^2\,(i=1,\dots,K)$

Step5: repeat the procedure of step 4 and set the scales of Gaussian as $\sigma_i^n=\frac{\sigma_i^0}{n-1}(i=1,2,\dots,K)$, the filtered result is then represented as $r_i^n\,(i=1,\dots,K)$.

Step6: extracting the extreme points of $r_i^1, r_i^2,\dots, r_i^n\,(i=1,\dots,K)$, and checking the extreme points that have disappeared. Those across many layers are selected as robust extreme points. And then the corresponding locations are recorded.

Remark 1: the reason for step 6 is based on the understanding that the robust extreme point has disappeared slowly. Those points that disappeared quickly (that is remained in a few layers) are not the real extreme points. The geometric illustration in one dimensional case is given in Figure 4. From Figure 4, we see that the points labelled by star symbols ("*") have weakened slowly, while the points labelled by cross symbols ("+") have disappeared quickly, so we have reason to conclude that the star points are really the extreme points while the cross points are false extreme points or noise points.

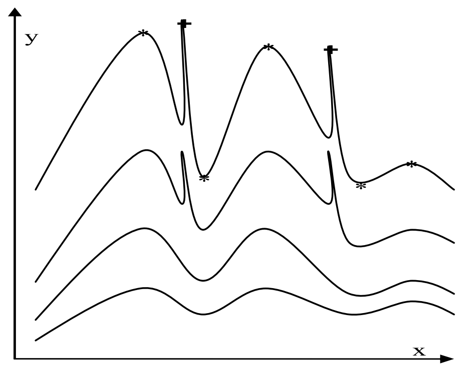

Figure 4. Extreme Point across Layers

Remark 2: in practice, the number of the filtered layers should be considered. That is the filtered time used must be determined. There are two simple strategies for this problem. One is to give a filtered time in advance, and another is that the filtering time can be given by comparing two adjacent filtered results, if the difference of their extreme points is smaller than a given threshold, then the filtered procedure can be phased out. Certainly, the filtered times in each sub-region may be different.

4.3 Fitting

We proposed different filters for different sub-areas and obtained the function curve of the measuring data after filtering the error. After the Gauss filtering, we can get several peak points on the curve filtered, which represents the true location of the topographic data contrasted against the measuring data with error.

Although some extreme points on the measuring data will be weakened, and even be eliminated after the filtering, we can say the remaining extreme points are the real extremes of the topographic data with high confidence.

After we extract several extreme points from the measuring topographic data in each sub-region, the next task is to fit these extreme points into a dense point set by a particular method. In fact, there are many methods that can be employed, such as interpolation techniques and other fit methods. For sake of simplicity, the bi-cubic interpolation method which is used frequently in many applications is adopted. Its formula is expressed as:

$$\begin{aligned} z &= a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 \\ &\quad + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 \end{aligned} \tag{2}$$

4.4 Estimation of the Real Topographic Data

By the steps introduced in section 4.3, approximated sub topographic data is obtained. As mentioned before, on the basis of prior knowledge about the rough error distribution on the topographic data, we have divided the map into several sub-areas. In order to assess the integrated accuracy of the topographic data, we have to put all the fitted functions together to represent the integrated representative expression of the whole topographic data.

Remember that the above interpolation method is implemented on each of the sub-regions, then for each location on the boundary of each sub-region, there are two interpolated results, which interpolated result should be accepted? A simple method is to take the average value of these two results. An illustration for this point is given in Figure 5.

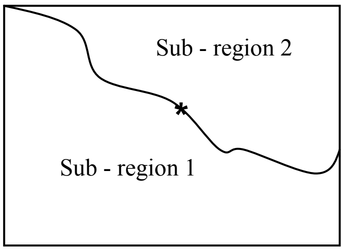

Figure 5. Illustration of Boundary Point

5. CONSTRUCTION OF ERROR SURFACE

5.1 Configuration of Error Surface

After fitting the true function for the real data, denoted as $F_1(x)$; and we already have the function of the measured data, denoted as $F_0(x)$, and then we can obviously obtain the error surface for the research data, which can be denoted as $E = F_0(x) - F_1(x)$.

5.2 Search of Sample Points

During the spatial sample on the topographic data, we would like to select the points which are representative of the population to the most extent. To satisfy this, such points in the sample areas will be characteristic of the large change in the error value. We would not like to use the maximum error or the minimum error to represent the whole error of the topographic data, as it will magnify or shorten the actual error and can not reflect the true situation.

We have thought the error is similar to some extent and we have treated the error by this way; in another words, the error in one area has the equivalent floatability. If we can find the error with obvious change corresponding to the adjacent one, we can describe the amount of the error more accurately for the topographic data.

The function description of the error is obtained by the method introduced in section 5.1. The problem of finding an error with a sharp change is equal to the problem of finding the peak points on the error surface. Consequently, we can select such points as our sample points. So a key problem is to how to select the peak on the error surface.

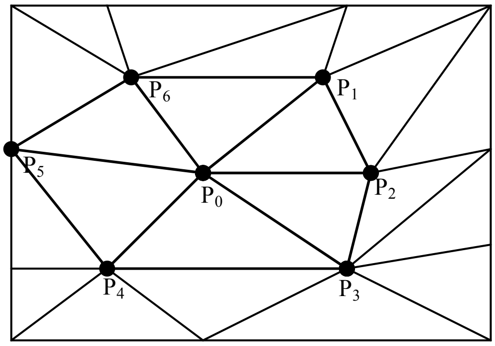

Figure 6. Extraction of Peaks on Error Surface

A method can be designed as follows:

Step1: for the points located on the error surface, construct a Delauney triangle network.

Step2: for a point p, compare it with the points linked to this point, if it is the smallest or the biggest, then it is a peak, otherwise, it is not a peak.

An example is given in Figure 6, if we want to determine whether the point P_0 is the peak point, the points $P_i\,(i = 1,\dots 6)$ should be compared with the point P_0 , if the point P_0 is the smallest or the biggest among these six points, then the point P_0 is a peak point. This procedure is implemented over each of the points, and then all the peak points can be extracted.

The peak points extracted are considered as the representative of the accuracy of the topographic data. As the error contribution from such points can be used to describe the integrated accuracy, they can be used to decide the selection of the sample points in this study. In other words, if we can find out these peak points, the spatial sampling scheme can be decided.

6. CONCLUSIONS

In this paper, we propose a novel spatial sampling scheme. The proposed approach aims at sampling as few points as possible, as well as reflecting the error of the topographic data. Based on the proposed method, the appropriate sample points would hope to be selected not only to reflect the true error situation of the topographic data, but also to provide a description of the error distribution. Compared with existing spatial sampling methods for spatial data in GIS, the approach presented in this paper provides a possible method to give a generic solution for error removing.

The novel idea of this method is that the error of the topographic map is considered and regarded as a noise in the signal viewpoint. On the basis we suppose that the error obeys the Gaussian distribution, we try to remove the error as if removing the noise using different Gaussian filters in signal processing. The error-free data can help us to obtain the error surface. The optimal sample points would hope to be selected based on the searching the extreme points on the error surface. Furthermore, the error surface can rightly reflect the error distribution of the topographic data.

An important application of this method is to assess the quality of the topographic data. On the condition when we only have the measured topographic data at hand and prior knowledge of the rough error distribution of the topographic data, we could design a good solution for spatial sampling to select the right sample points to reflect the accuracy and the error distribution of the topographic data.

REFERECNES

American Society for Photogrammetry and Remote Sensing (ASPRS) Specifications and Standards Committee, 1990. ASPRS Accuracy Standards for Large-Scale Maps. *Photogrammeric Engineering and Remote Sensing*, 56(7), pp. 1068-1070.

Angulo, J.M. and Bueso, M.C., 2001. Random perturbation methods applied to multivariate spatial sampling design. *Environmetrics*, 12, pp. 631-646.

Angulo, J.M., Ruiz-Medina, M.D., Alonso, F.J. and Bueso, M.C., 2005. Generalized approaches to spatial sampling design. *Environmetrics*, 16, pp. 523-534.

Banjevic, M. and Switzer, P., 2002. Bayesian network designs for fields with variance as a function of the location. *Proceedings of the 2002 JSM Conference*.

Bogaert, P. and Russo, D., 1999. Optimal spatial sampling design for the estimation of the variogram based on a least squares approach. *Water Resources Research*, 35, pp. 1275-1289.

Bolstad, P.V., Gessler, P. and Lillestand, T.M., 1990. Positional uncertainty in manually digitized map data. *International Journal of Geographical Information System*, 5, pp. 159-168.

Devillers, R. and Jeansoulin, R., 2006. *Fundamentals of Spatial Data Quality*. ISET Ltd, Britain and United States, pp. 15.

Federal Geographic Data Committee, 1998. Part 3, National Standard for Spatial Data Accuracy, Geospatial Positioning Accuracy Standards. FGDC-STD-007.3-1998: Washington, D.C., Federal Geographic Data Committee, 24 p.

Goodchild, M.F., 1991. Keynote address: Symposium on Spatial Database Accuracy. In *Proceedings of Symposium on Spatial Database Accuracy*, edited by G.J. Hunter (Melbourne: Department of Geometrics, University of Melbourne, Australia), pp. 1-16.

Jennifer, C. and Walsby, 1995. The causes and effects of manual digitizing on error creation in data input to GIS. *Em Innovations in GIS 2*. Editado por Peter Fisher, Taylor and Francis, pp. 113-122.

Meng, X. L., Shi, W.Z. and Liu, D.J., 1998. Statistical tests of the distribution of errors in manually digitized cartographic lines. *Geographic Information Sciences*, 4, pp.52-58.

Muller, W.G., Zimmerman, D.L., 1999. Optimal designs for variogram estimation. *Environmetrics*, 10, pp. 23-37.

Royle, J.A., 2002. Exchange algorithms for construction large spatial designs. *Journal of Statistical Planning and Inference*, 100(2), pp. 121-134.

Shi, W.Z., Cheung, C.K. and Tong, X.H., 2004. Modelling error propagation in vector-based overlay analysis. *ISPRS Journal of Photogrammetry & Remote Sensing*, 59, pp. 47-59.

Shi, W.Z., Cheung, C.K. and Zhu, C.Q., 2003. Modelling error propagation in vector-based buffer analysis. *International Journal of Geographical Information Science*, 17(3), pp. 251-271.

Shi, W.Z. and Liu W.B., 2000. A stochastic process-based model for the positional error of line segments in GIS. International Journal of Geographical Information Science, 14, pp. 51-66.

Shi, W.Z., 1994. Modeling positional and thematic error in integration of GIS and remote sensing. ITC Publications, Enschede, 22, 147p.

Shi, W.Z., 1997. Statistical modeling uncertainties of three-dimensional GIS features. *Cartography and Geographic Information Systems*, 24, pp. 21-26.

Shi, W.Z., 1998. A generic statistical approach for modeling error of geometric features in GIS. *International Journal of Geographical Information Science*, 12, pp. 131-143.

U.S. Bureau of the Budget, 1947. United States National Map Accuracy Standards. U.S. Bureau of the Budget, Washington, D.C..

Van Groenigen, J.W. and Stein, A. 1998. Constrained optimization of spatial sampling using continuous simulated annealing. *Journal of Environmental Quality*, 27(5), pp. 1078-1086.

Wiens, D.P., 2005. Robustness in spatial studies i: Minimax design. *Environmetrics*, 16(2), pp. 191-203.

Zhou, S.J., Lu, T.D., Guan, Y.L. and Zang D.Y, 2003. The Analytical Collocation of the Measurement Errors Distribution. *Acta Metrologica Sinica*, 24(3), pp. 250-253.

Zhu, Z., Stein, M., 2005. Spatial sampling design for parameter estimation of the covariance function. *Journal of Statistical Planning and Inference*, 134(2), pp. 583-603.

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