A FUZZY SPATIAL REGION MODEL BASED ON FLOU SET

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KEY WORDS: Spatial Region Model, Fuzzy Region, Flou set, GIS, Membership Value

ABSTRACT:

Uncertainty modeling for geometric data is currently an important problem in spatial reasoning, geographic information systems (GIS) and spatial databases. In many geographical applications spatial regions do not always have homogeneous interiors and sharply defined boundaries but frequently their interiors and boundaries are fuzzy. This paper provides a fuzzy spatial region model based on flou set. The model is different from the original fuzzy model that uses membership value but makes use of the relative relations between point sets, and represents a fuzzy region as a flou set. The model can obtain the properties and relations of fuzzy regions by using the operational properties of flou set. The fuzzy region model based on flou set is valuable in the fields such as GIS, geography and spatial database.

1. INTRODUCTION

Spatial region is a kind of geographic entities in many fields such as geography, geology, environment and soil, etc. These entities will often be fuzzy when they are represented as spatial objects. This kind of entities is continuous in space, and two fuzzy spatial regions may share a common gradual boundary, e.g., the transition zones between a desert and a prairie, among different soil strata, or between a hill and a valley. It is hard to decide the positions of boundaries for the fuzzy spatial entities, such as forests, prairies, climate regions and habitats of animals. This kind of spatial objects is usually called *fuzzy objects*.

At present, there are two common perspectives in which spatial objects can be considered as spatially fuzzy (Kulik, 2003). One view is called *ontic fuzziness*, where the objects themselves are fuzzy. The other view is called *semantic fuzziness*, where the concepts or representations of the objects are fuzzy. Spatial fuzziness is considered as a variant of semantic fuzziness in this paper, i.e., a term, such as " forest " , does not refer to a single fuzzy object in space but its spatial location will be represented as a fuzzy region.

In most current geographic information systems and spatial databases the spatial extensions of geographic objects are often modeled as sharp regions that have a unique boundary, and therefore are unable to represent and deal with fuzzy objects that have fuzzy boundary. It has been one of the fundamental problems to define and represent fuzzy spatial regions in geographic information systems with fuzzy objects. The formalization of fuzzy regions also plays an increasingly important role in geographic information systems. In computer science and geographic information science, spatial fuzziness is often modeled by employing fuzzy approaches. Based on fuzzy set theory, fuzzy approaches use the concept of different degrees of membership to describe fuzziness.

This paper presents a fuzzy spatial region model based on flou set. It can well formalize fuzzy spatial region and can be used conveniently in practical applications. It is different from the original fuzzy model that uses membership values. The flou-set model makes use of the relative relation between sets, and represents a fuzzy region as a flou set. This model can obtain the property and relation of fuzzy spatial regions using the operational property of flou set. The rest of the paper is organized as follows. Firstly, the fundamental concepts and operations of flou set are showed in section 2. Then in section 3, the representation and properties of fuzzy region model based on flou set are presented in detail. It is also illuminated that the method is effective and practicable through a practical case in section 4. In the following section the method also is compared with relative work. Finally the conclusions of this paper and future work are given in section 6.

2. FLOU SET

Flou set stems from some linguistic considerations of Yves Gentilhomme about the vocabulary of a natural language (Gentilhomme, 1968). Now flou set is often used to model fuzzyness and uncertainty. The fundamental concepts, properties and operations of flou set are introduced as follows (Kerre, 1999).

Definition 1: A *flou set* in a universe U is a pair (E, F) of subsets of U such that $E \subseteq F$. E is called *the certain zone*, F is called *the maximal zone* and $F \setminus E$ is called *the flou zone* (see Fig. 1). The class of flou sets in a universe U will be denoted by $FS(U)$, namely $FS(U) = \{(E,F) \mid E \subseteq U, F \subseteq U, E \subseteq F\}$

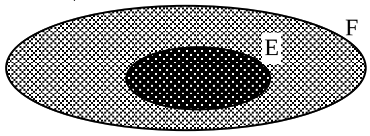

Fig. 1. Illustration of a flou set

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A flou set $A=(E,F)$ can be explained as follows: E is the set of center elements in A, $F \setminus E$ is the set of surrounding elements, and the elements in E are said more belonging to A than the elements in $F \setminus E$.
For example, let U be the set of all possible heights expressed in cm for a person, e.g., an interval [0, 300]. The class of high persons could be then represented as a flou set ([180, 300], [160, 300]) indicating that every height ≥ 180 can be accepted as tall, every height ≤ 160 is considered as not tall, and every 160 ≤ height ≤ 180 is classified as doubtful with respect to the label tall.
Two flou sets $A=(E,F)$ and $B=(E',F')$ in $FS(U)$ may be provided with the following binary (AND and OR) and unary (NOT) operations:

$$\mathrm{A} \cap \mathrm{B} = (E,F) \cap (E',F') = (E \cap E', F \cap F')$$
$$\mathrm{A} \cup \mathrm{B} = (E,F) \cup (E',F') = (E \cup E', F \cup F')$$
$$\overline{A} = \overline{(E,F)} = (\overline{F}, \overline{E})$$

One can easily verify that $(FS(U), \cap, \cup, ^{-})$ is a *Morgan algebra*, in which the smallest element is $(\varnothing,\varnothing)$ and the largest element is (U,U).
The corresponding order relation on $FS(U)$ is defined as $(E,F) \subseteq (E',F') \Leftrightarrow (E \subseteq E', F \subseteq F')$.
Definition 2: An *m-flou set* in a universe U ($m \geq 2$) is an m-tuple $(E_1, E_2, \cdots, E_m)$ of subsets of U such that $E_1 \subseteq E_2 \subseteq \cdots \subseteq E_m$. The class of m-flou sets in a universe U will be denoted by $FS_m(U)$.
The class $FS_m(U)$ of all m-flou sets in U can be endowed with the following binary and unary operations:

$$\begin{aligned} A \cap B &= (E_1, E_2, \cdots, E_m) \cap (F_1, F_2, \cdots, F_m) \\ &= (E_1 \cap F_1, E_2 \cap F_2, \cdots, E_m \cap F_m) \\ A \cup B &= (E_1, E_2, \cdots, E_m) \cup (F_1, F_2, \cdots, F_m) \\ &= (E_1 \cup F_1, E_2 \cup F_2, \cdots, E_m \cup F_m) \\ \overline{A} &= \overline{(E_1, E_2, \cdots, E_m)} = (\overline{E_m}, \cdots, \overline{E_2}, \overline{E_1}) \end{aligned}$$

It is obvious that $(FS_m(U), \cap, \cup, ^{-})$ is also a Morgan algebra.
The corresponding order relation on $FS_m(U)$ is defined as

$$(E_1, E_2, \cdots, E_m) \subseteq (F_1, F_2, \cdots, F_m) \Leftrightarrow \forall i \in \{1, \cdots, m\} (E_i \subseteq F_i).$$

3. FUZZY REGION MODEL

3.1 Fuzzy region representation

In some applications, for a fuzzy concept such as a forest, a forest region can be characterized by several different criteria, thus we can get different sharp regions to represent forest regions. Any sharp region may be regarded as a crisp description for the fuzzy concept of a forest. Thus, a forest fuzzy region is characterized as a family of sharp regions.
Here we use flou set to represent fuzzy region, and subsets (sharp regions) of a flou set must satisfy nested relation from smaller to larger. When some sharp regions of a fuzzy region are not included in each other, if all regions share at least a common intersection, then we can construct a new family of nested sharp regions that will form a flou set.
If a fuzzy region is described as finite sharp regions $D_i, i \in \{1, 2, \cdots, m\}$ that share a common intersection, we can construct some number of nested regions $E_i, i \in \{1, 2, \cdots, m\}$ in the following manner.

$$E_i = \bigcap_{\{j_1, \cdots, j_i\} \subseteq \{1, \cdots, m\}} \left(\bigcup_{k=1}^{i} D_{j_k} \right) \tag{1}$$

For example, in the case m = 3, we would obtain three new regions from three sharp regions D_1, D_2, D_3 that have a common subset.

$$E_1 = D_1 \cap D_2 \cap D_3$$
$$E_2 = (D_1 \cup D_2) \cap (D_1 \cup D_3) \cap (D_2 \cup D_3)$$
$$E_3 = D_1 \cup D_2 \cup D_3$$

These nested subsets form a flou set $A=(E_1, E_2, \cdots, E_m)$, which can be used to represent a fuzzy region of a fuzzy concept as shown in Fig. 2. In the following, we assume the *core* of a fuzzy region *A* is $Core(A)=E_1$ and the *hull* of *A* is $Hull(A)=E_m$. After fuzzy regions are modeled as flou sets, we can obtain new fuzzy regions form the original fuzzy regions using the operations of intersection, union and complement. We can also deduce the spatial relationships between fuzzy regions through the relations on flou set. Cohn's Egg-Yolk model can be regarded as a special case of the flou set model, where it is a fuzzy spatial region model based on flou set when m equals 2.

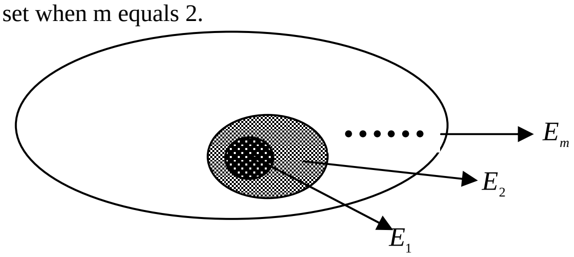

Fig. 2. A fuzzy spatial region based on flou set

The *alpha-cut* of fuzzy set theory is a geometric structure specified by the above definitions of fuzzy regions. Let $\tilde{A}$ be a fuzzy set on universe U, $\alpha \in [0, 1]$, then the alpha-cuts of $\tilde{A}$ is $A_\alpha = \{u | \mu_{\tilde{A}}(u) \geq \alpha, u \in U\}$. For $1=\alpha_1 > \alpha_2 > \cdots > \alpha_n = 0$, it is obvious that $A_{\alpha_1} \subseteq A_{\alpha_2} \subseteq \cdots \subseteq A_{\alpha_n}$, then $A=(A_{\alpha_1}, A_{\alpha_2}, \cdots, A_{\alpha_n})$ is a fuzzy region that is described by flou set. The sharp regions A_1 and A_0 are the core and hull of fuzzy region A respectively.

3.2 Properties of fuzzy region

In this subsection, some properties and relations of fuzzy region based on flou set will be presented.
Definition 3: For two fuzzy regions $A=(E_1, E_2, \cdots, E_m)$ and $B=(F_1, F_2, \cdots, F_m)$, the *equality relation* can be described as $\forall i \in \{1, 2, \cdots, m\} [E_i = F_i] \Rightarrow A = B$, namely a fuzzy region is uniquely determined by the corresponding flou set.

Definition 4: For two fuzzy regions $A=(E_1,E_2,\cdots,E_m)$ and $B=(F_1,F_2,\cdots,F_m)$, the *contained relation* is defined as $\forall i\in\{1,2,\cdots,m\}[E_i\subseteq F_i]\Rightarrow A\subseteq B$, namely the contained relation of two fuzzy regions is equivalent to that of the corresponding flou sets.

In some applications of fuzzy regions, the characterization of fuzzy regions by sharp regions allows us to compare points in a relative manner regarding their degree of membership to a fuzzy region. It is not necessary to decide an absolute membership value of an individual point but a relative value. Here the relative membership values can be easily obtained through comparing the relative membership relations of two points in a fuzzy region.

Definition 5: The *relative membership value* of two points P and Q in a fuzzy region $A=(E_1,E_2,\cdots,E_m)$ can be defined as follows:

A point P belongs to a fuzzy region A at least to the same degree as a point Q, if $\forall i\in\{1,2,\cdots,m\}[Q\in E_i\Rightarrow P\in E_i]$, which is denoted by $P\geq_A Q$.

A point P belongs to a fuzzy region A at most to the same degree as a point Q, if $\forall i\in\{1,2,\cdots,m\}[P\in E_i\Rightarrow Q\in E_i]$, which is denoted by $P\leq_A Q$.

Point P and Q belongs to the same degree to a fuzzy region A, if $\forall i\in\{1,2,\cdots,m\}[P\in E_i\Leftrightarrow Q\in E_i]$, which is denoted by $P\approx_A Q$.

A point P belongs more to a fuzzy region A than a point Q, if $(P\geq_A Q)\wedge\neg(Q\geq_A P)$, which is denoted by $P>_A Q$.

For the relative membership value of points in a fuzzy region, we introduce the following axioms.

$$\forall A\forall P\forall Q[P>_A Q\Leftrightarrow\exists i\in\{1,2,\cdots,m\}[P\in E_i\wedge Q\notin E_i]]$$
$$\forall A\forall P\forall Q[P\geq_A Q\vee Q\geq_A P]$$
$$\forall A\forall P\forall Q[P\in Core(A)\Rightarrow P\geq_A Q]$$
$$\forall A\forall P\forall Q[P\notin Hull(A)\Rightarrow Q\geq_A P]$$

For some tasks such as the visualization of a fuzzy region, an absolute representation of fuzziness is necessary. An absolute representation assigns a fixed value to every point of a fuzzy region. The absolute membership values of individual points to a fuzzy region can be calculated with the properties of flou set. Here we assume that a fuzzy region consists of a set of finite sharp regions, namely the dimension m of flou set is a finite number.

In the following, two alternatives of obtaining an absolute representation of fuzziness are presented. In the first method, we assume that a fuzzy region consists of a set of finite sharp regions. The degree of membership of a point P associated with a fuzzy region $A=(E_1,E_2,\cdots,E_m)$ (denoted as $\mu_A(P)$) can be characterized by the number of subsets of a flou set that contain the point, relative to the number of all subsets of a flou set.

$$\mu_A(P)=\|E_i|P\in E_i,\, i\in\{1,2,\cdots,m\}\|/m \qquad (2)$$

Using the above equation, we have

$$\begin{cases}\mu_A(P)=1 & if\quad P\in E_1;\\ \mu_A(P)=1/m & if\quad P\in E_m\wedge P\notin E_{m-1};\\ \mu_A(P)=0 & if\quad P\notin E_m\end{cases}$$

Alternatively, another method is using the Hausdorff metric of sets [15] to calculate the membership value of a point in a fuzzy region. This metric is zero, if and only if the two sets are identical. To characterize the Hausdorff metric, we first introduce the ε*-neighborhood*. The ε-neighborhood of a set T, abbreviated as $N_\varepsilon(T)$, is the set of all points with a distance smaller than ε, namely

$$N_\varepsilon(T)=\{P\mid\exists Q\in T\,[dist(P,Q)<\varepsilon]\},$$

where $dist(\ )$ is a distance function between two points.

The Hausdorff metric H of two sets T_1 and T_2 is defined as the following equation:

$$H(T_1,T_2)=\inf\{\varepsilon\mid T_1\subset N_\varepsilon(T_2)\wedge T_2\subset N_\varepsilon(T_1)\}$$

The absolute membership value of a point P in a fuzzy region $A=(E_1,E_2,\cdots,E_m)$ is defined by the following equation:

$$\mu_A(P)=1-\inf\left\{\frac{H(E_i,E_1)}{H(E_m,E_1)}\middle|P\in E_i,\ i\in\{1,2,\cdots,m\}\right\} \qquad (3)$$

For a fuzzy spatial region, the result of the second alternative may be different with that of the first alternative.

4. A PRACTICAL CASE

We have applied the fuzzy region model in the system of synthesis information minerals prognosis. This system is to extract and synthesize the characteristic information of multi-source data in certain area, which includes geography, geology, geophysics, geochemistry, remote sensing and so on.

In this application, the observed spatial data are not often precise, but rather varying in an interval range. The data may also contain errors to some extent, such as data of chemical element, aeromagnetic and gravitational survey. These make the degree of the spatial data satisfy the fuzzy concepts to be an interval value, and the region determined by the fuzzy concepts is an undetermined fuzzy region as well.

In the process of minerals prognosis, the data of chemical element copper and iron are often interval values. The content of copper and iron in the soil may imply the mineral. Let us illustrate a fuzzy region defined by a fuzzy concept " Target area" , according to the content of copper and iron. Target area means that the contents of copper and iron are relatively high. The sample points (*P*) are $\{P_1,P_2,\cdots,P_{18}\}$ and the values of copper (*C*) and iron (*I*) are given in Table 1.

P	C (mg)	I (mg)
1	[216, 221]	[958, 1052]
2	[218, 222]	[1080, 1116]
3	[221, 225]	[1280, 1378]
4	[267, 272]	[955, 1020]
5	[228, 234]	[913, 998]
6	[240, 245]	[1000, 1080]
7	[212, 218]	[1138, 1216]
8	[224, 230]	[1198, 1286]
9	[235, 239]	[1244, 1368]
10	[257, 262]	[2188, 2256]
11	[207, 212]	[744, 868]

12	[222, 227]	[239, 336]
13	[107, 114]	[3166, 3289]
14	[175, 180]	[1206, 1398]
15	[213, 219]	[332, 424]
16	[167, 172]	[1732, 1812]
17	[231, 236]	[1256, 1372]
18	[261, 267]	[976, 1064]

Table 1. The values of copper and iron

Because the concept " Target area " is fuzzy, three sharp regions of target area are estimated by three individual experts respectively as follows:

$$D_1 = \{1, 2, 5, 7, 8, 11, 17\}$$
$$D_2 = \{1, 2, 3, 5, 6, 14, 17\}$$
$$D_3 = \{1, 2, 6, 7, 8, 9, 11\}.$$

Now we can further represent the target area as a fuzzy region through transforming the above three sharp regions according to equation (2). Then we can get three new sharp regions that will form a flou set.

$$E_1 = D_1 \cap D_2 \cap D_3 = \{1, 2\}$$
$$E_2 = (D_1 \cup D_2) \cap (D_1 \cup D_3) \cap (D_2 \cup D_3)$$
$$= \{1, 2, 5, 6, 7, 8, 11, 17\}$$
$$E_3 = D_1 \cup D_2 \cup D_3 = \{1, 2, 3, 5, 6, 7, 8, 9, 11, 14, 17\}$$

We thus use a flou set (E_1, E_2, E_3) to represent the fuzzy region of target area. Its core is E_1, and hull is E_3. For the visualization task, we may compute the absolute membership values for sample points of the fuzzy concept " Target area " using equation (3). The absolute membership result for the eighteen sample points is $\{1, 1, 1/3, 0, 2/3, 2/3, 2/3, 2/3, 1/3, 0, 2/3, 0, 0, 1/3, 0, 0, 2/3, 0\}$.

In the system of synthesis information minerals prognosis, we represent the fuzzy regions as flou sets and we can use these flou sets to obtain the property and relation of fuzzy regions using the operational property of flou set.

In the original fuzzy models, the membership functions associated with fuzzy concepts are often hard to determine and the numerical values for individual points are not necessary in some cases. In the above case, we can see that rather than using the membership values directly, the flou set model employs the relative relations between point sets to represent a fuzzy region as a flou set. One advantage of the model is that it is easy to model a fuzzy region using different views of fuzzy concepts. Furthermore, we can analyze topological relations between fuzzy regions based on flou sets. Such model can also be well used in the case of contour maps because many contour lines will form the sharp regions of a fuzzy region and they satisfy the nested relation from smaller to larger as a flou set.

5. RELATIVE WORK

In computer science and geographic information science, spatial fuzziness is often modeled by employing fuzzy approaches. Based on fuzzy set theory, fuzzy approaches use the concept of different degrees of membership to describe fuzziness. Most of relative work model fuzzy region as a classical fuzzy set and based on membership functions and cut sets, the relationships between fuzzy regions can be worked out.

Burrough considers fuzzy set theory as an appropriate method to deal with spatial fuzziness in mathematical or conceptual models of empirical phenomena (Burrough, 1996). The fuzzy model proposed by Zhan uses fuzzy sets to represent indeterminate regions where every point is assigned a membership value within interval [0, 1] and every α-cut level region is a determinate region (Zhan, 1998). Brown applies fuzzy set theory to represent vegetation area as a continuous spatial area (Brown, 1998). Altman represents fuzzy region and calculates the spatial relation based on operations of fuzzy set (Altman, 1994). Tinghua Ai defines the boundary of fuzzy region as a wide one where the degree a certain point belonging to the thematic region is defined by a fuzzy membership function (Tinghua, 1998). Molenaar et al. try to model fuzzy objects using classical fuzzy set where the similarity degree is calculated based on the intersection of two fuzzy sets (Molenaar, 2000). Based on the fact that the spatial objects in GIS thematic map as used in practice are usually plotted according to predefined threshold of attribute membership degree, Liu et al. build up the conformation description mode, i.e., the boundary, interior and exterior of a fuzzy region (Wenbao, 2002). Guesgen et al use fuzzy techniques to model imprecise and qualitative spatial relations between geographic objects (Guesgen, 2000). Schneider introduces fuzzy points, fuzzy lines, and fuzzy regions (Schneider, 1999) and studies their algebraic and topological properties on a discrete geometric structure (Schneider, 2003).

In many practical applications it is often difficult to acquire precise data, and the membership value of objects to fuzzy concept is also not easy to make sure. So in some cases the original fuzzy model is not suitable. The fuzzy region model based flou set in this paper is suitable for the fuzzy character of spatial data in applications. The idea and result of this model is compatible with the cognition of human beings. It can model and analyze the classical fuzzy region and crisp region uniformly.

6. CONCLUSIONS AND FUTURE WORK

To solve the problem of modeling fuzzy objects in GIS applications and spatial databases, this paper presents a *fuzzy region model based on flou set*. It is different from the original fuzzy model that uses membership values but uses the relative relation of sets to represent a fuzzy region as a flou set. The flou-set model makes use of the calculation properties of flou set to obtain the properties and relative relations. In this paper, we analysis the possible attributes of a fuzzy region, and present two alternatives to calculate relative membership values and absolute membership values. The results of this model in the system of synthesis information minerals prognosis show that it is valuable in the fields such as GIS, geography and spatial databases.

Future work includes: modeling high order fuzziness; and analyzing the topological relations between fuzzy regions based on flou set and the relations between fuzzy regions based on different dimensions. In addition, the application of this model in spatial information systems and spatial databases is still a long-term goal.

ACKNOWLEDGEMENT

The research is supported by NSFC Major Program/China under Grant No.60496321, NSFC/China under Grant Nos. 60373098, 60173006, the National High-Tech Research and Development Plan of China under Grant No.2003AA118020, the Major Program of Science and Technology Development Plan of Jilin Province under Grant No. 20020303, the Science and Technology Development Plan of Jilin Province under Grant No. 20030523.

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