AN ALGEBRA FOR MOVING OBJECTS

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ABSTRACT:

Qualitative representing and reasoning for road network moving objects is an important problem in spatio-temporal reasoning, but it is till open. We propose a new spatial algebra for road network moving objects based on Interval Algebra Theory. Renz has developed an algebra DIA to represent one dimensional moving objects by giving time interval a spatial interpretation. We extend DIA to road network, propose RNDIA. We give an algebra formalization for RNDIA compatible to DIA. Then the reasoning problems such as RSAT are primarily discussed. Our work proposes an executable qualitative representation and reasoning method of road network moving object.

1. INTRODUCTION

Spatio-temporal reasoning (STR), the researching fields aiming spatial and/or temporal questions, has wide variety of potential applications in Artificial Intelligence (such as spatial information systems, robot navigation, natural language processing, visual languages, qualitative simulation of physical processes and commonsense reasoning) and other fields (such as GIS and CAD)00.

But most time variant spatial objects are hard or costly to track or record except moving objects. Existing technology has made it possible to track down movements of target objects in the air (e.g., airplanes), on the land (e.g., vehicles, wild animals, people, etc.) and ocean (e.g., ships, animals), we call them moving objects. The position of object is usually detected by GPS and presented by a single point. Since point has no shape or area, its spatio-temporal attribute is simple.

There are a wide range of research issues towards moving objects, including modeling / representation, query language design, indexing techniques and query optimization0000.
In common with other spatio-temporal areas, we believe that moving objects research should generally be characterized as: techniques for interpreting the semantic content of spatio-temporal data, extracting and reasoning the spatial knowledge have lagged behind the techniques for storage and query spatio-temporal data. Further more, prior work on spatio-temporal reasoning is relevant but insufficient for moving objects. Therefore, the formal theories especially the qualitative knowledge representation models for moving objects are needed to solve this problem.

There are two types of moving objects: those having a free trajectory such as a bird flying through the sky and those with a constrained trajectory in which the movement of an object in space is strongly restricted due to physical constraints, e.g. a vehicle driving through a city. Since vehicles equipped with GPS are quite common in these days, the constrained trajectory, or just call it road network moving object (RNMO), is more practical.

Very little researches have been performed towards the RNMO reasoning aspect. Weghe and Cohn build a calculus of relations between disconnected network-based mobile objects in 2004, the connected topological relation are not considered. Renz give an algebra DIA to represent one dimensional moving objects in 20010. DIA is developed form Interval Algebra, and includes both topology and direction information. He call this the first step phase of spatial odyssey. But no one extends his work to 2D free or restricted movements as yet.

RNMO is similar to one dimensional moving object in a certain extent. In this paper, we extended the DIA to RNDIA. We give a compatible algebra formalization for RNDIA. Later the composition reasoning and SAT reasoning are discussed.

The rest of the paper is organized as follows. In section 2, we introduce Renz's Directed Intervals Algebra. In section 3 and section 4, Road Network Directed Intervals Algebra is presented. In section 5, reasoning method of RNDIA is discussed. Section 6 concludes the paper.

2. DIRECTED INTERVALS ALGEBRA

When looking at vehicles on one particular way, vehicles and their regions of influence (such as safety margin, braking distance, or reaction distance) could be represented as intervals on a line which represents the possibly winded way. Similar to the well-known Interval Algebra developed for temporal intervals 0, it seems useful to develop a spatial interval algebra for RNMO.

There are several differences between spatial and temporal intervals which have to be considered when extending the Interval Algebra towards dealing with spatial applications.

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(1) spatial intervals can have different directions, either the same or the opposite direction as the underlying line.

(2) ways usually have more than one lane where vehicles can move, i.e., it should be possible to represent that intervals are on different lanes and that one interval is, e.g., left of, right of, or beside another interval.

(3) it is interesting to represent intervals on way networks instead of considering just isolated ways.

(4) intervals such as those corresponding to regions of influence often depend on the speed of vehicles, i.e., it should be possible to represent dynamic information.

Renz developed an algebra DIA for qualitative spatial representation and reasoning about directed intervals, identify tractable subsets, and show that path-consistency is sufficient for deciding consistency for a particular subset which contains all base relations0. The 26 base relations of DIA are list in Table 1.

Directed Intervals Base Relation	Symbol	Pictorial Example
x behind$_=$ y	$b_=$	-x-> -y->
y in-front-of$_=$ x	$f_=$	
x behind$_{\neq}$ y	$b_{\neq}$	<-x- -y->
x in-front-of$_{\neq}$ y	$f_{\neq}$	-x-> <-y-
x meets-from-behind$_=$ y	$mb_=$	–x–> –y–>
y meets-in-the-front$_=$ x	$mf_=$	
x meets-from-behind$_{\neq}$ y	$mb_{\neq}$	<–x– –y–>
x meets-in-the-front$_{\neq}$ y	$mf_{\neq}$	–x–> <–y–
x overlaps-from-behind$_=$ y	$ob_=$	—x—> —y—>
y overlaps-in-the-front$_=$ x	$of_=$	
x overlaps-from-behind$_{\neq}$ y	$ob_{\neq}$	<—x— —y—>
x overlaps-in-the-front$_{\neq}$ y	$of_{\neq}$	—x—> <—y—
x contained-in$_=$ y	$c_=$	–x–>
y extends$_=$ x	$e_=$	—y—>
x contained-in$_{\neq}$ y	$c_{\neq}$	<–x–
y extends$_{\neq}$ x	$e_{\neq}$	—y—>
x contained-in-the-back-of$_=$ y	$cb_=$	–x–>
y extends-the-front-of$_=$ x	$ef_=$	—y—>
x contained-in-the-back-of$_{\neq}$ y	$cb_{\neq}$	<–x–
y extends-the-back-of$_{\neq}$ x	$eb_{\neq}$	—y—>
x contained-in-the-front-of$_=$ y	$cf_=$	–x–>
y extends-the-back-of$_=$ x	$eb_=$	—y—>
x contained-in-the-front-of$_{\neq}$ y	$cf_{\neq}$	<–x–
y extends-the-front-of$_{\neq}$ x	$ef_{\neq}$	—y—>
x equals$_=$ y	$eq_=$	—x—> —y—>
x equals$_{\neq}$ y	$eq_{\neq}$	—x—> <—y—

Table 1: The 26 base relations of the directed intervals algebra

3. ROAD NETWORK DIRECTED INTERVALS ALGEBRA

The Directed Intervals Algebra surely took a big step towards the real world applications of qualitative spatial reasoning, the large body of work and the large number of results obtained on the Interval Algebra such as algorithms and complexity results also be applied to a spatial interpretation of the Interval Algebra. Unfortunately, the DIA is only suitable for one dimensional line, the free or restricted 2D movements are not supported. So there is still a long way towards the final goal of putting them into practice. In this paper, we extend DIA to Road Network Directed Intervals Algebra (RNDIA) which could be used to model moving cars in Road Network (or other types of restricted 2D movements) .

Since road network is an undirected graph, to apply DIA on it we must define "direction" on road network. The first step is to set up a frame of reference. So the origin and order is surely required. The origin is a point in the road network, may be the center of the city or the south west corner of city or the start/end point of highways. The selection of origin is critical for model the scene, choosing a meaningful origin is important. For instance, when discuss "the highways near Beijing", choosing Beijing as the origin is a better choice.

It's more difficult to choose the order, since the road network is an undirected graph. To solve this problem, we use the shortest path. Assume S is the origin of road network $\Re$, SP(S, x) indicates the length of the shortest path in $\Re$ from S to x. Note that both S and x are the points on the edge , not the vertexes of the graph. If S and x are not on the vertexes of $\Re$, SP(S, x) can be calculated by adding S and x as new vertexes to $\Re$ and performing the classical shortest path algorithm.

For two points x and y in $\Re$, if SP(S, x) < SP(S, y) then $x< y$, If SP(S, x) ≤ SP(S, y) then $x \leq y$. (Fig. 1)

Figure 1. Road network directed intervals algebra

Another type of order is defined by two points, such as "the highways from Beijing to Shanghai". The metric value of x from S to T is defined by SP(S,T,x) (Fig. 2):

$$SP(S,T,x) = \frac{SP(S,x)}{SP(S,x)+SP(T,x)} SP(S,T)$$

Without loss of generality, we only use the first definition of SP in the latter part of this paper.

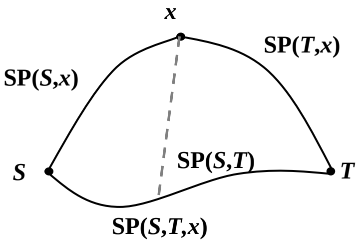

Figure 2. Two points order

In DIA , the car movement is represented by a straight line which interpreted as vehicles and their regions of influence (such as safety margin, braking distance, or reaction distance). It can also be interpreted as car movement track during a period of past time or in the near future. Future track is estimated by current speed and direction. What ever the case, DIA can not tell us when the car is turning around or two cars moving on parallel/crossed streets. In our work, car movement track is represented by a directed curve embedded in road network.

Track $X=<x_1, x_2>$ is the movement track from x_1 to x_2. The direction of track is defined:

$$D(<x_1,x_2>)=\begin{cases} + & \text{if } \forall p \in <x_1,x_2> [x_1 \le p \le x_2] \\ - & \text{if } \forall p \in <x_1,x_2> [x_2 \le p \le x_1] \\ * & \text{otherwise} \end{cases}$$

The reversed direction operator is ~ (Table 2).

	+	-	*
~	-	+	*

Table 2. The reversed direction operator

X is “simplex directed” iff $D(X) \neq \{*\}$ and $\forall A \subset X\ [D(A) = D(X)]$ (Fig. 3).

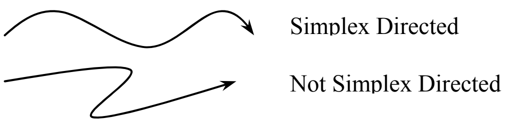

Figure 3. Simplex directed

In this paper, we assume that **all tracks are simplex directed**. This assumption is reasonable if we only investigate the short tracks (the movement within a small period of past/future time).

The order of two tracks $X=<x_1, x_2>$, $Y=<y_1, y_2>$ is defined as follows:

If $\max\{x_1, x_2\} < \min\{y_1, y_2\}$ then $X < Y$.
If $\max\{x_1, x_2\} \le \min\{y_1, y_2\}$ then $X \le Y$.

The union of *X* and *Y* has no branch iff

$$\forall A \subset X, \forall B \subset Y\ [A \cap B = \varnothing \rightarrow A < B \ \vee \ B < A]$$

,

denoted by NB(*X* , *Y*) (Fig. 4).

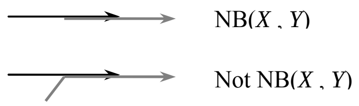

Figure 4. No branch

NB(*X* , *Y*) means the *X* and *Y* can be treated as one dimensional directed intervals since $X \cup Y$ has no branch, otherwise it is not the case.

Proposition 1: *X* is simplex directed → NB(*X* , *X*)
Proof. We assume that $\forall <a_1,a_2>\ <b_1,b_2> \subset X$. Without loss of generality, we assume that $D(X)=\{+\}$ and $a_1 < b_1$. So $a_1 < a_2$ and $b_1 < b_2$. $\{<a_1,a_2>\} \cap \{<b_1,b_2>\} = \varnothing \rightarrow a_2 < b_1 \rightarrow <a_1,a_2> < <b_1,b_2>$.

Proposition 2: *X*,*Y* are simplex directed**,** $X \subseteq Y$ or $Y \subseteq X \rightarrow$ NB(*X*,*Y*)
Proof. We assume that $X \subseteq Y$, NB(*X* , *Y*)= NB(*X* , *X*).

Proposition 3: *X*,*Y* are simplex directed**,** $X \le Y$ or $Y \le X \rightarrow$ NB(*X*,*Y*)
Proof. Given two simplex directed tracks $X=<x_1, x_2>$, $Y=<y_1, y_2>$. We assume that $X \le Y$. $\forall A=<a_1,a_2> \subset X$, $\forall B=<b_1,b_2> \subset Y$, $\max\{a_1, a_2\} \le \max\{x_1, x_2\} \le \min\{y_1, y_2\} \le \min\{b_1, b_2\}$. So $A \le B$.

Proposition 4: If NB(*X*,*Y*) the relation between *X* and *Y* is equal to the relation between two intervals in one dimension line.
Proof. We build a mapping from *X* ,*Y* to two intervals in one dimensional line . $\delta(p)$: $p \in X,Y \rightarrow$ SP(*S*, *p*). Every $p \in X,Y$ have one and only one $\delta(p)$. No two points *p*,*q* of *X* ,*Y* map to same value ($\delta(p) = \delta(q)$) , otherwise the small subtracks <*p*><*q*> contained *p* and *q* , have ¬NB(<*p*>,<*q*>) , it will lead to ¬NB(*X* , *Y*). So δ is one to one mapping. And the relation of *X* and *Y* can be exclusively represented by the two intervals.

Next we will give the base RNDIA relations of two simplex directed track by the predications defined above.

4. BASE RNDIA RELATIONS

Firstly, those RNDIA base relations are divided into two parts: NB(*X*,*Y*) and not NB(*X*,*Y*). If NB(*X*,*Y*) is true, *X*,*Y* can be simply treated as the one dimensional directed intervals , RNDIA is equivalent to DIA. In other cases we have to define some new relations.

The points set of track *X* is {*X*}. {*X*} is an infinite set. Unlike *X* , {*X*} has no direction.

Given two simplex directed tracks $X=<x_1, x_2>$, $Y=<y_1, y_2>$.

By Proposition 2 we know that when one track is totally contained in the other there is surely no branch, there is no need to define new relations under the conditions (1) to (3).

(1) {*X*}={*Y*}
If $D(X) = D(Y)$ then $X\ eq_{=}\ Y$
Else $X\ eq_{\neq}\ Y$

(2) $\{X\} \subset \{Y\}$
If D(X) = D(Y)
If x_2=y_2 then X cf$_=$ Y
Else If x_1=y_1 then X cb$_=$ Y
Else X c$_=$ Y;
Else
If x_1=y_2 then X cf$_\neq$ Y
Else If x_2=y_1 then X cb$_\neq$ Y
Else X c$_\neq$ Y;

(3) $\{Y\} \subset \{X\}$
If D(X) = D(Y)
If x_2=y_2 then X eb$_=$ Y
Else If x_1=y_1 then X ef$_=$ Y
Else X e$_=$ Y;
Else
If x_2=y_1 then X ef$_\neq$ Y
Else If x_1=y_2 then X eb$_\neq$ Y
Else X e$_\neq$ Y;

(4) $\{X\} \cap \{Y\}$ is line
(4.1) NB(X , Y)
$Z = X \cap Y$. Z is connected (has no detached parts) and Z is not in the middle of X or Y, otherwise there will be branches (Fig. 5).

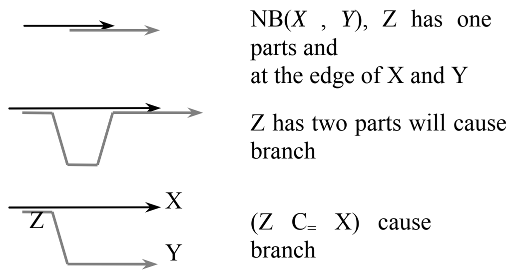

Figure 5. Overlay Relation

So we have 4 relations equivalent to DIA relations:
If D(X) = D(Y)
If (D(X) = {+} and $x_2 < y_2$) or (D(X) = {-} and $y_2 < x_2$) then X ob$_=$ Y
Else X of$_=$ Y ;
Else
If (D(X) = {+} and $x_2 < y_1$) or (D(X) = {-} and $y_1 < x_2$) then X ob$_\neq$ Y
Else X of$_\neq$ Y ;

(4.2) not NB(X , Y)
Tow new relations are proposed:
If D(X) = D(Y)
X po$_=$ Y ;
Else
X po$_\neq$ Y ;

The latter two case in Fig. 5 are all po$_\neq$.

(5) $\{X\} \cap \{Y\}$ is one or more points
In this case NB(X,Y) iff $X \le Y$ or $Y \le X$ (Fig. 6). And two new relations are defined when NB(X,Y) is false(Fig. 7). So we defined relations as:

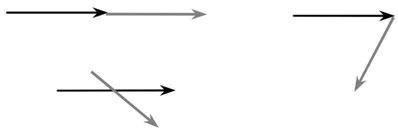

Figure 6. Intersection is point

If D(X) = D(Y)
If (D(X) = {+} and $Y \le X$) or (D(X) = {-} and $X \le Y$) then X mf$_=$$Y$;
ElseIf (D(X) = {+} and $X \le Y$) or (D(X) = {-} and $Y \le X$) then X mb$_=$$Y$;
Else X cr$_=$ Y;
Else
If (D(X) = {+} and $X < Y$) or (D(X) = {-} and $Y < X$) then X mf$_\neq$$Y$;
ElseIf (D(X) = {+} and $Y < X$) or (D(X) = {-} and $X < Y$) then X mb$_\neq$$Y$;
Else X cr$_\neq$ Y;

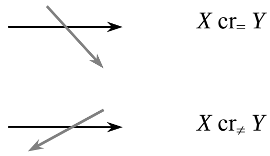

Figure 7. Cross Relations

(6) $\{X\} \cap \{Y\} = \varnothing$
Similar to (5) NB(X,Y) iff $X < Y$ or $Y < X$.

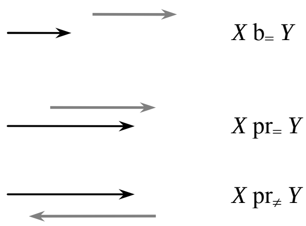

Figure 8. Disconnected Relations

If D(X) = D(Y)
IF (D(X) = {+} and $Y < X$) or (D(X) = {-} and $X < Y$) then X f$_=$$Y$;
IF (D(X) = {+} and $X < Y$) or (D(X) = {-} and $Y < X$) then X b$_=$$Y$;
Else X pr$_=$ Y
Else
IF (D(X) = {+} and $X < Y$) or (D(X) = {-} and $Y < X$) then X f$_\neq$$Y$;
IF (D(X) = {+} and $Y < X$) or (D(X) = {-} and $X < Y$) then X b$_\neq$$Y$;
Else X pr$_\neq$ Y

{ eq$_=$,eq$_\neq$, cf$_=$, cb$_=$, c$_=$, cf$_\neq$, cb$_\neq$, c$_\neq$,ef$_=$, eb$_=$, e$_=$, ef$_\neq$, eb$_\neq$, e$_\neq$, ob$_=$, of$_=$, ob$_\neq$, of$_\neq$, po$_=$, po$_\neq$,mf$_=$, mb$_=$, mf$_\neq$, mb$_\neq$, cr$_=$, cr$_\neq$, b$_=$, f$_=$, b$_\neq$, f$_\neq$, pr$_=$, pr$_\neq$ } are 32 base relations of RNDIA. From the above procedure, we know they are JEPD (jointly exhaustive and pair-wise disjoint) relations. According to Proposition 4, { eq$_=$,eq$_\neq$, cf$_=$, cb$_=$, c$_=$, cf$_\neq$, cb$_\neq$, c$_\neq$,ef$_=$, eb$_=$, e$_=$, ef$_\neq$, eb$_\neq$, e$_\neq$, ob$_=$, of$_=$, ob$_\neq$, of$_\neq$, mf$_=$, mb$_=$, mf$_\neq$, mb$_\neq$, b$_=$, f$_=$, b$_\neq$, f$_\neq$} are equivalent to DIA since they have no branch (NB(X , Y)).
When the road network $\Re$ has no branch, such as a single highway without exit, and S is the start point of the highway, NB(X,Y) is always true. In this case , from the definition we can

deduce that NRDIA is equal to DIA. So DIA is a special case of RNDIA (DIA $\subset$ RNDIA).

5. SPATIO-TEMPORAL REASONING OF RNDIA

Two main topics of STR are representation and reasoning. Composition reasoning is the basic operation for further reasoning works such as CSP. The composition is to determine *xTz* by *xRy* and *ySz*. In most cases, *T* can't be unique. And {*} means the union of all base relations. The composition of relation S,T is represented by S ○ T. The composition of DIA has been specified by Renz0.

The composition of RNDIA is quite difficult than DIA. Because the DIA relations are not close in RNDIA . For instance, $b_= \circ f_= = \{*\}_=$ (If x behind y and y in front of z then x and z can be any relation without consider directions).

Unlike DIA, we have no idea about automatically calculate composition of RNDIA. By enumerating all possible compositions, we give the composition table of RNDIA. But it is too big to list here.

Directions {= , ≠} are not considered in composition table of RNDIA, they are determined by the following rule:

$R_p, S_q \in$ RNDIA

$$R_p \circ S_q = \begin{cases} (R \circ S)_p & \text{if } q=\{=\} \\ (R \circ S)_{\neg q} & \text{if } q=\{\neq\} \end{cases}$$

Constraint satisfaction problem (CSP) as typical reasoning model is also very important in STR. Most constraint satisfaction problem we studied in STR is RSAT problem which is formally defined as :

Given a set Θ of constraints of the form *x*R*y* , where x,y are variables and R is an spatio-temporal relation such as DIA or RNDIA. Deciding the consistency of Θ is called RSAT. An assignment of spatio-temporal regions such that all the constraints are satisfied is a model of Θ. A set of spatio-temporal relation R^* is tractable if RSAT of R^* can be solved in polynomial time.

Every pair of variables has constrain in Θ. If no information is given about the relation holding between two variables x and y, then the universal constraint x{*}y is contained in Θ.

We say that a set of constraints Θ' is a refinement of Θ if and only if the same variables are involved in both sets, and for every pair of variables x,y, if xR'y $\in \Theta'$ and xRy $\in \Theta$ then R'⊆R. Θ' is a consistent refinement of Θ if and only if Θ' is a refinement of Θ and both Θ' and Θ are consistent. A consistent scenario Θ_s of a set of constraints Θ is a consistent refinement of Θ where the relation of every constraint in Θ_s is a basic relation. The model of a consistent constraints Θ could be described by Θ_s for all the basic relations between the variables are determined.

There are several ways of deciding consistency of a given set of constraints over a set of relations. The most common way is to use backtracking over Θ . This is done by applying for each triple of constraints *xTz* , *xRy* , *ySz* the operation $T = T \cap (R \circ S)$.

If the empty relation is not contained, the resulting set is path consistent. But the RSAT of RNDIA is clearly NP-hard since the consistency problem of the DIA and Interval Algebra are already NP-hard. Give the tractable subset of RNDIA is our next work.

6. CONCLUSION

The main contribution of this paper is a methodology for representing and reasoning road network moving object. We extended the Directed Interval Algebra to Road Network Directed Interval Algebra for dealing with road network moving object. By applying a shortest path based order system, we establish a mapping from road network to one dimensional line. RNDIA base relations are defined with compatible to DIA relations. DIA is a special case of RNDIA. Then the reasoning problems such as RSAT are primarily discussed. Our work improves the application of Interval Algebra theoretical results, proposes an executable qualitative representation and reasoning method of road network moving object.

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